

1 ϕ^3 Theory

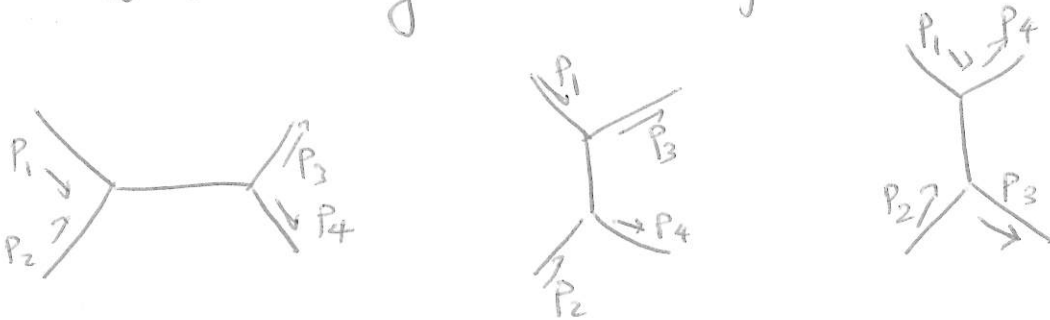
$$L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{3!} \phi^3$$

1.  = $\frac{i}{p^2 - m^2 + i\epsilon}$ propagator

 = $-ig$ vertex

2. - No diagrams to order g .

- To order g^2 : 3 diagrams:



the amplitude of each of these is

$$A \propto g^2 \Rightarrow |A|^2 \propto g^4$$

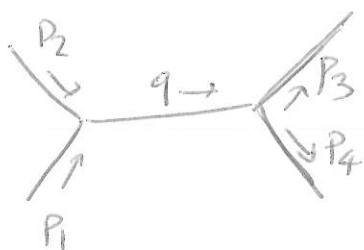
$\rightarrow \frac{d\sigma}{d\Omega} \propto g^4$ ~~and~~ $[\sigma] = L^2 = \frac{1}{E^2}$ and $[g] = E$

→ by dimensional analysis

$$\sigma \propto \frac{g^4}{E^6}$$

where E is the typical energy involved in the process (e.g. $(P_1^0 + P_2^0) = E_{cm}$)

3.



$$q = P_1 + P_2 = P_3 + P_4$$

$$= (-ig)^2 \frac{i}{q^2 - m^2 + i\epsilon} (2\pi)^4 \delta^{(4)}((P_1 + P_2) - (P_3 + P_4))$$

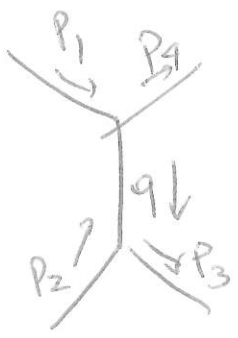
$$A_1 = \frac{-ig^2}{(P_1 + P_2)^2 - m^2 + i\epsilon}$$



$$= (-ig)^2 \frac{i}{q^2 - m^2} (2\pi)^4 \delta^{(4)}((P_1 + P_2) - (P_3 + P_4))$$

$$q = P_1 - P_3$$

$$A_2 = \frac{-ig^2}{(P_1 - P_3)^2 - m^2 + i\epsilon}$$



$$= (-ig)^2 \frac{i}{q^2 - m^2} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - (p_3 + p_4))$$

$$q = p_1 - p_4$$

$$A_3 = \frac{-ig^2}{(p_1 - p_4)^2 - m^2 + i\epsilon}$$

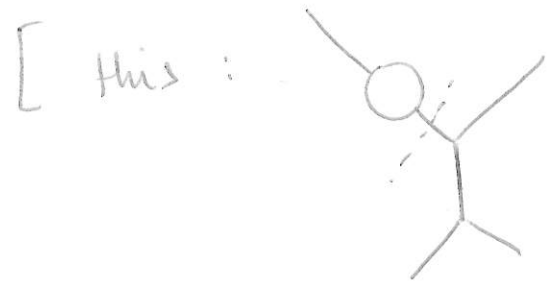
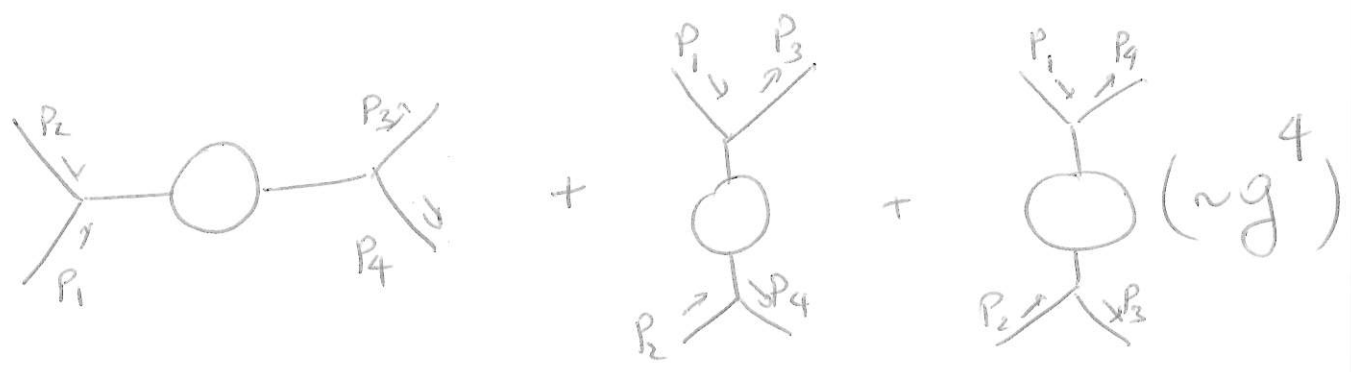
$$A_{tot} = A_1 + A_2 + A_3 =$$

$$= -ig^2 \left[\frac{1}{(p_1 + p_2)^2 - m^2} + \frac{1}{(p_1 - p_3)^2 - m^2} + \frac{1}{(p_1 - p_4)^2 - m^2} \right]$$

$$= -ig^2 \left[\frac{1}{s - m^2} + \frac{1}{t - m^2} + \frac{1}{u - m^2} \right]$$

$$s \equiv (p_1 + p_2)^2 \quad t = (p_1 - p_3)^2 \quad u = (p_1 - p_4)^2$$

4.

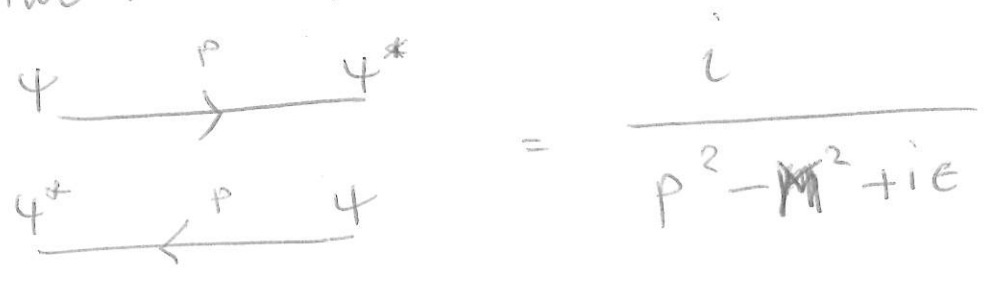


[This does not count, since we can amputate the loop by cutting one external leg]

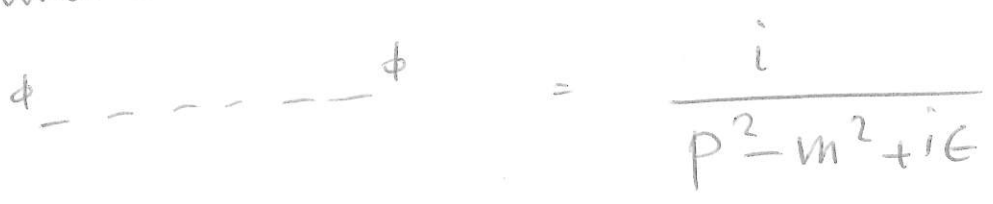
2. Scalar Yukawa theory

$$L = (\partial^\mu \psi)^\dagger \partial_\mu \psi - M^2 \psi^\dagger \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \underbrace{g \phi \psi^\dagger \psi}_{\text{interaction part.}}$$

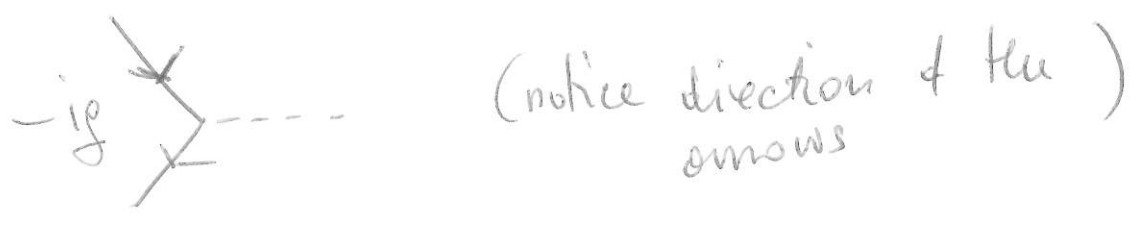
1. ψ now is a complex field, so there are two kinds of lines:



ϕ is a real field so the lines are unoriented



The vertex joins one ϕ , one ψ and one ψ^\dagger

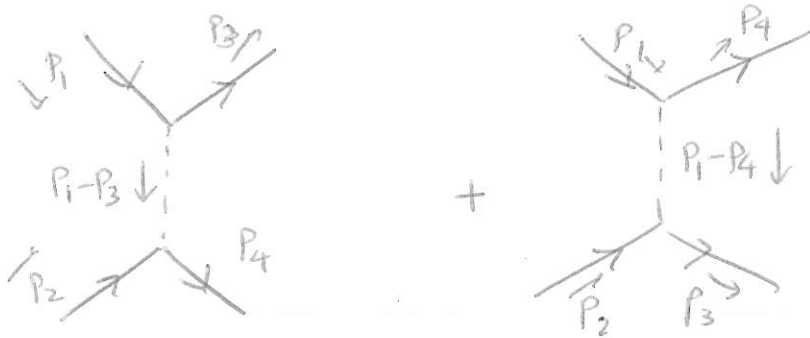


(notice direction of the arrows)

The arrows may be thought of the direction the charge flows (i.e. the conserved $U(1)$ charge of the fermions)

⇒ the net charge entering a vertex must add up to zero (charge conservation) 5

2. $\psi\psi \rightarrow \psi\psi$



$$= (-ig)^2 \left[\frac{i}{(p_1 - p_3)^2 - m^2} + \frac{i}{(p_1 - p_4)^2 - m^2} \right]$$

$$\frac{d\sigma}{d\Omega} \sim \frac{g^4}{Q^6} \quad Q = \text{momentum transferred}$$

$\psi\phi \rightarrow \psi\phi$



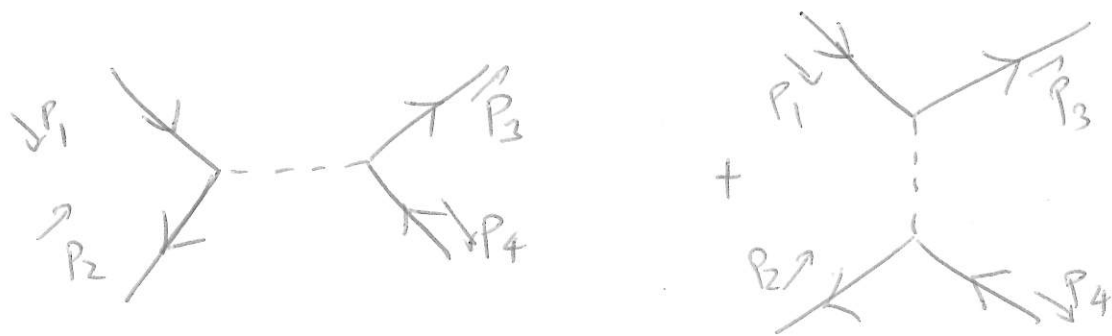
$$= (-ig)^2 \left[\frac{i}{(p_1 - p_3)^2 - M^2} + \frac{i}{(p_1 + p_2)^2 - M^2} \right]$$

$$- \psi \bar{\psi} \rightarrow \phi \phi$$



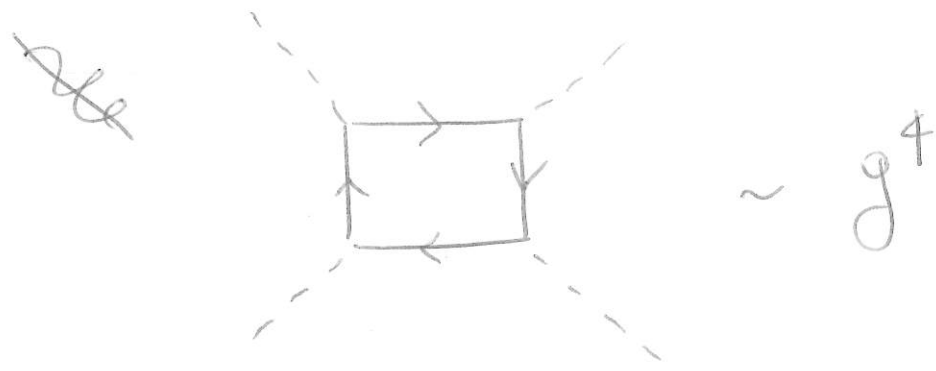
$$= (-ig)^2 \left[\frac{i}{(p_1 - p_3)^2 - M^2} + \frac{i}{(p_1 - p_4)^2 - M^2} \right]$$

$$- \psi \bar{\psi} \rightarrow \psi \bar{\psi}$$



$$= (-ig)^2 \left[\frac{i}{(p_1 + p_2)^2 - M^2} + \frac{i}{(p_1 - p_3)^2 - M^2} \right]$$

3. $\phi\phi \rightarrow \phi\phi$ cannot happen at tree-level because there are no vertices involving only ϕ (ϕ^4 or ϕ^3) so we need some ψ 's and $\bar{\psi}$'s somewhere. Since they are not in the final states, they must be in loops. the lowest order process is:



4. take for example $\psi\psi \rightarrow \psi\psi$ scattering: from point 2



now suppose that all the external legs have energies $\ll m \Rightarrow (p_1 - p_3)^2 - m^2 \approx -m^2$ etc

$\approx \frac{g^2}{m^2}$ = some vertex one would have with $\frac{g^2}{m^2} \psi\psi^\dagger \psi\psi^\dagger$

$$L_{\text{eff}} (p^2 \ll m^2) = \partial_\mu \psi^\dagger \partial^\mu \psi - \underbrace{M^2}_{\lambda} \psi^\dagger \psi - \frac{g^2}{m^2} \psi \psi^\dagger \psi \psi^\dagger$$

$\lambda = \frac{g^2}{m^2}$ effective quartic coupling of a theory with nucleons only

ii. λ is dimensionless, because $[g] = 1$
 $[m] = 1$

iii. since $[\lambda] = 0$ perturbation theory is OK, in principle, even at energies higher than m .

However the predictions of the "effective" theory differ from those of the original theory which includes ϕ , at energies $E > m$.