Correction for exercices I and II

Exercice I

Questions on the lecture

1. *Björken scaling invariance and its violation.* Reminder: In deep inelastic scattering experiment (an electron colliding on a nucleon), we note Q^2 the momentum squared exchanged between the electron and a parton inside the nucleon, and x the fraction of the nucleon longitudinal momentum carried by the interacting parton.

   Börken scaling invariance refers to the fact that the structure functions of the nucleons (for instance F_2) are independent, for high energy, of the Q^2 of the photon absorbed by the parton inside the nucleon. This is due to the fact that to first order, at high energy, the partons are approximately free inside the nucleon. This can be written as, for the structure function F_2 : $F_2(x, Q^2) = F_2(x)$.

   This scaling is violated at low x due to the fact that the parton inside the nucleon can emit or absorb soft collinear gluons. The cross section for this to happen being logarithmically divergent for soft gluons, this is breaking the Björken scaling invariance and the partons inside the nucleon can not be considered as free anymore. The DGLAP equations allow to relate the parton structure functions between two different Q^2, so one can use the pdfs measured at low energy by Hera experiments (or by fixed target experiments) at the LHC.

2. *Solar neutrinos.* Solar neutrinos are produced by nuclear reactions in the sun, mainly the fusion process $pp \to d\ e^+\nu_e$, which is followed by further fusion processes into heavier nuclei (He3, He4, Be7, Li7 and B8). All the neutrinos that are produced in these processes are electron neutrinos. The energy of most solar neutrinos is ~ 0.5 MeV, and it goes up to ~ 15 MeV for fusion processes with the heaviest nuclei. The standard solar model can predict the flux of neutrinos and their spectrum. The flux of solar neutrinos is much larger, and their typical energies much smaller, comparing to atmospheric neutrinos. The human body, for instance, is crossed by approximately 3×10^5 solar neutrinos per second. The number of electron neutrinos detected on earth is roughly 1/3 of the prediction ("the solar neutrino problem"). It was solved when neutrinos oscillations were established. In particular, the SNO experiment measured both the electron neutrino rate and the total neutrino rate. The latter was compatible with the prediction of the standard solar model. Most of the solar electron neutrinos oscillate into muon neutrinos, and thus the study of the oscillation rate allows measuring θ_{12} and Δm^2_{12}. Most of these oscillations occur inside the sun (the oscillation rate in matter is larger than in vacuum). Different neutrino experiments are sensitive to different neutrino energy ranges. We talked about several experiments:

   - Homestake (Davis) experiment: large amount of chlorine in a gold mine underground. Inverse beta reaction of solar electron neutrinos with the chlorine produces radioactive argon (relatively long lived), which is than collected and counted.

- Several detectors that used gallium (SAGE, Gallex, GNO), used a similar logic, germanium is produced.
- Super K: water tank and PMTs (Cerenkov-light based). Exploits elastic scattering of solar neutrinos and electrons. Sensitive to all flavors of neutrinos.
- SNO: 1000 t of heavy water with light detector to measure Cerenkov light. Several different reactions with the deuteron are sensitive to different kinds of neutrinos.

Exercice II

Higgs boson natural width

1. Part 1: Higgs boson production at the LHC

   (a) There is no direct coupling Higgs-g-g due to the fact that gluons are massless. The dominant diagram process via a loop of top quark as shown in Fig. 1 since the coupling is proportional to the mass and the top quark is much heavier that any other quarks, this is the only loop to consider.

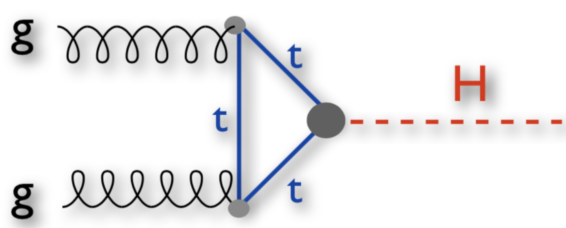

Figure 1: Inclusive Higgs boson production at the LHC from the gluon fusion mode.

   (b)

$$\begin{aligned}\hat{s} &= (x_1\,p_1 + x_2\,p_2)^2 = x_1^2\,k_1^2 + x_2^2\,p_2^2 + 2\,x_1 x_2\,p_1\,p_2 \\ &= 0 + 0 + 2x_1x_2\left(\sqrt{s}/2 \quad 0 \quad 0 \quad \sqrt{s}/2\right)\left(\sqrt{s}/2 \quad 0 \quad 0 \quad -\sqrt{s}/2\right) \\ &= 2x_1x_2(s/4 + s/4) \\ &= x_1\,x_2\,s\end{aligned} \tag{1}$$

   (c) The parton distribution functions gives the probability that parton i carries x_i fraction of the longitudinal momentum of the proton. This probability depends in principle on the factorisation scale (this dependence is neglected in this exercise). So by construction of the pdfs, the total cross section is given by:

$$\sigma(pp \to h) = \iint dx_1 dx_2 f_g(x_1) f_g(x_2) \sigma(g(k_1)g(k_2) \to h) \tag{2}$$

plugging in the formula for the gluon-gluon cross section, and the fact that $\hat{s} = x_1 x_2 s$

$$\begin{aligned}\sigma(pp \to h) &= \iint dx_1 dx_2 f_g(x_1) f_g(x_2) \frac{\pi^2}{8\, m_h}\, \Gamma_{gg} \delta(x_1 x_2 s - m_h^2) \\ &= \frac{\pi^2}{8\, m_h}\, \Gamma_{gg} \iint dx_1 dx_2 f_g(x_1) f_g(x_2) \delta(x_1 x_2 s - m_h^2)\end{aligned} \tag{3}$$

Then using the fact that $\delta(f(x)) = \delta(x - x_0)/|f'(x_0)|$ for $f(x) = x \times s - m_h^2$ (so $f'(x) = s$ and $x_0 = m_h^2/s$. We end up:

$$\sigma(pp \to h) = \frac{\pi^2}{8\, m_h}\, \Gamma_{gg} \times \frac{1}{s}\, l_{gg}\left(\frac{m_h^2}{s}\right) \tag{4}$$

(d) Since l_{gg} is dimensionless, in the units $\hbar = c = 1$, $[L_{gg}] = E^-2$ (1/energy^2), so to put L_{gg} in L^2 (length^2), we need to multiply by $(\hbar\, c)^2$. s

(e) At $\sqrt{s} = 13$ TeV, we get $\tau_{13} = 125^2/(13000)^2 = 9.2\, 10^{-5}$, and from the figure, $l(\tau_{13}) \approx 4100, 10^3$. Using the fact that $\Gamma_{gg} = \mathcal{B}(h \to gg) \times \Gamma_h$, the value $(\hbar c)^2 = 0.39$ GeV^2.mbarn, the cross section is therefore:

$$L_{gg}\left(\frac{m_h^2}{s}\right) = 4100 \times 10^3 \times \frac{0.39}{13000^2}\ \ [\text{mbarn}] \approx 0.01 \text{ mbarn} \tag{5}$$

(f) The cross section is therefore given by

$$\begin{aligned}\sigma(pp \to h) &= \frac{3.14^2}{8 \times 125} \times 0.10 \times 4.1\, 10^{-3} \times 4100 \times 10^3 \times \frac{0.39}{13000^2}\ \ [\text{mbarn}] \\ \sigma(pp \to h) &= 38 \times 10^{-12}\ \ [\text{barn}] \approx 38\ \text{pb}\end{aligned} \tag{6}$$

(g) The increase cross section from 8 TeV to 13 TeV is given by the ratio L_{13}/L_8. At 8 TeV, $\tau_8 = \frac{125^2}{8000^2} = 2.4\, 10^4$, giving the ratio

$$\begin{aligned}\frac{L_{13}}{L_8} &= \frac{8^2}{13^2} \frac{l_{gg}(9.2\, 10^{-5})}{l_{gg}(2.4\, 10^{-4})} \approx \frac{8^2}{13^2} \frac{4250}{950} \\ \frac{L_{13}}{L_8} &\approx 1.7\end{aligned} \tag{7}$$

The cross section increases by a factor of approximately 1.7. This is due to the increase of phase space from 8 TeV to 13 TeV. The average x of the interacting gluons is lower at 13 TeV and the proton contains more or more gluons for smaller and smaller x. Another way to say this is that the gluon pdf are increasing exponentially when x is decreasing.

2. Higgs boson production in a leptonic machine.

(a) The diagram is given on Fig. 2

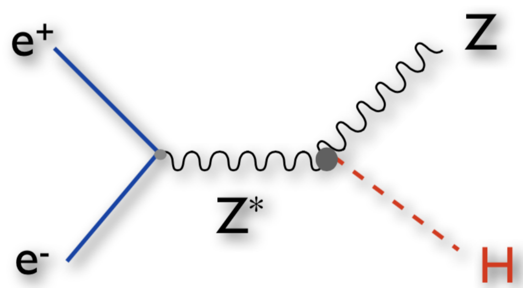

Figure 2: Reaction $e^-e^+ \to ZH$ expecteds in a leptonic machine.

(b) The total cross section is the sum of all the polarised cross sections averaged other the helicities of the incoming leptons. This average factor is $1/2 \times 1/2 = 1/4$ since each leptons have only two potential helicities. Therefore:

$$\sigma(e^-e^+ \to Zh) = \frac{1}{4} \times [\, \sigma(e_L^- e_L^+ \to Zh) + \sigma(e_R^- e_R^+ \to Zh) + \sigma(e_L^- e_R^+ \to Zh) + \sigma(e_R^- e_L^+ \to Zh)\,] \tag{8}$$

This is a vectorial interaction so both leptons needs to be of opposite chirality which is the same as (resp. opposite to) their helicity since we are dealing with ultra-relativistic electrons (resp. positrons). Therefore

$$\sigma(e_L^- e_L^+ \to Zh) = \sigma(e_R^- e_R^+ \to Zh) = 0 \tag{9}$$

(c) The ratio σ_L/σ_R is solely due to the different couplings of the Z boson to left and right fermions, *i.e.*

$$\begin{aligned}\frac{\sigma_L}{\sigma_R} &= \frac{\left(I_3^e - Q_e \sin^2\theta_w\right)^2}{\left(-Q_e \sin^2\theta_w\right)^2} \\ &= \frac{(-0.5 + 0.23)^2}{0.23^2} \\ &= 1.38\end{aligned} \tag{10}$$

Thus, the total cross section can be written as a function of σ_L as $\sigma = 0.25 \times (\sigma_L + \sigma_R) = 0.25 \times \sigma_L\,(1 + 1/1.38)$, which gives:

$$\sigma_L = \frac{4}{1 + \frac{1}{1.38}} \times 200 = 464 \text{ pb} \tag{11}$$

(d) The partial width Γ_{ZZ} is proportional to the coupling $|g_{hZZ}|^2$ and to a phase space element which can be expressed solely as a function of m_h and m_Z. Therefore when writing the cross section, this coupling can be replaced by Γ_{ZZ}, this is the only unknown in the matrix element (the other part is due to the coupling of the Z to electrons which can be absorbed in the factor A

$$\sigma(e^-e^+ \to Zh) \propto g_{hZZ}^2 \propto \Gamma_{ZZ} \tag{12}$$

3. Total Higgs boson width measurement

   (a) We have $\sigma(i \to h(\to Y)) = \sigma(i \to h) \times \mathcal{B}(h \to Y)$ and therefore by definition of the branching ratio

$$\sigma(i \to h(\to Y)) = \sigma_i \mathcal{B}(h \to Y) = A_{F_i} \frac{\Gamma_{F_i} \Gamma_Y}{\Gamma_h} \tag{13}$$

   (b) Using the previous question, for $Y \equiv F_i$ we have:

$$\sigma_{ii} = A_{F_i} \frac{\Gamma^2_{F_i}}{\Gamma_h} \tag{14}$$

   replacing Γ_{F_i} by the σ_i / A_{F_i}

$$\sigma_{ii} = \frac{1}{A_{F_i}} \frac{\sigma_i^2}{\Gamma_h} \tag{15}$$

   Hence, the total width can be measured in a model independent way, measuring independently σ_i and σ_{ii}.

$$\Gamma_h = \frac{1}{A_{F_i}} \times \frac{\sigma_i^2}{\sigma_{ii}} \tag{16}$$

   (c) At the LHC, one can not measure σ_i in a model independent way for several reasons. First, a priori we do not know all the decay modes (in case of new physics), so we can no reconstruct the Higgs boson in all its potential decay modes. Second, several decay modes are extremely difficult to measure, this is especially true for $h \to gg$ which is by far dominated by the di-jet production coming from QCD.

4. Precision on the Higgs boson width measurement

   (a) The total number of ZHv events is given by $\sigma \times L = 300 \times 500 = 150000$.

   (b) The total number of events in the 3 bosons final state is given by

$$N_{3Z} = \sigma \times \mathcal{B} \times L = 150000 \times 0.01 = 1500.$$

   (c) The width is given by Eq. 16, assuming there is no error on A_{F_i}, the relative precision on the width is therefore:

$$\frac{\delta \Gamma}{\Gamma} = \sqrt{4 \times \left(\frac{\delta \sigma_i}{\sigma_i}\right)^2 + \left(\frac{\delta \sigma_{ii}}{\sigma_{ii}}\right)^2} \tag{17}$$

   Since the cross section is proportional to a the number of events measured, to the yields, then

$$\begin{aligned} \frac{\delta \sigma_i}{\sigma_i} &= \frac{1}{\sqrt{150000}} = 2.6\ 10^{-3} \\ \frac{\delta \sigma_{ii}}{\sigma_{ii}} &= \frac{1}{\sqrt{1500}} = 2.6\ 10^{-2} \end{aligned} \tag{18}$$

   and therefore (note that in the exam we were mentioning only the second term which is dominant):

$$\frac{\delta \Gamma_h}{\Gamma_h} = 2.6\ \% \tag{19}$$