# Mid-term exam of Particle Physics
# Thursday November $14^{\text{th}}$ 2019

*Duration: 3 hours*
*6 printed pages*

*Allowed material: PDG booklet, simple calculator.*
*Feynman rules for exercice II are provided at the end.*

**Solve on two separate sheets exercices I-II and exercices III-IV.**

*Approximate duration per exercice:*

| | | | |
|---|---|---|---|
| Ex. I: | 20 min. | Ex. II: | 70 min. |
| Ex. III: | 25 min. | Ex. IV: | 65 min. |

## Exercice I

*Questions on the lecture*

Reply shortly and succinctly to the questions below. The shortest answer that details in a comprehensive manner all the relevant arguments is the best.

1. Explain the theoretical and experimental logic behind the Cronin and Fitch experiment, which allowed the discovery of $CP$ violation in the neutral-kaons system in 1964.
2. Deep Inelastic Scattering (DIS) experiments use, for instance, the reaction $e^-p \to e^-X$, where $X$ is a system of hadrons, to probe the internal structure of the proton. Explain.

## Exercice II

*The BABAR experiment*

The BABAR experiment was an electron-positron collider aimed at studying $CP$ violation. The collider (PEP-II) properties are given in Tab. 1.

Table 1: Properties of the PEP-II collider (and naming convention for the exercise)

| Property | value |
|---|---|
| Electron beam (oriented along $+\vec{z}$ axis and named $A$ beam) | 9 GeV |
| Positron beam (oriented along $-\vec{z}$ axis and named $B$ beam) | 3.1 GeV |

We note throughout the exercise $\sqrt{s}$ the value of the total energy in the center of mass.

In this exercise, we consider the production of particle-antiparticle pairs ($F\bar{F}$):

$$e^+e^- \to F\bar{F}$$

This exercise is divided into 8 questions, which are mostly independent from one another with the exception of question 5.

1. *Cross section master formula* We remind the master formula:

$$d\sigma = \frac{1}{2\,E_A\;2\,E_B|\beta_A - \beta_B|} \times |\mathcal{M}|^2 \times (2\pi)^4\delta^{(4)}(p_A + p_B - \sum_{i=1}^{i\leq f} k_i) \prod_{i=1}^{i\leq f} \frac{d^3\vec{k}_i}{(2\pi)^3\;2\,E_i}, \qquad (1)$$

where $f$ is the total number of particles in the final state, $p_A \equiv (E_A, \vec{p}_A)$ and $p_B \equiv (E_B, \vec{p}_B)$ the quadri-momenta of the colliding particles in the lab frame, $\beta_A$ and $\beta_B$ the velocities of the colliding particles along the $\vec{z}$ axis, $k_i \equiv (E_i, \vec{k}_i)$ the quadri-momenta of the particles in the final state.

Separate Eq. 1 into a flux term, a matrix element term and a phase space term. Specify the property of each term under Lorentz transformation. Note that we consider the $\delta$-function ensuring 4-momentum conservation as part of the phase space term $dLIPS$.

2. *Phase space term $dLIPS$* as defined in part 1. Compute in the center of mass.
   (a) Integrate $dLIPS$ over $d^3\vec{k}_2$. What is the consequence on the value of $k_2$ ?
   (b) We note $k^*$ the fixed value of $|\vec{k}_1|$ in the center of mass. Explicit $k^*$ as a function of $\sqrt{s}$ and the mass of the particle $F$, $m_F$.
   (c) Integrate $dLIPS$ over the remaining $\delta$-function corresponding to the energy conservation. Give the value of $dLIPS$ as a function of $k^*$, $\sqrt{s}$ and $d\Omega$.

3. *Matrix element $\mathcal{M}$* (independent part).
   (a) Draw the Feynman diagram corresponding to the QED reaction $e^+e^- \to \mu^+\mu^-$.
   (b) Using the Feynman rules given in Fig. 2, write down the matrix element as a function of the bi-spinors $u$, $v$, $\bar{u}$, $\bar{v}$ and $\alpha$ (the hyperfine structure constant). Specify the quadri-momentum of the bi-spinors, noting respecitlvely $p_A$, $p_B$, $k_1$ and $k_2$ the quadri-momenta of $e^-$, $e^+$, $\mu^-$ and $\mu^+$. The actual computation gives in the center of mass

$$|\mathcal{M}|^2 = (4\pi)^2\;\alpha^2\;\left\{\left(1 + \frac{4\,m_F^2}{s}\right) + \left(1 - \frac{4\,m_F^2}{s}\right)\cos^2\theta^*\right\}, \qquad (2)$$

   where $\theta^*$ is the angle of the $\mu^-$ with respect to the $e^-$ beam axis in the center of mass.

4. *Flux term $\mathcal{F}$* as defined in part 1. We assume $m_A = m_B = m_e \approx 0$.
   (a) Write the Lorentz transformation ($4 \times 4$ matrix) from the lab frame to the center of mass frame (use the $\gamma$ and $\beta$ parameters of the center of mass). Give $E_A^*$, the energy of particle $A$ in the center of mass, as a function of $\gamma$, $\gamma\beta$, $E_A$ and $\vec{p}_A$.
   (b) Give the value of $E_A^* \times E_B^*$ as a function of $E_A \times E_B$.
   (c) Using this property find the flux factor as a function of $\sqrt{s}$

5. *Differential cross section* (requires part 1 to part 4).
   (a) Putting all the pieces together, find the expression of the $2 \to 2$ differential cross section $\frac{d\sigma}{d\Omega}$ as a function of $\sqrt{s}$, $k^*$ and the matrix element $\mathcal{M}$.
   (b) Using Eq. 2, find out that the total cross section at leading order (LO):

$$\sigma_{tot} = \frac{4\pi}{3}\;\frac{\alpha^2}{s}\sqrt{1 - \frac{4\,m_F^2}{s}}\;\left(1 + \frac{1}{2}\;\frac{4\,m_F^2}{s}\right) \qquad (3)$$

6. *Kinematics of the production.* (This part is completely independent.)

   (a) Using Tab. 1, compute the value of $\sqrt{s}$.

   (b) From the PDG, what are the mass and width of the resonance $\Upsilon(4S)$? Does it play a role in the reaction $e^-e^+ \to F\bar{F}$ in the *BABAR* experiment?

   (c) What are the dominant decays of the $\Upsilon(4S)$ ?

7. *The $F\bar{F}$ productions.* This part is mostly independent, the last question requires part 6.

Table 2: Total production cross section from various physics processes in $e^+e^-$ collisions at $\sqrt{s}$ of PEP-II.

| Physics process | Cross section $[nb]$ |
|---|---|
| $b\bar{b}$ | 1.11 |
| $u\bar{u}$ | 1.61 |
| $d\bar{d}$ | 0.40 |
| $s\bar{s}$ | 0.38 |
| $c\bar{c}$ | 1.30 |
| $\mu^+\mu^-$ | 1.15 |

   (a) What is the approximation of Eq. 3 for $2\,m_F \ll \sqrt{s}$. Compute the value of the expected cross section $\sigma(e^+e^- \to \mu^-\mu^+)$ at LO using this approximation. Compare to the number in Tab. 2 and give one potential reason for the difference.

   (b) From the value of $\sigma(e^+e^- \to \mu^-\mu^+)$, assuming that $2\ m_u \ll \sqrt{s}$, $2m_d \ll \sqrt{s}$, give the expected cross section for $\sigma(e^+e^- \to u\bar{u})$ and $\sigma(e^+e^- \to d\bar{d})$. Compare to the result in Tab. 2, give one potential source for the difference ?

   (c) What do you expect for the ratio $\sigma(e^+e^- \to b\bar{b})/\sigma(e^+e^- \to d\bar{d})$ at LO in QED? Give a quantitative estimate.

   (d) Find a the reason why the ratio $\sigma(e^+e^- \to b\bar{b})/\sigma(e^+e^- \to d\bar{d})$ is so large compared to the LO QED expectation (previous question)?

8. *Available statistics.* (This part is completely independent.)

   (a) The instantaneous luminosity of the machine was $\mathcal{L} \approx 10 \times 10^{33}\ cm^{-2}s^{-1}$. Assuming a month is effectively only 20 days, what was the integrated luminosity collected per month (express your result in $\text{fb}^{-1}$/month) ?

   (b) In total the *BABAR* experiment collected 433 $\text{fb}^{-1}$. Using the cross sections given in Tab. 2, what is the total number of $b\bar{b}$ pairs recorded by the experiment? How many $B^0\overline{B}^0$ pairs and $B^+B^-$ pairs were collected knowing the $b\bar{b}$ pairs were produced via the $\Upsilon(4S)$ resonance?

## Exercice III

*Allowed and forbidden processes, Feynman diagrammes*

For each of the processes below, determine whether it is allowed or forbidden. For the forbidden processes, explain why, giving *all the possible reasons* (here we do not require to take into account multiplicative quantum numbers). For the allowed processes, specify and justify by which *dominant* interaction they occur and draw the corresponding Feynman diagrams (one per process). Note on the diagram all the particles (including virtual particles).

1. $\Sigma^+ \longrightarrow \Lambda\mu^+\nu_\mu$
2. $D^+ \longrightarrow p\tau^-\nu_\tau$
3. $e^-p \longrightarrow e^-n\pi^+$
4. $B^0 \longrightarrow \tau^+\tau^-$
5. $e^+e^- \longrightarrow \gamma\gamma$

## Exercice IV

*The exotic state* $X(3872)$

The exotic state $X(3872)$ was discovered by the Belle experiment in the year 2003 in the decay mode $J/\psi\,\pi^+\pi^-$. Since its discovery, other experiments, in particular LHCb, confirmed its existence, and physicists are trying to understand and determine its nature, properties and quantum numbers.

1. A spectrum published by the LHCb experiment in 2013 is given in Fig. 1. It shows a clear peak that corresponds to the X(3872). Briefly explain and comment the *whole* figure using relevant numbers from the PDG. Estimate, *using only the information given in the figure*, the mass and the width of the $X(3872)$ (with a precision of 10 MeV).

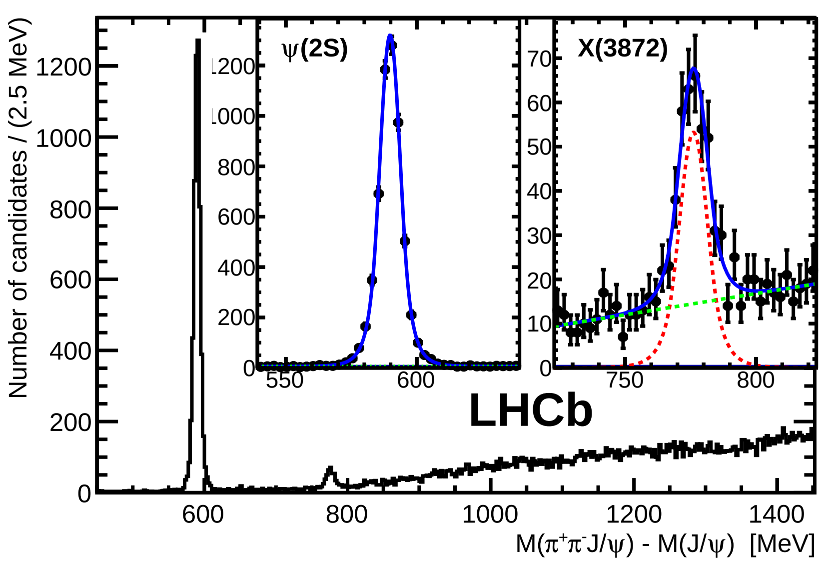


Figure 1: Spectrum of the variable $M(J/\psi\,\pi^+\pi^-) - M(J/\psi)$, published by LHCb in 2013. The peak corresponding to the $X(3872)$ is clearly visible. It is enlarged in the right-hand side inset. The dashed line shows the signal component in the fit to data.

2. Give all the relevant arguments to show that the decay $X(3872) \rightarrow J/\psi\,\pi^+\pi^-$ is due to the strong interaction. Give, with a brief explanation, all the additive quantum numbers (electric charge, baryon number, flavours...) of the $X(3872)$.

3. Draw the Feynman diagram of the decay, supposing that $X(3872)$ is a $c\bar{c}$ state.

4. Under the hypothesis that the $X(3872)$ is a $c\bar{c}$ state, what is its isospin? Determine the ratio of partial widths of its decay to $J/\psi\,\pi^+\pi^-$ and $J/\psi\,\pi^0\pi^0$. Explain.

There exist several indications that the $X(3872)$ is not a meson or a baryon. It could be a molecule of two $c\bar{q}$ mesons (where $q$ is a $u$, $d$ or $s$ quark), or a $c\bar{c}q\bar{q}$ state, bound by strong interaction, which is called tetraquark. To determine the nature of the $X(3872)$, it is useful to measure its spin, parity and charge conjugation.

5. Knowing that the decay $X(3872) \to J/\psi\,\gamma$ was observed, determine the quantum number $C$ (charge conjugation) of the $X(3872)$. Briefly explain.

6. We now go back to the decay $X(3872) \to J/\psi\,\pi^+\pi^-$. Using the answer to the previous question, find the quantum number $C$ of the $\pi^+\pi^-$ pair, noted $C_1$.

7. Using all the information that you have until now, conservation laws and relevant information from the PDG, show that *if* the $\pi^+\pi^-$ pair originates in a decay of a resonance, this resonance can be either the $\rho^0(770)$ or the $\omega(782)$ *and nothing else*. In this case, why can the contribution from the $\omega(782)$ be neglected?

If the $\pi^+$ and the $\pi^-$ do not originate in a resonance, we say that they constitute a non-resonant state. Even in this case, we can consider the $\pi^+\pi^-$ pair as a system and obtain its quantum numbers.

8. Establish a relation between the eigenvalue $C_1$ of the $\pi^+\pi^-$ pair and its parity $P_1$. Deduce a constraint on the relative angular momentum, $\ell_1$, between the two pions, and the total angular momentum, $J_1$, of the pair.

The LHCb experiment, exploiting the angular distributions of all the final-state particles, showed that the $J^{PC}$ of the $X(3872)$ is $1^{++}$. We will now deduce all the relevant quantum numbers of final state $J/\psi\,\pi^+\pi^-$.

9. What is the relation between the parity of the $X(3872)$ and the relative angular momentum, $\ell_2$, between the pair $\pi^+\pi^-$ and xthe $J/\psi$?

10. Using a simple argument, explain why it is reasonable to suppose that $\ell_2$ takes the smallest possible value? What is this value?

11. Examine the coherence of this result with parity conservation in the process $X(3872) \to J/\psi\,\pi^+\pi^-$ and confirm the parity $P_1$ of the $\pi^+\pi^-$ pair found in question 8.

12. Give all the possibilities for the combination of the angular momenta $\vec{J_1} + \vec{J}_{J/\psi}$, where $J_1$ is the total angular momentum of the $\pi^+\pi^-$ pair and $J_{J/\psi}$ is the spin of the $J/\psi$.

13. Finally, is a resonant state of the pair $\pi^+\pi^-$ possible? Same question for the non-resonant state. Briefly argue.

Figure 2: QED Feynman rules. $s$ refers to the (anti-)fermion spin and $\lambda$ to the photon helicities.

| | | | |
|---|---|---|---|
| Incoming fermion: |  | $\rightarrow$ | $u\ (\vec{p}, s)$ |
| Incoming antifermion: |  | $\rightarrow$ | $\overline{v}\ (\vec{p}, s)$ |
| Outgoing fermion: |  | $\rightarrow$ | $\overline{u}\ (\vec{p}, s)$ |
| Outgoing antifermion: |  | $\rightarrow$ | $v\ (p, s)$ |
| Incoming photon: | $\mu$  | $\rightarrow$ | $\epsilon_\mu(\vec{k}, \lambda)$ |
| Outgoing photon: |  $\mu$ | $\rightarrow$ | $\epsilon_\mu(\vec{k}, \lambda)^*$ |