**M2 NPAC - Particle physics (2019-2020)**

# Exercise sheet № 1 - Relativistic kinematics

**I -** Time dilatation - ultrarelativistic case

The collision of a proton beam with a target produces a large quantity of $\pi$ mesons. $N_0$ pions with a precise kinetic energy are selected by a magnetic dipole. The frame $\mathcal{R}'$ is related to these mesons, and the corresponding Lorentz factor is $\gamma = 1000$. The mesons pass through a linear tunnel of $L_0 = 3$km between their production point and a detector. The frame $\mathcal{R}$ is related to the tunnel. The proper lifetime of $\pi$ mesons is $\tau_0 = 2.6 \times 10^{-8}$s.
**a-** Compute the mean lifetime of the mesons in the frame $\mathcal{R}$ of the tunnel.
**b-** Compute the length of the tunnel observed in the frame $\mathcal{R}'$ related to the $\pi$ mesons.
**c-** How long does it take the pions to cross the distance $L_0$?
**d-** The mesons decay in flight. How many of them reach the detector?
**e-** What is the corresponding number if the computation is done in the frame related to the pions?

In the following exercises, use the masses of particles from the PDG booklet when needed. We consider the nutrinos as massless.

**II -** Decay at rest

For the following decays, write the momentum-energy 4-vector for each of the particles in the frame where the initial state particle is at rest. Make a judicious choice of the axes, in order to get simple expressions for the 4-vectors. Compute the energy and momentum of each of the final-state particles as a function of the masses of the particles in the process.
**a-** $\pi^+ \longrightarrow \mu^+\nu_\mu$ ; also compute the kinetic energy of the $\mu^+$
**b-** $^{12}\text{C}^* \longrightarrow$ $^1 2\text{C}\gamma$ ($^{12}C^*$ is an excited state of $^{12}C$; its excitation energy is 4.433 MeV.)
**c-** $n \longrightarrow pe^-\overline{\nu_e}$ ; in this 3-body decay compute the maximum energy of the electron and the proton. Comment.

**III -** Decay in flight

A $\pi^+$ meson decays in flight to $\mu^+\nu_\mu$ (see II.a). Find the maximum- and minimum-values of the $\mu^+$ energy in the laboratory frame as a function of the Lorents factor $\gamma$ of the $\pi^+$.

**IV -** Threshold of particle production

**a-** *i.e.* Production of $\pi$ mesons
Consider the reactions of $\pi$ meson production in nucleon-nucleon collisions:

$$p \ \ p \longrightarrow p \ \ p \ \ \pi^0 \quad , \quad p \ \ p \longrightarrow p \ \ n \ \ \pi^+$$

Compute the threshold energy of protons that hit a fixed target of protons for these reactions to take place (what would you suggest to use as a fixed proton target?)
**b-** *i.e.* Nucleon-antinucleon pair production
In 1952, Emilio Ségré's team discovered antiprotons by producing them in a reaction of the type:

$$p \ \ p \longrightarrow p \ \ p \ \ \overline{p} \ \ p,$$

using a fixed target. Compute the energy of the proton beam at the threshold of this reaction and compare it to the mass of the produced $p\overline{p}$ pair.
**c-** *i.e.* Production of a $W^+W^-$ pair
We want to create a pair of $W^+W^-$ bosons in the reaction $e^-e^+ \longrightarrow W^+W^-$ ($m_{W^\pm} = 80.1$ GeV/$c^2$).

1. Determine the threshold energy of a positron beam in the case of a fixed electron target.
2. Same question in the case of a symmetric collider. Compare and comment the results.

## V - Photoproduction of $\pi$ mesons

Photoproduction of $\pi^+$ mesons is the following interaction of a photon and a proton:

$$\gamma \ \ p \longrightarrow n \ \ \pi^+$$

**a-** What is the minimal energy of a photon, necessary for this reaction to take place with a proton at rest?
**b-** The cosmic microwave background radiation (CMBR) is a form of electromagnetic radiation filing the universe. This radiation, which has a thermal black body spectrum at a temperature of 3 K, is well explained as radiation left over from an early stage in the development of the universe. For a cosmic ray proton, what is the minimal energy to produce the above reaction with a CMBR photon (at 3 K)?
We recall that the Boltzman constant $k$ is $8.617 \times 10^{-5}$ eV K$^{-1}$.
**c-** For a cosmic-ray photon, what is the minimal energy to produce with a CMBR photon the reaction:

$$\gamma \ \ \gamma \longrightarrow e^- \ \ e^+$$

**d-** Explain in more detail the origin of CMBR.

## VI - The decay $\pi^0 \rightarrow \gamma\gamma$

**a-** Write an expression for the angle between the momenta of the two photons in the reaction $\pi^0 \rightarrow \gamma\gamma$ as a function of their energies and the $\pi^0$ mass. Show that the minimum value of this energy ($\theta_{min}$) is obtained for a symmetric decay, where the photons have equal energies.
**b-** Compute $\theta_{min}$ for a $\pi^0$ at rest, then for an ultrarelativistic $\pi^0$ ($\gamma >> 1$) and finally for the case where the kinetic energy of the $\pi^0$ is equal to its rest energy.

## VII - Two-body decay

In this exercise we consider the following decay mode of the $B^-$ meson:

$$B^- \longrightarrow J/\psi \ \ K^-$$

($m_B = 5.3$ GeV/$c^2$, $m_{J/\psi} = 3.1$ GeV/$c^2$, $m_K = 0.5$ GeV/$c^2$)
**a-** Calculate the energies of the $J/\psi$ and of the $K^-$ in the rest frame of the $B^-$.
**b-** In the laboratory, where the energy of the $B^-$ is 45 GeV:

1. calculate the minimal and maximal energy of the $J/\psi$ and the $K^-$;
2. give the relation between the directions of the $J/\psi$ emission in the laboratory and in the rest frame of the $B^-$.