Structure formation and Baryon Acoustic Oscillations

James Rich

DPhP-IRFU
CEA-Saclay
91191 Gif-sur-Yvette
james.rich@cea.fr

January, 2020
Outline

- Observed inhomogeneities
  - Large-Scale Structure (LSS)
  - Clusters of galaxies
- Spherical collapse model
  - CDM-only
  - CDM + baryons
- Determination of $\Omega_M$ and $\Omega_\Lambda$ with BAO
- Description of the inhomogenous universe
  - Power spectra of density and potential fluctuations
- Time evolution of Fourier modes
The universe is not homogeneous

Slice of the nearby universe ($z < 0.14$) from Sloan Digital Sky Survey (SDSS)

Universe homogeneous when averaged over distances corresponding to

$\Delta z \sim 0.05$

$\Delta r \sim 0.05c/H_0$

$\sim 60h^{-1}\text{Mpc}$

$h = H_0/100\text{km s}^{-1}\text{Mpc}^{-1}$

$\sim 0.7$

$d_H \equiv c/H_0 = 2998h^{-1}\text{Mpc}$
Bound Structures

- Galaxies: $10^8 M_\odot < M \leq 10^{13} M_\odot$
  \[ \frac{\rho}{\bar{\rho}} \approx 10^5 \quad \frac{v_{rot}^2}{c^2} \sim \frac{1}{2} \frac{GM}{Rc^2} \approx 10^{-6} \]

- Galaxy clusters: $M \leq 10^{15} M_\odot$
  \[ \frac{\rho}{\bar{\rho}} \approx 10^3 \quad \langle v^2 \rangle / c^2 \sim \frac{1}{2} \frac{GM}{Rc^2} \approx 10^{-5} \]

- Stars: $10^{-1} M_\odot < M < 20 M_\odot$
  \[ \frac{\rho}{\bar{\rho}} \approx 10^{29} \quad \frac{GM_\odot}{R_\odot c^2} = 2 \times 10^{-6} \]

- Black holes: $10 M_\odot (?) < M < 10^6 M_\odot$
  \[ 10^{46} > \frac{\rho}{\bar{\rho}} > 10^{28} \quad \frac{GM}{ Rc^2} = 1 \]

The density is very inhomogeneous but space-time is very homogeneous

- metric $= (-1, 1, 1, 1) + \text{order } \Phi$

$\Rightarrow$ use of RW metric justified?
Galaxy clusters: largest bound objects

Coma Cluster:

> 1000 galaxies

\[ M_{\text{coma}} \sim 10^{15} M_{\odot} \]

\[ R_{\text{coma}} \sim 1 \text{ Mpc} \]

\[ \rho_{\text{coma}}/\rho_0 \sim 10^3 \]

Gravitational potential:

\[ \Delta \Phi_g \sim GM_{\text{coma}}/R_{\text{coma}}c^2 \]

\[ \sim 2\langle v^2 \rangle/c^2 \sim 2 \times 10^{-5} \]

Note: \( \Delta \Phi_g \leq 2 \times 10^{-5} \) for all objects in the universe except neutron stars and black holes.
Coma Cluster: the galaxies
Coma Cluster: hot gas $\Rightarrow$ x-rays

Coma Cluster of galaxies

Image courtesy of V. Briel, MPI Garching, Germany

European Space Agency
Bullet Cluster: dark matter, hot gas, galaxies

Gas separated from dark matter and galaxies in collision between two clusters.
Spherical collapse model

A critical matter-only universe with a small spherical expanding region with $\rho > \rho_c$.

Overdense region acts like a mini-closed universe: Gravity excess stops the its expansion starting a contraction phase.
Spherical collapse model

Spherical symmetry $\Rightarrow$ dynamics of $R(t)$ independent of rest of universe. Conservation of energy of test particle at a boundary:

$$\left(\frac{1}{2}\right)\ddot{R}^2 - \frac{GM}{R} = - \frac{GM}{R_{\text{max}}}$$

$M =$ mass contained in spherical region (time-independent)

$$dt = \frac{dR}{\sqrt{2\Phi_g} \sqrt{R_{\text{max}}/R - 1}}$$

Model characterized by $R_{\text{max}}$ and $\Phi_g = GM/R_{\text{max}} < \sim 10^{-5}$
Spherical collapse model: small time behavior

\[ dt = \frac{dR}{\sqrt{2\Phi_g} \sqrt{R_{\text{max}}/R - 1}} \]

\[ t \to 0 \Rightarrow R \ll R_{\text{max}} \]

\[ R(t) \approx \left( \frac{9\Phi_g R_{\text{max}}}{2} \right)^{1/3} t^{2/3} \]

\[ a(t) \text{ also } \propto t^{2/3} \]
$R(t)$: expansion, collapse, virialization

\[ t_{\text{max}} = \frac{R_{\text{max}}}{\sqrt{2} \Phi_g} \int_0^1 \frac{dx}{\sqrt{x^{-1} - 1}} \]

⇒ Small objects (small $R_{\text{max}}$) form before large objects.
(“bottom-up” structure formation)
Density contrast at $t_{\text{max}}$

\[
a(t) \propto t^{2/3}
\]

Normalization:
\[
a(t) = R(t) \text{ for } t \to 0
\]

\[
\frac{a(t_{\text{max}})}{R(t_{\text{max}})} = \left( \frac{3\pi}{4} \right)^{2/3} \approx 1.8
\]

\[
\frac{\rho}{\bar{\rho}}(t_{\text{max}}) = \frac{9\pi^2}{16} \approx 5.5
\]
$R(t_{\text{max}})$ is small compared to Hubble distance

\[ a(t) \propto t^{2/3} \]
Normalization:
\[ a(t) = R(t) \text{ for } t \to 0 \]
\[ \Rightarrow \frac{\dot{a}}{a} = \frac{2}{3}t \]

\[ \frac{c}{H(t_{\text{max}})} \approx \frac{1}{\sqrt{\Phi_g}} \]
$R(a)$: Hubble entry

\[ R(t) \propto (\Phi_g R_{\text{max}})^{1/3} t^{2/3} \]
\[ d_H(t) \propto t \]
\[ R_{\text{enter}} \sim \Phi_g R_{\text{max}} \]
Gravitational potential

Potential fluctuation at \( t \ll t_{\text{max}} \) equals potential at maximum expansion:

\[
G \left( \frac{4\pi R^3}{3} \right) \Delta \rho = \frac{GM}{R(t)} = \frac{GM}{R_{\text{max}}}
\]

and \( \approx \) depth of virialized potential well:

\[
\frac{GM}{R_{\text{vir}}} \approx 2 \frac{GM}{R_{\text{max}}}
\]
\[ \frac{\Delta \rho}{\bar{\rho}} = \frac{(\rho - \bar{\rho})}{\bar{\rho}} \]

Pre-collapse:

\[ \frac{\Delta \rho}{\rho} \propto a(t) \]

\[ \Delta \Phi \propto \bar{\rho} R^2 \frac{\Delta \rho}{\rho} \]

\[ \propto a^{-3} \times a^2 \times a^1 \]

(time independent)
Summary of spherical-collapse model

- Small fluctuations lead to bound objects only if $\Omega_M \sim 1$
  - If $\Omega_M < 1$ small fluctuations insufficient to give $\rho > \rho_c$
  - If $\Omega_M > 1$ the whole universe collapses.
  - $\Omega_M \sim 1$ for $3 \times 10^{-4} < a/a_0 < 0.5$ (our universe).

- During matter epoch ($\Omega_M \sim 1$):
  - $\Delta \rho/\rho \propto a(t)$
  - $\Delta \Phi_g \propto \bar{\rho} \times (\Delta \rho/\rho) \times R^2$ is time independent

- Baryon perturbations escape over-dense region with sound wave.
  - $\Rightarrow$ Baryon perturbations do not grow until recombination.
An initial over-density:
\[ t = 0 \]
\[ c_s \sim c/\sqrt(3) \]
\((\gamma,p,e \text{ plasma})\)

\[ c_s \rightarrow 0 \text{ at recombination} \]
\((r \sim 150\text{kpc})\)

Today: Enhanced correlation at \( r = 147.5\text{Mpc} \)
BAO Peak in galaxy-galaxy correlation function

Galaxy-galaxy correlation function at two redshifts

Baryon Oscillation Spectroscopy Survey
BAO Peak $\Rightarrow D_M(z)/r_d$ and $(c/H(z))/r_d$

Galaxy positions are found in $(z, \theta, \phi)$ space. For an ensemble of galaxies near redshift $z$, the BAO peak in the correlation function in the radial and transverse directions are

$$\Delta z_{BAO} = \frac{r_d}{c/H(z)}$$
$$|\Delta \vec{\theta}_{BAO}| = \frac{r_d}{D_M(z)}$$

The measured values of $\Delta z_{BAO}$ and $|\Delta \vec{\theta}_{BAO}|$ determines $D_M(z)/r_d$ and $(c/H(z))/r_d$. 
ΛCDM parameters from BAO

Expansion rate and Hubble distance:

\[ H(z) = \frac{c}{d_H(z)} = H_0 \left[ \Omega_\Lambda + \Omega_M (1 + z)^3 + \Omega_k (1 + z)^2 + \ldots \right]^{1/2} \]

where \( \Omega_k = 1 - \Omega_M - \Omega_\Lambda \).

Distance to \( z \):

\[ d(z) = \int_0^z d_H(z)dz \]

Angular diameter distance to \( z \):

\[ d_M(z) = d_c S(d(z)/d_c) \quad d_c = \frac{c/H_0}{\sqrt{|\Omega_k|}} \quad S = \begin{cases} \sin & \text{for } \Omega_k < 0 \\ \sinh & \text{for } \Omega_k > 0 \end{cases} \]

⇒ \( d_M(z)/r_d \) and \( d_H(z)/r_d \) are functions of \( z \) and \( (\Omega_M, \Omega_\Lambda, r_d H_0) \)
$d_H(z)/r_d$ vs. $z$

Models:
standard ΛCDM
$(\Omega_M, \Omega_\Lambda) = (1, 0)$
$(\Omega_M, \Omega_\Lambda) = (0, 0)$
\[ \dot{a} \propto \frac{H(z)}{1 + z} \text{ vs. } z \]

deceleration: \( z > 0.6 \)
acceleration: \( z < 0.6 \)
$d_M(z)/r_d$ vs. $z$

Models:
- standard $\Lambda$CDM
  $$ (\Omega_M, \Omega_\Lambda) = (1, 0) $$
- $$(\Omega_M, \Omega_\Lambda) = (0, 0) $$
BAO and SNIa constraints

BAO results:
\[ \Omega_M = 0.288 \pm 0.033 \]
\[ \Omega_\Lambda = 0.695 \pm 0.115 \]
\[ \Omega_k = 0.02 \pm 0.14 \]
\[ H_0 r_d = 147.33 \text{Mpc} \times (68.5 \pm 1.5) \text{km s}^{-1} \text{Mpc}^{-1} \]

(de Sainte Agathe et al, 2019)
Fourier expansion of density in box, $V = L^3$

$$\rho(\vec{r}) = \bar{\rho} \left[ 1 + \sum_{\vec{k}} \delta_\vec{k} \exp(i \vec{k} \cdot \vec{r}) \right]$$

$$\vec{k} = \frac{2\pi \vec{n}}{L}, \quad \delta_{-\vec{k}} = \delta^*_{\vec{k}}$$

Density dispersion:

$$\frac{\bar{\rho}^2 - \bar{\rho}^2}{\bar{\rho}^2} = \sum_{\vec{k}} |\delta_\vec{k}|^2 = \int_0^{\infty} \frac{dk}{k} \frac{k^3 P(k)}{2\pi^2}$$

$$P(k) = V \langle |\delta_{\vec{k}}|^2 \rangle$$

For now, the expansion is done at a fixed time, $t$. Later we will add the time dependence of the amplitude of co-moving modes:

$$\delta_{\vec{k}}(t), \quad \lambda_k(t) = \frac{2\pi}{k} \frac{a(t)}{a_0}$$
Power spectrum in standard $\Lambda$CDM

Mean square amplitude maximum at $k \sim 0.02 (\text{Mpc}/h)^{-1}$

High $k$ modes have small amplitude but there are many of them!

Standard $\Lambda$CDM : $(\Omega_{cdm}, \Omega_b) = (0.269, 0.0484)$, $\Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b$, $h = 0.674$, $A_s = 2 \times 10^{-9}$
Density fluctuation vs. scale

\[ (\frac{\Delta \rho}{\rho})^2 \sim \int \frac{dk}{k} k^3 P(k) \frac{2\pi^2}{k} \]

Universe is \( \sim \) homogeneous on scales \( k < 0.1(Mpc/h)^{-1} \)

\( (\lambda = 2\pi/k > 60Mpc/h) \)

Standard \( \Lambda \)CDM : \( (\Omega_{cdm}, \Omega_b) = (0.269, 0.0484) \), \( \Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b \), \( h = 0.674, A_s = 2 \times 10^{-9} \)
Gravitational potential fluctuations: $\leq 2 \times 10^{-5}$

Gravitational potential fluctuation:

$$\phi_k \sim \frac{4\pi G \bar{\rho}}{k^2} \delta_k$$

$$\sqrt{V k^3 \langle \phi_k^2 \rangle} \sim 2 \times 10^{-5}$$

at small $k$

(Primordial scale-invariant potential fluctuations from inflation.)

Standard $\Lambda$CDM: $(\Omega_{cdm}, \Omega_b) = (0.269, 0.0484)$,

$\Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b$, $h = 0.674$, $A_s = 2 \times 10^{-9}$
Hubble exit, then Hubble entry

\[ d_H(a) = \sqrt{\frac{3}{8\pi G \rho(a)}} \]

\[ \lambda_k(a) = \frac{2\pi}{k} \frac{a(t)}{a_0} \]

\( \lambda_1 \) and \( \lambda_2 \) “leave” the Hubble volume during inflation and then “enter”.

\( \lambda_1 \) enters during radiation epoch

\( \lambda_2 \) enters during matter epoch
Inflation $\Rightarrow$ scale-independent fluctuations

$\rho(a) \sim$ constant during inflation

$\Rightarrow$ fluctuation amplitude scale-independent at Hubble-exit

Super-Hubble dynamics preserves amplitude

$\Rightarrow$ All scales enter Hubble radius with equal amplitude.
Radiation epoch: potential decay

The gravitational potential of modes that enter during the radiation epoch decays because acoustic oscillation prevents increase of $\Delta \rho/\rho$:

$$\Phi_g \sim G \bar{\rho} \frac{\Delta \rho}{\rho} \lambda^2 \sim a^{-4} \times a^2$$
Gravitational potential fluctuations: $\leq 2 \times 10^{-5}$

Short wavelength modes have Hubble-entry during radiation epoch resulting in decay of gravitational potential.

Long wavelength modes enter during matter epoch and therefore preserve primordial potential fluctuation from inflation.

$k_{eq}$: mode with Hubble entry at matter-radiation equality:

$$\frac{\lambda(a_{eq})}{2\pi} = \frac{a_{eq}/a_0}{k_{eq}} = d_H(a_{eq}) \Rightarrow k_{eq} \sim 0.01\text{Mpc}/h^{-1}$$
A mode with $a_{\text{enter}} \sim 10^{-4}a_0 < a_{\text{eq}}$

CDF growth suppressed while $a < a_{\text{eq}} \sim 3 \times 10^{-4}$

$\delta \propto a(t)$ for $a > a_{\text{eq}}$

CDF growth slows when $\Lambda$ begins to dominate ($a > 0.5$)

Baryons oscillate until recombination ($a \sim 10^{-3}$)
A mode with $a_{\text{enter}} \sim 10^{-4} a_0 < a_{\text{eq}}$
Galaxies are a “biased” tracer of matter at large scale \((k < 0.05)\):

\[
P(k)_{\text{gal}} = b_{\text{gal}}^2 P(k)_{\text{matter}}
\]

At small scale \((k > 0.05)\) non-linear growth complicates the galaxy power spectrum.

“Wiggles” due to BAO are believed to be robustly positioned, but they are more easily seen in the correlation function.
Correlation function

\[ \xi(\mathbf{r}) = \left\langle \delta(\mathbf{r}') \delta(\mathbf{r}' + \mathbf{r}) \right\rangle \]

\[ \sim \int e^{i \mathbf{k} \cdot \mathbf{r}} P(k) \, d^3 k \]

\( \xi(r) \) has a peak at \( r = r_d \)

(sound horizon)

Standard ΛCDM: \((\Omega_{cdm}, \Omega_b) = (0.269, 0.0484), \]

\( \Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b, \quad h = 0.674, \quad A_s = 2 \times 10^{-9} \)
Correlation function

\[ \xi(\vec{r}) = \langle \delta(\vec{r}') \delta(\vec{r'} + \vec{r}) \rangle \sim \int e^{i \vec{k} \cdot \vec{r}} P(k) \, d^3 k \]

\( \xi(r) \) has a peak at \( r = r_d \) (sound horizon)

Standard \( \Lambda \)CDM: \( (\Omega_{cdm}, \Omega_b) = (0.269, 0.0484), \)
\( \Omega_{\Lambda} \sim 1 - \Omega_{cdm} - \Omega_b, \, h = 0.674, \, A_s = 2 \times 10^{-9} \)
\( H_0 \) from distance ladder

\[ v = H_0 D \text{ (for } z \ll \sim 0.1) \]
\[ v = \text{recession velocity from redshift} \]
\[ \text{need small “peculiar” velocity } \Rightarrow v/c = z > 0.02 \]
\[ D = \text{distance from photon flux from objects of known luminosity} \]
\[ F = \frac{L}{4\pi D^2} \]

Latest result: \( H_0 = 73.5 \pm 1.4 \text{km s}^{-1}\text{Mpc}^{-1} \)

Objects of known luminosity:
- Type Ia Supernovae (SNIa) (calibrated with cepheids)
- Cepheid variable stars (calibrated with parallax/xallarap)
Distance Ladder: three steps

Hubble-flow SNI ⇒ $H_0 L_{SNIa}^2$:

$$F_{SNIa} = \frac{L_{SNIa}}{4\pi (zc/H_0)^2}$$

$(0.01 < z < 0.05)$

Cepheids in SNIa hosts ⇒ $L_{SNIa}/L_{ceph}$:

$$\frac{L_{SNIa}}{L_{ceph}} = \frac{F_{SNIa}}{F_{ceph}}$$

Cepheids of known D ⇒ $L_{ceph}$:

$$F_{ceph} = \frac{L_{ceph}}{4\pi D_{ceph}^2}$$
Distance Ladder: mostly small statistics

\[ F_{\text{SNIa}} = \frac{L_{\text{SNIa}}}{4\pi(zc/H_0)^2} \]

Hundreds of SNIa in Hubble flow \((0.01 < z < 0.05)\)

\[ \frac{L_{\text{SNIa}}}{L_{\text{ceph}}} = \frac{F_{\text{SNIa}}}{F_{\text{ceph}}} \]

18 SNIa in galaxies with cepheid distances

\[ F_{\text{ceph}} = \frac{L_{\text{ceph}}}{4\pi D_{\text{ceph}}^2} \]

four galaxies of known distance and observed cepheids

NGC4258: one maser-black hole binary
M31: two detached stellar binaries
LMC: eight detached stellar binaries
Milky Way: 15 cepheid parallaxes

Systematics: do the cepheids in these four galaxies have the same luminosities and photometry as the cepheids in the 18 galaxies hosting SNIa?