Final exam of Particle Physics
Monday February 4$^{\text{th}}$ 2019

Duration: 3 hours
6 printed pages

Allowed material: PDG booklet, simple calculator.
Solve on two separate sheets exercices I-II and III-IV.

Approximate duration per exercice:			
Ex. I:	15 min.	Ex. II:	70 min.
Ex. III:	25 min.	Ex. IV:	70 min.

Exercice I

Questions on the lecture

Reply shortly and succinctly to the questions below. The shortest answer that details in a comprehensive manner all the relevant arguments is the best.

1. Explain what is Björken-scaling invariance and why is it violated.
2. What are the main features of solar neutrinos? What did their study allow to learn about which observables? Briefly describe the principle of one experiment that provided information on solar neutrinos.

Exercice II

Higgs boson natural width

In this exercise we will study the potential ways to measure the total Higgs boson width. We consider a Higgs boson with the properties:

$$m_h = 125 \text{ GeV} \quad , \quad \Gamma_h^{SM} = 4.1 \text{ MeV} \quad , \quad \mathcal{B}^{SM}(h \to gg) \approx 0.10 \qquad (1)$$

where Γ_h^{SM} is the total Higgs boson width expected in the standard model (SM). For a given Higgs boson decay to the final state X, we will note $\Gamma_X \equiv \Gamma(h \to X)$, for instance $\Gamma_{gg} = \Gamma(h \to gg)$.
The parts 1-4 below are independent of each other.

1. Higgs boson production at the LHC

   We consider proton-proton collisions at the LHC, and we note the total energy in the center of mass $\sqrt{s}$. The 4-momenta of the protons in each beam are respectively:

$$p_1 \equiv (\sqrt{s}/2, 0, 0, +\sqrt{s}/2) \ , \qquad p_2 \equiv (\sqrt{s}/2, 0, 0, -\sqrt{s}/2). \qquad (2)$$

(a) The dominant contribution of the Higgs boson production at the LHC is coming from the interaction of two gluons. Draw the corresponding Feynman diagram in the standard model.

(b) We note k_i the 4-momentum of the interacting gluon from proton i and x_i the corresponding fraction of the proton momentum carried by this gluon, hence $k_i \approx x_i\, p_i$. The cross section is given by:

$$\sigma(g(k_1)g(k_2) \to h) = \frac{\pi^2}{8\, m_h}\, \Gamma_{gg}\delta(\hat{s} - m_h^2) \tag{3}$$

where $\sqrt{\hat{s}}$ is the total energy in the center of mass of the colliding gluons, *i.e.* $\hat{s} = (k_1 + k_2)^2$. Express $\hat{s}$ as a function of x_1, x_2 and s.

(c) Using the definition of the parton density function and Eq. 3 show that the total cross section $\sigma(pp \to h)$ can be written as [1]:

$$\sigma(pp \to h) = \frac{\pi^2}{8\, m_h}\, \Gamma_{gg} \times L_{gg}\left(\frac{m_h^2}{s}\right), \tag{4}$$

where L_{gg} is called partonic luminosity and is given by:

$$L_{gg}(\tau) = \frac{1}{s} \times l_{gg}(\tau) \quad \text{with} \quad l_{gg}(\tau) = \int_0^1 dx_1 \int_0^1 dx_2\ f_g(x_1) f_g(x_2) \delta(x_1 x_2 - \tau) \tag{5}$$

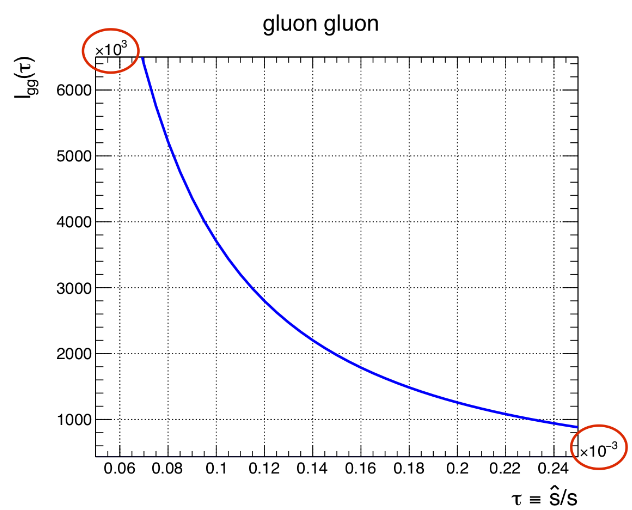


Figure 1: Universal $l_{gg}(\tau)$ function used in the definition of the partonic luminosity

(d) The partonic luminosity function, $L_{gg}(\tau)$, is obtained here in the natural-unit system ($\hbar = c = 1$). Restore the coefficients $\hbar$ and c to represent $L_{gg}(\tau)$ in standard units, knowing that it is homogeneous to a cross section. Note that the function $l_{gg}(\tau)$ is dimensionless.

(e) Using Fig. 1, compute the partonic luminosity (in mb) corresponding to the Higgs boson production at $\sqrt{s} = 13$ TeV. You may use the PDG for the numerical value of $\hbar c$ in the appropriate units.

(f) From this, estimate the total cross section $\sigma(pp \to h)$ (in pb) expected in the standard model.

[1] We remind that $\delta(f(x)) = \delta(x - x_0)/|f'(x_0)|$ with x_0 such as: $f(x_0) = 0$

(g) Compute the cross section $\sigma_{13\,\text{TeV}}/\sigma_{8TeV}$ arising from the raise of the LHC energy from $\sqrt{s} = 8$ TeV to $\sqrt{s} = 13$ TeV. Explain qualitatively what is the physical reason for this change in the cross section.

2. Higgs boson production at an electron-positron collider (*e.g.* the ILC)

We now consider the Higgs boson production in a e^+e^- machine in the reaction $e^+e^- \to ZH$.

(a) Draw the Feynman diagram of the reaction $e^+e^- \to ZH$.

(b) The total cross section in an unpolarised collider is $\sigma(e^+e^- \to ZH) = 200$ fb. Express it as a function of $\sigma(e^-_L e^+_L \to ZH)$, $\sigma(e^-_L e^+_R \to ZH)$, $\sigma(e^-_R e^+_L \to ZH)$ and $\sigma(e^-_R e^+_R \to ZH)$.

(c) We note $\sigma_L \equiv \sigma(e^-_L e^+_R \to ZH)$ and $\sigma_R \equiv \sigma(e^-_R e^+_L \to ZH)$. Compute the ratio σ_L/σ_R using $\sin^2\theta_w \approx 0.23$. Deduce the value of σ_L.

(d) Argue that $\sigma(e^+e^- \to ZH) = A_{ZZ} \times \Gamma_{ZZ}$ with $\Gamma_{ZZ} = \Gamma(h \to ZZ^*)$ and A_{ZZ} a computable factor including phase space and the well measured coupling g_{Zee}.

3. Measuring the total Higgs boson width

We have demonstrated that at the LHC $\sigma(pp \to h) = A_{gg}\Gamma_{gg}$ and that at the ILC $\sigma(e^+e^- \to Zh) = A_{ZZ}\Gamma_{ZZ}$. We will generically note these reactions $i \to h$ and their corresponding cross sections $\sigma_i \equiv \sigma(i \to h) = A_{F_i} \times \Gamma_{F_i}$, where F_i designates *only* the final states gg at the LHC and ZZ at the ILC, and A_{F_i} is a pure kinematic factor.

(a) We consider the decay $h \to Y$, where Y designates a generic final state of the Higgs-boson decay. Write the cross section $\sigma\,(i \to h(\to Y))$ as a function of σ_i and the branching ratio $\mathcal{B}(h \to Y)$. Express it as a function of A_{F_i}, Γ_{F_i}, $\Gamma_Y \equiv \Gamma(h \to Y)$ and the total Higgs boson width Γ_h.

(b) Assuming that we have a measurement of σ_i, show that measuring the cross section of the process $i \to h(\to F_i)$ (noted σ_{ii}), we can measure Γ_h by expressing Γ_h as a function of σ_i, σ_{ii} and A_{F_i}.

(c) At the ILC, it is possible to measure $\sigma_{ILC} \equiv (e^+e^- \to ZH)$ using a method that is independent of the Higgs boson decay. At the LHC, one needs to reconstruct the Higgs boson in a given final state Y in order to record the event. To obtain the total $\sigma(pp \to h)$ one would need to measure all the possible final decays $h \to Y$. Give several reasons, both experimental and theoretical, why this is not possible.

4. Precision of the width measurement

We consider the reaction $e^+e^- \to ZH$ at the ILC which has polarised beams, achieving $\sigma_{ILC}(e^+e^- \to ZH) \approx 300$ fb. We assume a sample corresponding to a luminosity of $500\,\text{fb}^{-1}$.

(a) What is the number of ZH produced in this sample?

(b) To measure Γ_h, we need to consider the reaction $e^+e^- \to Z(H \to ZZ)$, with 3 Z bosons in the final state. Using the branching ratio $\mathcal{B}(h \to ZZ^*) \approx 1$ % (including reconstruction effects), what is the total number of events reconstructed in this 3 Z final state?

(c) The precision on Γ_h is directly proportional to the precision of the measurement $\sigma(e^+e^- \to Z(H \to ZZ))$. What is the expected *relative* statistical precision on Γ_h (i.e. $\delta(\Gamma_h)/\Gamma_h$) with this machine? We recall that the Poissonian uncertainty on a counted quantity N is $\sqrt{N}$.

Exercice III

Allowed and forbidden processes, Feynman diagrammes

For each of the processes below, determine whether it is allowed or forbidden in the standard model. For the forbidden processes, explain why, giving *all the possible reasons* (here we do not require to take into account multiplicative quantum numbers and angular momentum). For the allowed processes, specify and justify by which *dominant* interaction they occur and draw the corresponding Feynman diagrams (one per process). Note on the diagram the names of all real and virtual particles. When relevant, indicate near the vertex the CKM matrix elements that contribute and give their orders of magnitude in terms of $\lambda = \sin\theta_c$ (θ_c is the Cabibbo angle). Then, give the total order of magnitude of the diagram in terms of λ. In case of Penguin or box diagrams, do this all the possible intermediate quarks. As in general the tree process is favored, we will privilege it when several topologies are possible. Also, we will privilege flavor allowed to flavor suppressed diagrams.

1. $\pi^+ n \to K^+ \Lambda$
2. $\pi^+ n \to \pi^0 \Sigma^-$
3. $J/\psi \to \Sigma^0 \Lambda_b^0$
4. $J/\psi \to \tau^+ \tau^- \nu_\tau$
5. $B^+ \to \tau^+ \nu_\tau$
6. $\Lambda_b^0 \to p K^- \pi^0$
7. $\pi^+ n \to \pi^0 \Sigma_c^+$

Exercice IV

Charmless two-body decays in the BABAR experiment

In the *BABAR* experiment, the reaction

$$e^+ e^- \to \Upsilon(4S) \to B^0 \overline{B}^0 \tag{6}$$

was produced by an asymmetric collider. The two beams had different energies: $E(e^+) = 9\,\mathrm{GeV}$ and $E(e^-) = 3.1\,\mathrm{GeV}$, adjusted to produce the $b\bar{b}$ resonance $\Upsilon(4S)$. In this exercise we are interested in events containing the decays

$$B^0 \to K^+ \pi^-, \tag{7}$$

$$B^0 \to \pi^+ \pi^-. \tag{8}$$

The parts 2-4 below are independent of each other. It is strongly recommended to answer first the preliminary questions (part 1) using the PDG booklet.

1. Preliminary questions
   (a) What is the lifetime of the resonance $\Upsilon(4S)$?
   (b) Make a summary table recalling the spin, the parity, the mass, the lifetime and, when applicable, the quark content of each of the particles in the reactions (6), (7) and (8). Make a brief comment on the parity of the electron.
   (c) We indicate that the creation of the $\Upsilon(4S)$ in reaction (6) occurs by electromagnetic interaction. Explain why. Deduce which of the following helicity states of the interacting electron and positron are allowed: $e_R^+ e_R^-$, $e_L^+ e_L^-$, $e_L^+ e_R^-$, $e_R^+ e_L^-$.

2. Quantum numbers, interactions and conservation laws

   (a) Which is the dominant interaction through which decays the $\Upsilon(4S)$? the B^0? Give two arguments for each answer.

   (b) What are the parity and the charge conjugation of the state $B^0\overline{B}^0$ in reaction (6)?

   (c) What are the parities and the charge conjugations (when the latter are defined) of the final states of reactions (7) and (8)? Are they possible to obtain from conservation laws?

   (d) What is/are the possible value(s) of the relative angular momentum between the electron and the positron?

   (e) Draw the Feynman diagrams of the production and the decay of the $\Upsilon(4S)$ in (6).

3. Angular distribution

   In this part we analyze the angular distribution of the B mesons in the BABAR experiment. We will represent these distributions as a function of the angle θ^* between the directions of the outgoing B^0 and the incoming positron, in the center of mass of reaction (6) (in other words, this is the rest frame of the resonance $\Upsilon(4S)$).

   (a) Draw a scheme showing all the possible helicity states of the incoming electron and positron, and of the outgoing B mesons.

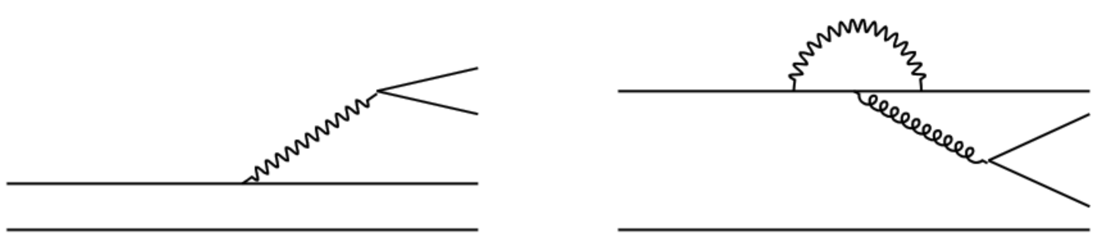
   (b) Associate to each scheme a differential cross section, as a function of θ^*. The helicity amplitude, noted A, is the same for all the helicity configurations. Explain why.

   (c) Compute the differential cross section of the process (6) as a function of θ^* and A.

   (d) Make a comment on the values of the differential cross section at $\theta^* = 0$ and $\theta^* = \pi$, regarding the angular momentum conservation in the reaction.

4. CKM matrix elements and CP violation

   (a) Each of the processes (7) and (8) has contributions from two types of diagrams, with the topologies given in the figures below. Name these topologies being as specific as possible. Draw the two diagrams corresponding to each process, noting near the vertices the relevant CKM matrix elements.



   (b) In both processes, (7) and (8), we are now interested in the intensity induced by the CKM matrix elements in each of the two diagrams above. Compare the corresponding amplitudes in terms of the $\mathcal{O}(\lambda^3)$ Wolfenstein parameterization. One of the processes has comparable contributions from the two diagrams in terms of the power of the parameter λ (sine of the Cabibbo angle). Which one is it? What is the consequence of this feature?

In the following, we consider 3 additional processes

$$B^0 \to K^- \pi^+, \tag{9}$$

$$\overline{B}^0 \to K^- \pi^+, \tag{10}$$

$$\overline{B}^0 \to \pi^+ \pi^-, \tag{11}$$

(c) Under the (wrong) hypothesis of no *CP* violation, which constrains can be set on some relations between numbers of expected events corresponding to the processes (7) to (11)?

(d) In reality, the number of events of the process (11) registered by *BABAR* is about 35% smaller than that of the process (8). Explain qualitatively (in some detail) the reasons for this observation.

(e) What types of *CP* violation are involved in different pairs of the decay modes (7) to (11): in mixing, in decay, in the interference between mixing and decay? Explain. Which observables can be used, in each case, for these studies?