Master NPAC

Cosmology homework #1

November 21st, 2019

Answers are to be sent by email to `nicolas.regnault-AT-lpnhe.in2p3.fr` by Thursday Nov 28th, 2019. Please use the following format (given as an example):

```
<your name>
 1. abbc
 2. aaaab
 3. bbc
 4. d
 5. b
 6. aa
```

We give:

- $M_\odot = 1.98855\ 10^{30}$ kg
- $L_\odot = 3.828\ 10^{26}$ W
- mass of the proton: 938.272 MeV
- 1 julian year = 365.25 days

The `scipy.constants` module may be useful. It is available on our `jupyterhub` server.

```
>>> import scipy.constants as C
>>> print C.eV, C.c, C.year
```

1. It is useful to know how to express the value of the Hubble constant $H_0 = 100h$km s^{-1} Mpc^{-1} in different units (keeping the h symbol in the result). Express the value of H_0 in:

(A) s^{-1}

a. $3.24\ 10^{-18}\ h$ **b.** $2.22\ 10^{-24}\ h^{-1}$ **c.** $3.45\ 10^{-11}\ h$ **d.** None of that

(B) Gyr^{-1} (with 1 year = 365.25 days):

a. $1.2223\ 10^{-5}\ h$ **b.** $0.10227\ h$ **c.** $1.1987\ 10^{3}\ h$ **d.** None of that

(C) Mpc^{-1}

a. $3.336\ 10^{-4}\ h$ **b.** $2.1276h$ **c.** $7.7543\ 10^{12}h$ **d.** None of that

(D) GeV

a. $1.167\ 10^{-3}h$ **b.** $1.765\ 10^{-12}h$ **c.** $2.133\ 10^{-42}h$ **d.** None of that

2. Same thing with the critical density $\rho_c = \frac{3H_0^2}{8\pi G}$.

(A) g cm^{-3}:

a. $1.88\ 10^{-29}\ h^2$ **b.** $1.27\ 10^{-12}\ h^{-2}$ **c.** $7.22\ 10^{-42}\ h$ **d.** None of that

(B) eV cm^{-3}:

a. $10537\ h^2$ **b.** $1.272\ 10^{3}\ h^2$ **c.** $1.276\ 10^{-4}\ h^{-2}$ **d.** None of that

(C) GeV^4:

a. $1.42\ 10^{-12}\ h^2$ **b.** $8.09\ 10^{-47}\ h^2$ **c.** $3.23\ 10^{11}\ h^2$ **d.** None of that

(D) protons m^{-3}:

a. $11.2h^2$ **b.** $1.123\ 10^{3}\ h^2$ **c.** $5.223\ 10^{-5}\ h^2$ **d.** None of that

(E) $M_\odot Mpc^{-3}$:

a. $3.22\ 10^{15}\ h^2$ **b.** $2.78\ 10^{11}\ h^2$ **c.** $5.55\ 10^{3}\ h^2$ **d.** None of that

3. The luminosity of a typical galaxy is $2\ h_{70}^{-2}\ 10^{10}\ L_\odot$ and the mean energy of stellar photons is $\sim 2\ eV$.

(A) Evaluate the photon flux (in $m^{-2}s^{-1}$) of a galaxy at a redshift $z \ll 1$

a. $\approx 100z^{-2}$ **b.** $\approx 10^5 z^{-1}$ **c.** ≈ 1000 **d.** None of that

(B) Compute the ratio of the photon flux of the nearest large galaxies ($d \sim 1Mpc$) over the photon flux of the nearest stars $\sim 1pc$.

a. $2\ 10^{-2}$ **b.** $5\ 10^{-5}$ **c.** $4\ 10^{9}$ **d.** None of that

4. The luminosity density of the Universe is $\sim 1.2 h_{70} 10^8\ L_\odot Mpc^{-3}$. Assuming stellar light output has been relatively constant over time, and assuming a mean energy of stellar photons of $\sim 2\ eV$:

(A) estimate the density of photons (in m^{-3}) that have been produced by stars, since the formation of the first stars, about one Hubble time ago

a. $2\ 10^3$ **b.** $4\ 10^{-5}$ **c.** $7\ 10^9$ **d.** None of that

(B) Compare it to the density of CMB photons (≈ 400 cm^{-3})

a. about the same **b.** much larger **c.** negligible

(C) Estimate the corresponding reduced density $\Omega_{\gamma_{\rm vis}}$

a. 0.04 **b.** 0.005 **c.** $7.7\ 10^{-7}$ **d.** None of that

5. On a sphere of radius R, what is the circumference of a cicle of radius L ?

(A) The Earth may be idealized as a perfect sphere of radius $R = 6371$ km. If you could measure distances with a precision of $\pm$1m, how large should you draw a circle on Earth to convince yourself that the Earth is not flat ?

a. 2.3 km **b.** 34 km **c.** 570 km **d.** None of that

6. On a sphere of radius R

(A) what is the angular size of an object of width ℓ placed at a distance d from the observer ?

a. $\delta\theta = \frac{\ell}{R \sin d/R}$ **b.** $\delta\theta = \ell/d^2$ **c.** None of that

(B) What happens when $d \to \pi R$?

a. $\delta\theta \to \infty$ **b.** $\delta\theta \to 0$ **c.** None of that