M2 NPAC 2019/2020

QFT **TD n°5**

Symmetry breaking and Higgs mechanism

1 $SU(2) \to \mathbf{1}$

Consider a theory with two complex scalar fields ϕ_1 and ϕ_2 with Lagrangian :

$$L = (\partial_\mu \phi_1)^* (\partial^\mu \phi_1) + (\partial_\mu \phi_2)^* (\partial^\mu \phi_2) - V(\phi_1, \phi_2), \tag{1}$$

where the potential is

$$V(\phi_1, \phi_2) = m^2 (\phi_1^* \phi_1 + \phi_2^* \phi_2) + \frac{\lambda}{2} (\phi_1^* \phi_1 + \phi_2^* \phi_2)^2 \qquad \lambda > 0. \tag{2}$$

1.1 Symmetries and currents

1. Rewrite L in a manifestly $SU(2)$-invariant way, by introducing an $SU(2)$ complex doublet Φ. Show that the Lagrangian is invariant under a global $SU(2)$ symmetry.
2. Write the infinitesimal transformation acting on Φ, using the infinitesimal generators of $SU(2)$, τ^a $(a = 1, 2, 3)$.
3. Write down the associated Nöther currents.

1.2 Global symmetry breaking

1. Suppose $m^2 > 0$.
   i. What are the classical stable vacua ?
   ii. Is the $SU(2)$ symmetry spontaneously broken ?
   iii. What is the mass spectrum of elementary excitations ?
2. Suppose now $m^2 < 0$.
   i. What are the classical stable vacua ?
   ii. Is the $SU(2)$ symmetry spontaneously broken, and if yes, is there a residual symmetry which leaves the vacuum invariant ?
   iii. What is the mass spectrum of elementary excitations ?

1.3 Local symmetry breaking

Now suppose the $SU(2)$ symmetry is promoted to a local symmetry.

1. Introduce a suitable number of vector fields, and corresponding covariant derivatives, and write the Lagrangian with local $SU(2)$-invariancve (including the kinetic terms for the gauge fields).
2. For $m^2 < 0$, give the spectrum of elementary excitations (masses, spins).

2 $SU(2) \to U(1)$

Consider a Lagragian in which the $SU(2)$ symmetry acts on a triplet of real scalar fields in the vector representation, $\vec{\phi} = (\phi^1, \phi^2, \phi^3)$:

$$L = \frac{1}{2} \sum_{a=1}^{3} (\partial_\mu \phi^a)(\partial^\mu \phi^a) - V(|\vec{\phi}|^2), \tag{3}$$

where the potential is

$$V = \frac{1}{2}m^2|\vec{\phi}|^2 + \frac{\lambda}{4}|\vec{\phi}|^4 \qquad \lambda > 0. \tag{4}$$

2.1 Adjoint action

1. Write the infinitesimal transformation acting on $\vec{\phi}$. What are the appropriate generators T^a ($a = 1, 2, 3$) in the vector representation of $SU(2)$ (i.e. they are three dimensional matrices which generate infinitesimal rotations on $\vec{\phi}$) ?
2. Write down the associated Nöther currents.

2.2 Global symmetry breaking

Suppose $m^2 < 0$.

1. Find the classical stable vacua and the residual symmetry group which leaves the vacuum invariant.
2. Find the spectrum of elementary excitations (their mass, and their transformation properties under the residual symmetry of the vauum).

2.3 Local symmetry breaking

Now suppose the $SU(2)$ symmetry is promoted to a local symmetry.

1. Introduce a suitable number of vector fields, and corresponding covariant derivatives, appropriate for the vector representation. Write the Lagrangian with local $SU(2)$-invariance (including the kinetic terms for the gauge fields).
2. For $m^2 < 0$, give the spectrum of elementary excitations (masses, spins and charges under the residual symmetry group).