Exercice III

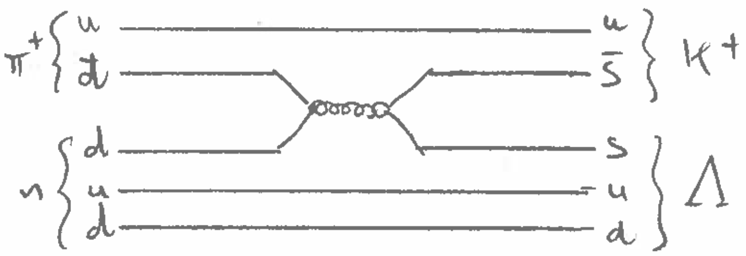
1. $\pi^+ n \to K^+ \Lambda$

$Q_i = Q_f = +1$
$B_i = B_f = +1$

L_l does not intervene

$\Rightarrow$ Allowed

Flavors are conserved $(S_i = S_f = 0)$
only hadrons participate $\Big\} \Rightarrow$ SI



2. $\pi^+ n \to \pi^0 \Sigma^-$

$Q_i = +1 \neq Q_f = -1$
$\Rightarrow$ forbidden

3. $J/\psi \to \Sigma^0 \Lambda_b^0$

$B_i = 0 \neq B_f = 2$
$M_i < M_f$
$\Rightarrow$ forbidden

4. $J/\psi \to \tau^+ \tau^- \nu_\tau$

$\mathcal{L}_{\tau,i} = 0 \neq \mathcal{L}_{\tau f} = +1$

$M_i < M_f$

$\Rightarrow$ forbidden

5. $B^+ \to \tau^+ \nu_\tau$

$M_i > M_f$

$Q_i = Q_f = 1$

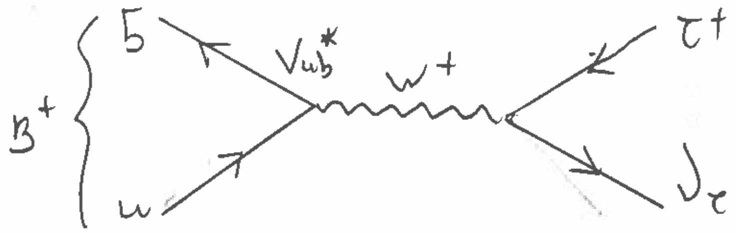
$\mathcal{L}_{\tau,i} = \mathcal{L}_{\tau f} = 0$

$\mathcal{B}$ and other $\mathcal{L}_e$ do not intervene

$\Rightarrow$ Allowed

B not conserved, ν present

$\Rightarrow$ WI



$\mathcal{M} \propto V_{ub}^* \sim \lambda^3$

6. $\Lambda_b^0 \to p K^- \pi^0$

$Q_i = Q_f = 0$

$\mathcal{B}_i = \mathcal{B}_f = +1$

$\mathcal{L}_e$ do not intervene

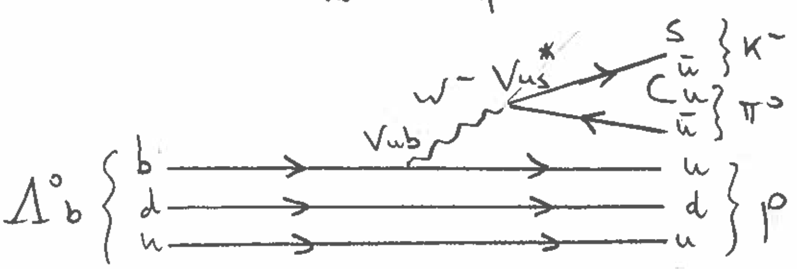
$M_i > M_f$

$\Rightarrow$ allowed

$S_i = 0 \neq S_f = -1$

$B_i = -1 \neq B_f = 0$

$\Rightarrow$ WI



$\mathcal{M} \propto V_{ub} V_{us}^* \sim \lambda^3 \lambda = \lambda^4$

7. $\pi^+ n \to \pi^0 \Sigma_c^+$

$Q_i = Q_f = +1$

$B_i = B_f = +1$

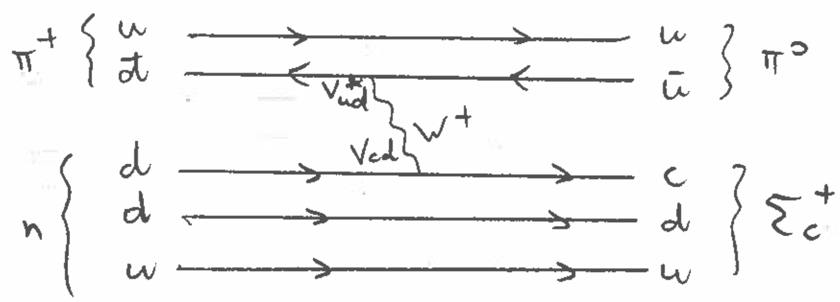
L_ℓ do not intervene

$M_i > M_f$

$\Rightarrow$ allowed

$C_i = 0 \neq C_f = +1$

$\Rightarrow$ WI



$\mathcal{M} \propto V_{cd} V_{ud}^* \sim \lambda \cdot 1 = \lambda$

Exercice IV

1.

(a) $\Gamma(\Upsilon(4S)) = 20.5$ MeV

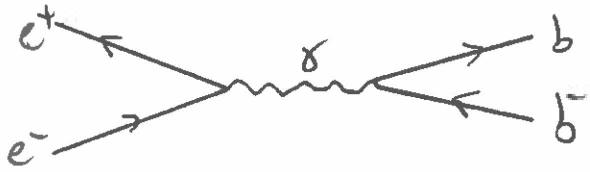
$$\tau = \frac{\hbar}{\Gamma} = \frac{6.58 \times 10^{-22}}{20.5} \simeq 3.2 \times 10^{-23} \text{ s}$$

(b)

	J	P	M (MeV)	τ(s)	quarks
e^+ (e^-)	1/2	−1 (+1)	0,511	stable	/
$\Upsilon(4S)$	1	−1	10579.4	3.2×10^{-23}	$b\bar{b}$
B^0 ($\bar{B}^0$)	0	−1	5279.6	1.52×10^{-12}	$\bar{b}d$ ($b\bar{d}$)
K^+ (K^-)	0	−1	493.7	1.2×10^{-8}	$u\bar{s}$ ($\bar{u}s$)
π^+ (π^-)	0	−1	139.6	2.6×10^{-8}	$u\bar{d}$ ($\bar{u}d$)

The parity of elementry (anti) fermions was fixed to +1 (−1).

(c) $e^+e^- \to \Upsilon(4S)$ occurs by the EM interaction because nothing forbids it (flavors are conserved) and as there are interacting leptons it cannot be the SI.
In addition, as $\sqrt{s} = M(\Upsilon(4S)) \ll M_Z$ the intermediate particle is dominantly a γ.



In the EM interaction the couplings: $e^+_R e^-_L$ and $e^+_L e^-_R$ exist.

2.

(a) $\Upsilon(4S) \to B^0 \bar{B}^0$ or $B^+ B^-$ (dominantly)

This is SI as indicated by
- The width (or lifetime) of the $\Upsilon(4S)$
- Only hadrons intervene and flavors are conserved.

B^0 decays via WI as
- The lifetime of the B^0 is long ($\sim 1 ps$)
- B^0 is the lightest meson carrying beauty. It can only "lose" its beauty via the WI.

(b) Method 1: conservation laws

$\Upsilon(4S) \to B^0 \bar{B}^0$ is a SI process

$\Rightarrow$ C and P are conserved

$$C(B^0\bar{B}^0) = C(\Upsilon(4S)) = -1$$
$$P(B^0\bar{B}^0) = P(\Upsilon(4S)) = -1$$

Method 2: computation

$$J_i = 1$$
$$\vec{J}_f = \underbrace{\vec{J}_{B^0} + \vec{J}_{\bar{B}^0}}_{0} + \vec{L}$$

$$\Rightarrow l = 1$$

$$P(B^0\bar{B}^0) = (P(B^0))^2 \times (-1)^l = +1 \times (-1) = -1$$

$B^0 \bar{B}^0$ is a particle - antiparticle system, with $J=0$

$$\Rightarrow C(B^0\bar{B}^0) = P(B^0\bar{B}^0) = -1$$

(c) In both cases it is the decay of a pseudoscalar particle into 2 pseudoscalar particles. Both decays are due to WI and thus the only relevant conserved quantity is angular momentum.

Angular momentum conservation:

$J_i = 0$

$\vec{J}_f = \underbrace{\vec{J}_1 + \vec{J}_2}_{0} + \vec{L}_{12} \Rightarrow l_{12} = 0$

For both reactions

$P_f = P_1 \cdot P_2 (-1)^0 = +1$

In reaction (7) the final state is not a C eigenstate.

In reaction (8) the final state is a particle - $\overline{\text{particle}}$ pair with spin 0.

$\hookrightarrow C = P = +1$

(d) EM $\Rightarrow$ P is conserved

$$\left.\begin{array}{l} P(\gamma) = -1 \\ P(e^+e^-) = (+1)(-1)(-1)^{l_{ee}} \end{array}\right\} \Rightarrow l_{ee} \text{ even}$$

Naively:

$$\left.\begin{array}{l} J(\Upsilon) = 1 \\ \vec{J}_{ee} = \underbrace{\vec{S}_{e^+} + \vec{S}_{e^-}}_{S_{ee} = 0 \text{ or } 1} + \vec{L}_{ee} \end{array}\right\} \begin{array}{l} S_{ee} = 0, \ l_{ee} = 1 \ (\text{forbidden}) \\ S_{ee} = 1, \ \underline{l_{ee} = 0 \text{ or } 2} \end{array}$$

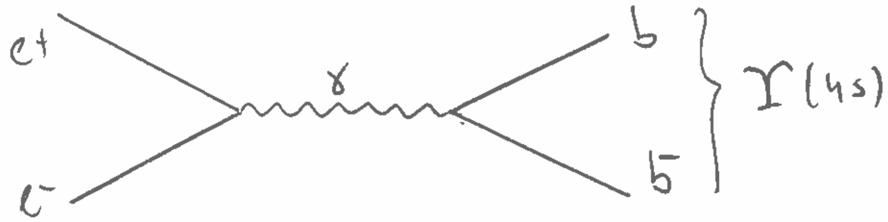
Also:

Given the allowed helicity states of the interacting e^+e^-:

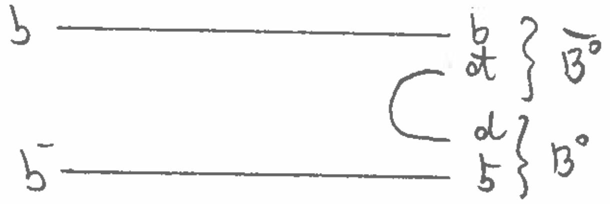
e^+ (or opposite spins) e^-

$\Rightarrow S_{ee} = 1, \ \underline{l_{ee} = 0 \text{ or } 2}$

(e) Production:



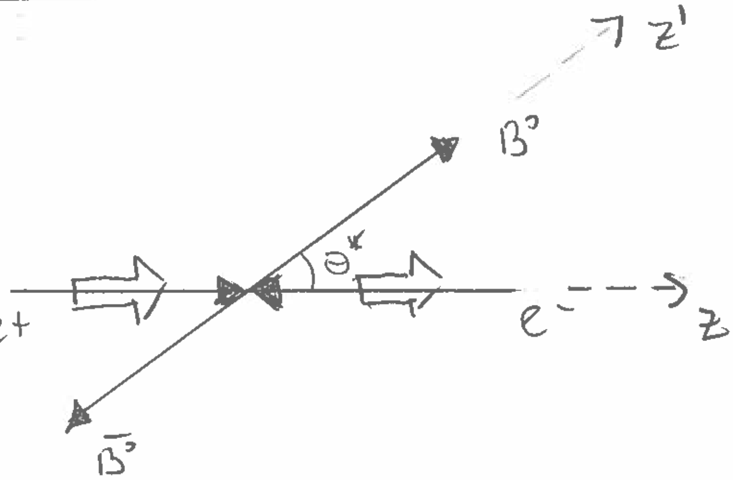
Decay:



3. Angular distribution

(a)

①



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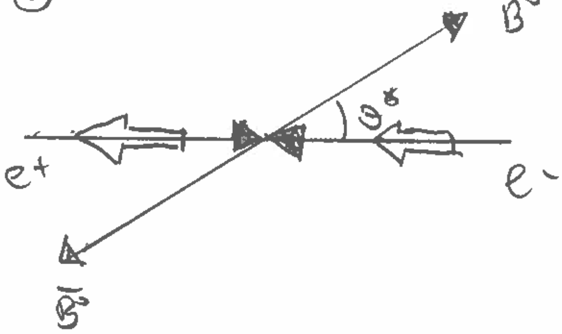


(b) $A_1 = A\, d^1_{10} = -\frac{A}{\sqrt{2}} \sin\theta^*$

$$A_2 = A\, d^1_{-10} = A\, d^1_{10} (-1)^{1-0} = \frac{A}{\sqrt{2}} \sin\theta^*$$

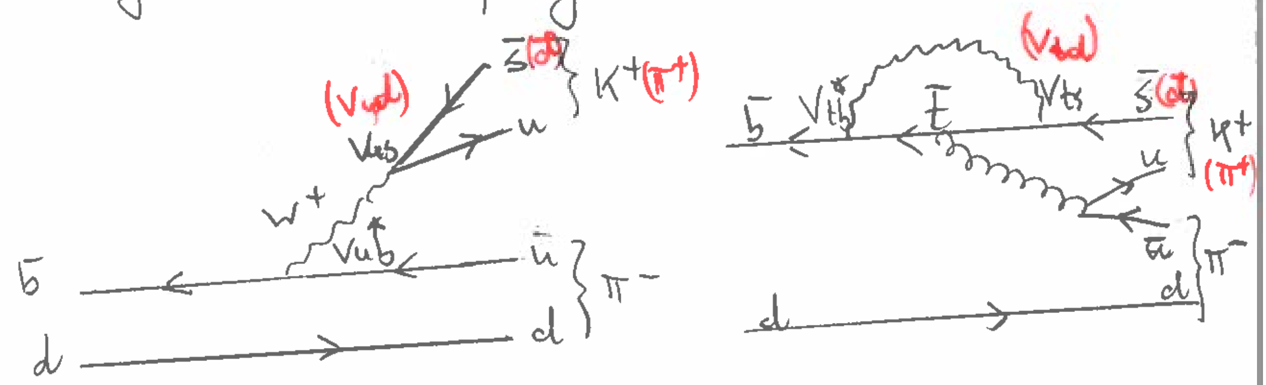
The two helicity amplitudes are strictly the same due to the parity conservation in EM and SI.

(c) $\sigma(\theta^*) = |A_1|^2 + |A_2|^2 = \frac{A^2}{2}\sin^2\theta^* + \frac{A^2}{2}\sin^2\theta^* = A^2 \sin^2\theta^*$

(d) In both cases $\sigma = 0$, otherwise the angular momentum cannot be conserved.

4. <u>CKM matrix elements and CP violation</u>

(a) left: color allowed tree diagram
right: Gluonic penguin



(b) Taking into account that the virtual top quark dominates in the penguin diagram:

$$A_{tree}(K\pi) \propto V_{ub}^* V_{us} \simeq A\lambda^3(\rho + i\eta)\lambda \sim \lambda^4$$

$$A_{penguin}(K\pi) \propto V_{tb}^* V_{ts} \simeq 1(-A\lambda^2) = -A\lambda^2 \sim \lambda^2$$

$$A_{tree}(\pi\pi) \propto V_{ub}^* V_{ud} \simeq A\lambda^3(\rho + i\eta)(1 - \frac{\lambda^2}{2}) \sim \lambda^3$$

$$A_{penguin}(\pi\pi) \propto V_{tb}^* V_{td} \simeq 1 \cdot A\lambda^3(1 - \rho - i\eta) \sim \lambda^3$$

$B^0 \to \pi^+\pi^-$ decays have similar contributions $(\mathcal{O}(\lambda^3))$ from tree and penguin diagrams.
This would usually result in a large CP violation due to the interference of the two corresponding amplitudes.

(c) $B^0/\bar{B}^0 \to \pi^+\pi^-$ ((8) & (11)) and $B^0/\bar{B}^0 \to K^\pm\pi^\mp$ ((7) & (10)) are CP conjugate. If there is no CP violation the number of events must be compatible.

(d) For each of the processes there is the interference between the tree and penguin diagrams, which causes this <u>direct</u> CP violation (visible in terms of number of events or more precisely the cosine term of the time-dependent CP asymmetry

(e) (6) $B^0 \to K^+\pi^-$ (10) $\bar{B}^0 \to K^-\pi^+$

Direct CPV (Observable: number of events)

(8) $B^0 \to \pi^+\pi^-$ (11) $\bar{B}^0 \to \pi^+\pi^-$

CPV in interference between decay and mixing (observable: time dependent CP asymmetry)