From nuclei to stars
Theoretical course

NPAC 2019-2020

Exercises 16/12/2019

1. Show that

$$(\vec{\tau}_1 \cdot \vec{\tau}_2)^2 |\alpha T M_T\rangle = (3 - 2\,\vec{\tau}_1 \cdot \vec{\tau}_2)|\alpha T M_T\rangle \,,$$

   where $|\alpha T M_T\rangle$ is a two-nucleon state with total isospin T, isospin projection M_T and other quantum numbers encapsulated in α.

2. Using second quantisation, demonstrate that $\langle\alpha\beta|\alpha\beta\rangle = 1$.

3. Write down a generic five-body operator in first and second quantisation.

4. Show that $|\alpha\beta\gamma\rangle$ is an eigenstate of the particle number operator and determine the associated eigenvalue.

5. Making use of Wick's theorem, compute the vacuum expectation value of

   a. The operator $a^+_\alpha a_\beta a^+_\gamma a_\delta a^+_\lambda a_\mu$;

   b. The operator $a_\alpha a^+_\beta a_\gamma a^+_\delta a_\lambda a^+_\mu$;

   c. The operator $a^+_\alpha a^+_\beta a^+_\gamma a^+_\delta$.

6. Prove that, for any Slater determinant $|\phi\rangle$, one has

$$E^\phi \equiv \frac{\langle\phi|H|\phi\rangle}{\langle\phi|\phi\rangle} \geq E_0 = \frac{\langle\Psi_0|H|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle} \,,$$

   where $|\Psi_0\rangle$ is the exact ground-state many-body wave function.

   Hint: Expand $|\phi\rangle$ in terms of the exact eigenstates of H, $|\Psi_k\rangle$.