Master NPAC

Cosmology homework #3

December 5, 2019

Answers are to be sent by email to `nicolas.regnault-AT-lpnhe.in2p3.fr` by Monday Dec 16th, 2019. Exercices 1, 2, 6 and 9 are multiple choice and are to be answered the usual way:

```
<your name>
 1. B
 2. A
 6. C
 9. A
```

For questions **3**, **4**, **5**, **7** and **8** you will need a computer. You may either use the `jupyterhub` server we have set up for this class, or your own computer and your favorite programming language – as long as it allows you to plot functions. Answers for each of these questions consist in plots. Save your plots in png or pdf format, name them from the question number (e.g. `3_a.png`, `3_b.png`, `4.png`, `5.png`, `7.png`, `8.png`) and attach them to your email.

We have seen in class that the expansion history is governed by the Friedmann equation:

$$H(a) = H_0 \left(\Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_r \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \underbrace{(1 - \Omega_m - \Omega_r - \Omega_\Lambda)}_{\Omega_k} \left(\frac{a_0}{a}\right)^2 \right)^{1/2} \tag{1}$$

which may be re-arranged as:

$$dt = \frac{1}{H_0} \frac{dx}{\left(\Omega_m/x + \Omega_r/x^2 + \Omega_\Lambda x^2 + \Omega_k\right)^{1/2}} \quad \text{with } x \equiv a/a_0 \tag{2}$$

We also have seen that the densities of the different types of fluids evolve differently as a function of the scale factor. The Universe was first dominated by radiation, then, by matter and now by vacuum-energy. In in homework, we will look into this a little more quantitatively.

Unless specified otherwise, we will consider a flat Universe, with $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$, $\Omega_r = 9\ 10^{-5}$, and $H_0 = 68\ \ \mathrm{km/s/Mpc}$.

1. Compute the redshift of matter-vacuum energy equality, z_{eq_Λ}, and matter-radiation equality z_{eq}:

(A) $z_{\mathrm{eq}_\Lambda} = 1.3 \qquad z_{\mathrm{eq}} = 3444$

(B) $z_{\mathrm{eq}_\Lambda} = 0.7 \qquad z_{\mathrm{eq}} = 3443$

(C) $z_{\mathrm{eq}_\Lambda} = 0.3 \qquad z_{\mathrm{eq}} = 3443$

(D) None of the above

2. It is interesting to note that the transition between decelerated and accelerated expansion occurred slightly before matter-vacuum-energy equality. For a flat Universe, dominated by matter and vacuum-energy, compute the redshift at which cosmic expansion started accelerating (use the second Friedmann equation, and $\ddot{a} = 0$)

(A) $1 + z_{\mathrm{acc}} = \left(\dfrac{2\Omega_\Lambda}{\Omega_m}\right)^{1/3} \qquad z_{\mathrm{acc}} \approx 0.65$

(B) $1 + z_{\mathrm{acc}} = \left(\dfrac{\Omega_\Lambda}{\Omega_m}\right)^{2/3} \qquad z_{\mathrm{acc}} \approx 0.71$

(C) $1 + z_{\mathrm{acc}} = \left(\dfrac{\Omega_\Lambda}{\Omega_m}\right)^{1/2} \qquad z_{\mathrm{acc}} \approx 0.49$

(D) None of the above

3. Write a function that returns the value of ρ_c (in $\mathrm{kg/m^3}$) as a function of H (in km/s/Mpc). Write a function that returns the value of H (in km/s/Mpc) as a function of a/a_0 and the cosmological parameters $(\Omega_m, \Omega_r, \Omega_\Lambda, H_0)$. Plot the evolution of ρ_c as a function of a/a_0 – you will probably want to produce a log-log plot (use the function `pl.loglog()` instead of `pl.plot()`).

4. Plot the evolution of ρ_m, ρ_r and ρ_Λ (in $\mathrm{kg/m^3}$) as a function of a/a_0 (again, you will quickly realize that it is convenient to use a log-log scale). Verify that the redshifts of matter-dark energy equality and matter-radiation equality are consistent with what you have found above.

5. Plot the evolution of $\Omega_m(a)$, $\Omega_r(a)$ and $\Omega_\Lambda(a)$ as a function of a/a_0 (use a semilog scale, e.g. `pl.semilogx()` instead of `pl.plot()`).

6. We can estimate the time since matter-dark energy equality by integrating analytically the Friedmann equation, neglecting the matter and radiation components. The time of matter-radiation equality may be estimated in a similar way. Do the computation. What do you obtain ?

(A) $t_{\mathrm{eq}_\Lambda} \approx 12.5$ Gyr $\quad t_{\mathrm{eq}} \approx 300000$ yr

(B) $t_{\mathrm{eq}_\Lambda} \approx 11.6$ Gyr $\quad t_{\mathrm{eq}} \approx 64000$ yr

(C) $t_{\mathrm{eq}_\Lambda} \approx 11.5$ Gyr $\quad t_{\mathrm{eq}} \approx 300000$ yr

(D) None of the above

7. A better alternative is to integrate numerically the Friedmann equation. We have done that in the "introduction to python" class by using a differential equation solver. Here, we will just estimate numerically the integral in equation 2.

Write a function that returns the integrand of equation 2 (beware of the units !). Then, write a function that computes the integral and returns the age of the Universe (in Gyr) as a function of a/a_0, or z, if you prefer (to perform the integration, you may use, for example, `scipy.integrate.trapz`. Plot the age of the Universe as a function of z.

8. Draw the evolution of a/a_0 as a function of time (in Gyr) (1) for our Universe (2) for $(\Omega_m = 2, \Omega_\Lambda = 0.)$, $(\Omega_m = 1, \Omega_\Lambda = 0.)$, and a $(\Omega_m = 0.3, \Omega_\Lambda = 0.)$.

9. Now, that you have a function that gives $t(z)$, re-estimate the time of matter-dark energy equality and matter-radiation equality.

(A) $t_{\mathrm{eq}_\Lambda} \approx 10.2$ Gyr $\quad t_{\mathrm{eq}} \approx 49900$ yr

(B) $t_{\mathrm{eq}_\Lambda} \approx 10.2$ Gyr $\quad t_{\mathrm{eq}} \approx 365400$ yr

(C) $t_{\mathrm{eq}_\Lambda} \approx 11.5$ Gyr $\quad t_{\mathrm{eq}} \approx 420000$ yr

(D) None of the above