Exercice II - allowed and forbidden processes...

Remark: in the "real" exam, write down values of the quantum numbers that intervene, and of the masses (or their ΔM) when needed.

1. $\Delta^{++} \to p^{+} p^{+}$

forbidden by B and mass (marginally, because of the widths of the Δ and the p.)

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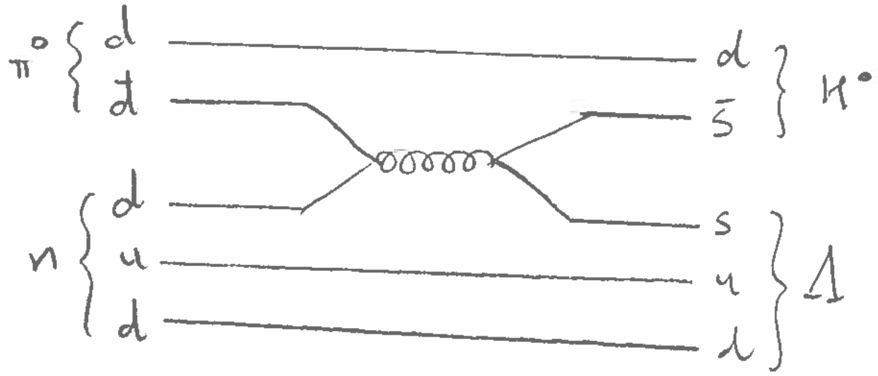
2. $\pi^{0} n \to K^{0} \Lambda$

Q, B, S are conserved.

Other flavors and lepton numbers do non intervene

There are only hadrons in the process

⇒ allowed, by strong int.



3. $D^{-} \to \tau^{-} \bar{\nu}_{\tau}$

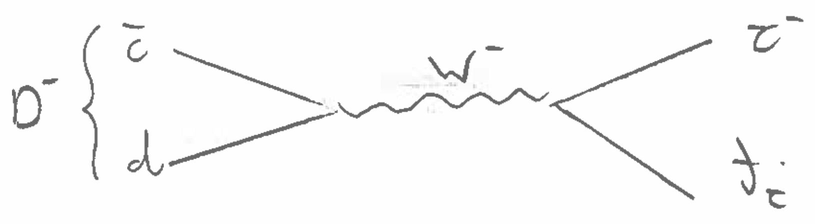
L_τ is conserved, as well as Q. $m_i > m_f$

C is violated (charm...)

Other charges do not intervene

A ν is present in the process

⇒ Allowed, by weak int.



4. $\bar{K}^0 \to \mu^+ \mu^-$

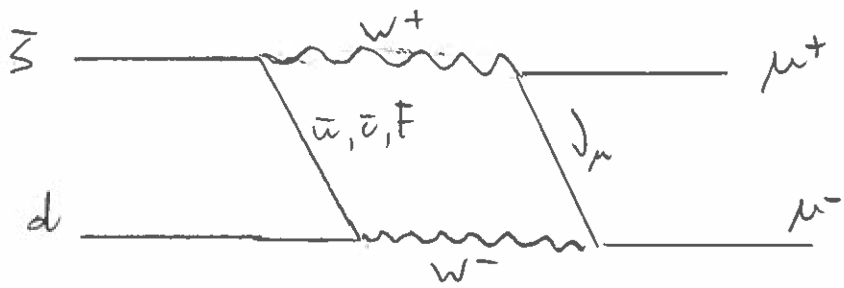
$m_i > m_f$

Q and L_μ conserved

S violated

Other charges do not intervene

$\Rightarrow$ Allowed by weak int.



5. $K^+ \to \pi^0 \tau^+ \nu_\tau$

$m_i < m_f$

L_τ is not conserved

$\Rightarrow$ Forbidden by L_τ and mass.

Exercice III

1. In the course, we learned:

$|\omega\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$

From the PDG:

$J = 1$

$m_\omega = 782,65$ MeV

$\Gamma_\omega = 8,49$ MeV

From the quark-content and the fact that the ω is a meson:

$\mathcal{B} = 0$

$S = C = B = 0$

$Y = 0$

3 main decay modes

(1) $\omega \to \pi^+ \pi^- \pi^0$ 89.2 %

(2) $\omega \to \pi^0 \gamma$ 8.28 %

(3) $\omega \to \pi^+ \pi^-$ 1.53 %

2. $\tau = \frac{\hbar}{\Gamma} = \frac{\hbar c}{\Gamma c} = \frac{197,3}{8,49 \times 3 \times 10^{23}} \simeq 8 \times 10^{-23}$ s $\sim 10^{-22}$ sec

Although this lifetime is slightly larger than the typical 10^{-23} sec, it is still larger by several orders of magnitude than the typical lifetime of a particle decaying via EM interaction. The strong interaction is the most intense, and thus, a single decay mode undergoing strong int. is enough to for the decaying particle to have the typical lifetime of the strong interaction.

3. Lifetime / width indicate that at least (1) occurs via strong interaction.

We notice the presence of a γ in (2), pointing clearly to the EM interaction

Comparing the BF of (3) and (2) points towards the fact that also (3) occurs by EM int, but this has to be justified.

Remark: there is no reason to attribute these interactions to the weak int. due to the following conjunction of arguments:

- no ν are present
- flavors are conserved
- relatively large BFs (smaller than (1), but not small enough for weak int.)

As we have to arbitrate between the SI and EM, it is useless to look at P and C conservation. On the other hand, G-parity can help.

for (1) $\quad G_i = G(\omega) = -1$

$$G_f = (G(\pi))^3 = (-1)^3 = -1$$

$\Rightarrow$ SI

for (3) $\quad G_i = -1$

$$G_f = (G(\pi))^2 = +1$$

$\Rightarrow$ EM

4. $\Gamma_\rho = 149{,}1 \text{ MeV} \simeq 17\, \Gamma_\omega$

The width of the ω is suppressed by phase space: ρ decays by SI mainly into 2π while ω into 3π.

5. P and C are conserved for (1), (2) and (3) as they occur by strong and EM interactions. The easiest mode to look at is (3). Mode (1) has 3 particles, thus l_{12} and l_3 ... Mode (3) has a γ, which has $J = 1$.

$J_i = J(\omega) = 1$

$\vec{J}_f = \underbrace{\vec{J}(\pi^+) + \vec{J}(\pi^-)}_{0} + \vec{L}$

$J_f = l$

$J_i = J_f \Rightarrow l = 1$

$P_f = (P(\pi))^2 (-1)^{-l} = -1$

$\Rightarrow P_i = -1$

We verify that $P(\omega) = -1$

6. π^0 is a boson, and thus the $\pi^0\pi^0$ system must be symetric with respect to the exchange between the two π^0.

In the case of two spin-0 particles the exchange only implies parity:

$P(\pi^0\pi^0) = +1 \Rightarrow l$ even

which is in contradiction with angular momentum conservation

7. ω is its own antiparticle (see quark content), thus a $\hat{C}$ eigenstate.

(3) $\omega \to \pi^0 \gamma$

(3) is an EM decay $\Rightarrow C_i = C_f$

$$C_f = C(\pi^0)\, C(\gamma) = (+1) \times (-1) = -1$$

$$\Rightarrow C_i = C(\omega) = -1$$

(verified in the PDG)

As $C(\pi^0\pi^0) = (C(\pi^0))^2 = +1$

$\omega \to \pi^0\pi^0$ is forbidden by the strong and EM interactions, which conserve C.

Remark: This statement is less strong than that of 6...

Exercice IV

1. (1) $K^- d \to \Sigma^- p$

(2) $K^- d \to \Sigma^0 n$

Isospin states:

$|K^-\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$ (K^- is an std meson ...)

$|d\rangle = |0\ 0\rangle$ (singlet)

$|K^- d\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$

$|\Sigma^-\rangle = |1\ -1\rangle$ (Σ^- is a part of the $\Sigma^+, \Sigma^0, \Sigma^-$ triplet)

$|\Sigma^0\rangle = |1\ 0\rangle$

$|p\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle$

$|n\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$

Using Clebsh-Gordan coeff's

$$|\Sigma^- p\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$$|\Sigma^0 n\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

Neglecting phase space effects

$$\frac{\sigma_{(1)}}{\sigma_{(2)}} \simeq \frac{|\langle i|H_F|f_{(1)}\rangle|^2}{|\langle i|H_F|f_{(2)}\rangle|^2} = \frac{\frac{2}{3}\ \cancel{f_{1/2}}}{\frac{1}{3}\ \cancel{f_{1/2}}} = 2$$

2. $|\pi^0\rangle = |1\ 0\rangle$; $|\pi^-\rangle = |1\ -1\rangle$

$$|n\pi^0\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \otimes |1\ 0\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$$|p\pi^-\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle \otimes |1\ -1\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

It is clear that

$$I_3(X) = I_3(p) + I_3(\pi^-) = I_3(n) + I_3(\pi^0) = -\tfrac{1}{2}$$

As X decays to an $I = 1$ and $I = \tfrac{1}{2}$ particles, its isospin may a-priori be $\tfrac{3}{2}$ or $\tfrac{1}{2}$.

To arbitrate between the states, we can use the ratio of BF, indicating:

$$\frac{\Gamma(X \to n\pi^0)}{\Gamma(X \to p\pi^-)} = \frac{1}{2}$$

This is verified by the state $|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$:

$$\frac{\langle \tfrac{1}{2}\ -\tfrac{1}{2}|H_F|n\pi^0\rangle}{\langle \tfrac{1}{2}\ -\tfrac{1}{2}|H_F|p\pi^-\rangle} = \frac{\tfrac{1}{3}\, \cancel{A_{1/2}}}{\tfrac{2}{3}\, \cancel{A_{1/2}}} = \frac{1}{2}$$

$\Downarrow$

$$|X(1520)\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

3. (a) To take into account the phase space, we need to compute p^* in the two reactions as a function of s.

Designating the final-state particles by 1 and 2:

$$P_i = (\sqrt{s}, 0)$$

$$P_1 = (E_1^*, p^*)$$

$$P_2 = (E_2^*, -p^*)$$

$$P_i = P_1 + P_2$$

$$P_2^2 = (P_i - P_1)^2$$

$$m_2^2 = (\sqrt{s} - E_1^*)^2 - (-p^*)^2$$

$$m_2^2 = s - 2\sqrt{s}E_1^* + \underbrace{E_1^{*2} - p^{*2}}_{m_1^2}$$

$$\underbrace{E_1^*}_{\sqrt{m_1^2 + p^{*2}}} = \frac{1}{2\sqrt{s}}(m_1^2 - m_2^2 + s)$$

$$p^* = \frac{1}{2\sqrt{s}}\left[(m_1^2 - m_2^2 + s)^2 - 4 s m_1^2\right]^{\frac{1}{2}} =$$

$$= \frac{1}{2\sqrt{s}}\left(m_1^4 + m_2^4 + s^2 - 2m_1^2 m_2^2 - 2m_1^2 s - 2m_2^2 s\right)^{\frac{1}{2}}$$

(b) ...

(c) Even without putting numerical values we can draw the following conclusions:
Final state of (1) is less massive than that of (2). There exists a $\sqrt{s}$ interval for which the first reaction is enabled but not the second. Phase space cannot be neglected.
When $s \gg m(\Sigma)^2$ the ratio of p^* becomes 1.
In this regime the approximation of same phase space factor holds