MID-TERM EXAM - SOLUTIONS

1) One could directly use quarks (and gluons) as basic degrees of freedom. The difficulty resides in the non-perturbative character of QCD at low energy, which makes its solution in the energy regime relevant to atomic nuclei extremely challenging.

2) A hypernucleus is a nucleus in which one of the nucleons is substituted with a strange baryon. They are of interest because they could provide additional (complementary) information on nucleon-nucleon (more in general baryon-baryon) interactions and because of the possible presence of hyper-nuclear matter in neutron stars.

3a) $|\vec{r}\,\vec{\sigma}\,\tau\rangle$

3b) $|\vec{r}_1\,\vec{\sigma}_1\,\vec{\tau}_1\,;\,\vec{r}_2\,\vec{\sigma}_2\,\vec{\tau}_2\rangle$

3c) $|JMLS\rangle \otimes |T\,M_T\rangle$

4a) $$P_\sigma = \vec{S}^2 - 1 = \frac{1}{4}\Big(\underbrace{\vec{\sigma}_1^{\,2}}_{=3} + \underbrace{\vec{\sigma}_2^{\,2}}_{=3} + 2\,\vec{\sigma}_1\cdot\vec{\sigma}_2\Big) - 1$$

$$= \frac{1}{2} + \frac{1}{2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$

4b) $P_\sigma = \vec{S}^2 - 1$ has eigenvalues $S(S+1) - 1 = \begin{cases} -1 & \text{if } S=0 \\ +1 & \text{if } S=1 \end{cases}$

$\Rightarrow P_\sigma^2\,|S\,M_S\rangle = |S\,M_S\rangle$

4c) Inverting

$$\begin{cases} |S=0\ M_S=0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \\ |1\ 1\rangle = |\uparrow\uparrow\rangle \\ |1\ 0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle\right) \\ |1\ -1\rangle = |\downarrow\downarrow\rangle \end{cases}$$

one has

$$\begin{cases} |\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|1 0\rangle + |0 0\rangle\right) \\ |\uparrow\uparrow\rangle = |1 1\rangle \\ |\downarrow\downarrow\rangle = |1 -1\rangle \\ |\downarrow\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|1 0\rangle - |0 0\rangle\right) \end{cases}$$

Using $P_\sigma |S=0\ M_S=0\rangle = -|00\rangle$ and $P_\sigma |S=1\ M_S\rangle = |1\ M_S\rangle$

one finds

$$P_\sigma |\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}\left(P_\sigma|10\rangle + P_\sigma|00\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|10\rangle - |00\rangle\right) = |\downarrow\uparrow\rangle$$

etc...

5) NN potential is typically adjusted on NN scattering data and properties of the deuteron.

NNN potential can be adjusted on properties of $A=3,4$ nuclei e.g. binding energy and charge radius of ^3H, ^3He, ^4He or n-d scattering data

6a) A non-negligible spin-orbit component can be deduced e.g. by looking at NN scattering phase shifts in the $L=1$ partial waves (P-waves).

6b) The presence of a strong tensor force can be inferred by comparing different P-wave phase shifts or by computing the magnetic moment of the deuteron

7) Consider the action of $\{a_\mu, a_\nu^\dagger\}$ on Slater determinant $|\alpha\beta\ldots\rangle$

Four cases

a) μ and ν are occupied in $|\alpha\beta\ldots\rangle$

b) μ and ν are unoccupied in $|\alpha\beta\ldots\rangle$

c) μ occupied and ν unoccupied in $|\alpha\beta\ldots\rangle$

d) μ unoccupied and ν occupied in $|\alpha\beta\ldots\rangle$

Now

a) $\{a_\mu, a_\nu^\dagger\}|\alpha\beta\ldots\rangle = (-1)^n a_\nu^\dagger |\alpha\beta\ldots\rangle$

$$= \begin{cases} 0 & \text{if } \nu \neq \mu \quad (\nu \text{ is already occupied}) \\ (-1)^{2n}|\alpha\beta\ldots\rangle & \text{if } \nu = \mu \end{cases}$$

$$= \delta_{\mu\nu} |\alpha\beta\ldots\rangle$$

b) $\{a_\mu, a_\nu^\dagger\}|\alpha\beta\ldots\rangle = a_\mu |\nu\alpha\beta\ldots\rangle$

$$= \begin{cases} 0 & \text{if } \nu \neq \mu \\ |\alpha\beta\ldots\rangle & \text{if } \nu = \mu \end{cases}$$

$= \delta_{\mu\nu} |\alpha\beta \dots \rangle$

c) $\{a_\mu, a_\nu^+\} |\alpha\beta \dots \rangle = a_\mu |\nu\alpha\beta \dots \rangle + (-1)^n a_\nu^+ |\alpha\beta \dots \rangle$

$= (-1)^{n+1} |\nu\alpha\beta \dots \rangle + (-1) |\nu\alpha\beta \dots \rangle$

$= 0$

d) $\{a_\mu, a_\nu^+\} |\alpha\beta \dots \rangle = \underbrace{a_\mu a_\nu^+ |\alpha\beta \dots \rangle}_{=0} + \underbrace{a_\nu^+ a_\mu |\alpha\beta \dots \rangle}_{=0}$

$= 0$