M2 NPAC — From Nuclei to Stars

Phenomenological aspects : final exam

February 6th 2019

1 Course questions (10 points over 20)

Question 1

Explain qualitatively why ^{235}U can fission after absorption of a thermal (vanishing kinetic energy) neutron (fissioning system ^{236}U), while it requires fast neutrons to fission ^{238}U (fissioning system ^{239}U).

Question 2

For numerical applications please use the following values for the liquid-drop formula parameters $a_s = 18.6$ MeV and $a_c = 0.72$ MeV.

(a) Give the definition of the fissility parameter x in terms the energy terms of the spherical liquid-drop binding energy formula.

(b) Explain what happens when $x < 1$ and $x > 1$ and the action of the terms you have given in (a).

(c) On slide "Saturation 2 - p. 13" one can see in the table that for ^{238}U $x = 0.769$. Is there a fission barrier ? If yes what is its height ? [Don't panic, the result you will obtain will be different from the one available in the table. Bonus question: try and guess why].

(d) What is the name of the point of the deformation path where the fissioning ^{238}U system possesses an axial quadrupole deformation parameter of $\beta_2 = 1.01$ as seen in the same table of slide "Saturation 2 - p. 13" ?

(e) Calculate the binding energy loss due to deformation the fissioning ^{238}U system has at the point defined in (d) assuming a purely axial shape.

(f) Calculate at first order in β the intrinsic quadrupole moment of the fissioning ^{238}U system at the point defined in (d). The result should be given in e.fm^2, use the definition of the radius of a sharp charge distribution $R_c = r_{0c}\, A^{1/3}$ where you will assume $r_{0c} = 1.2$ fm.

(g) Give the definition of the ellipticity η of an axial nuclear shape and give the expression of the intrinsic quadrupole moment as a function of the ellipticity.

(h) Use the result obtained in (f) to estimate the fraction of protons participating to the deformation of he fissioning ^{238}U system at the point defined in (d).

(i) Deduce the ratio major axis/minor axis of the associated ellipsoid shape.

(j) Go to "Saturation 2 - p. 9" : to what type of deformation is associated such a shape ?

Turn the page.

2 Problem: a basic shell model description of ^{90}Zr (10 points over 20)

The figure below shows (a) the low-energy part of the experimentally known level scheme of $^{90}_{40}\mathrm{Zr}_{50}$ and (b) the expected minimal valence space one may expect to be sufficient to describe (a).

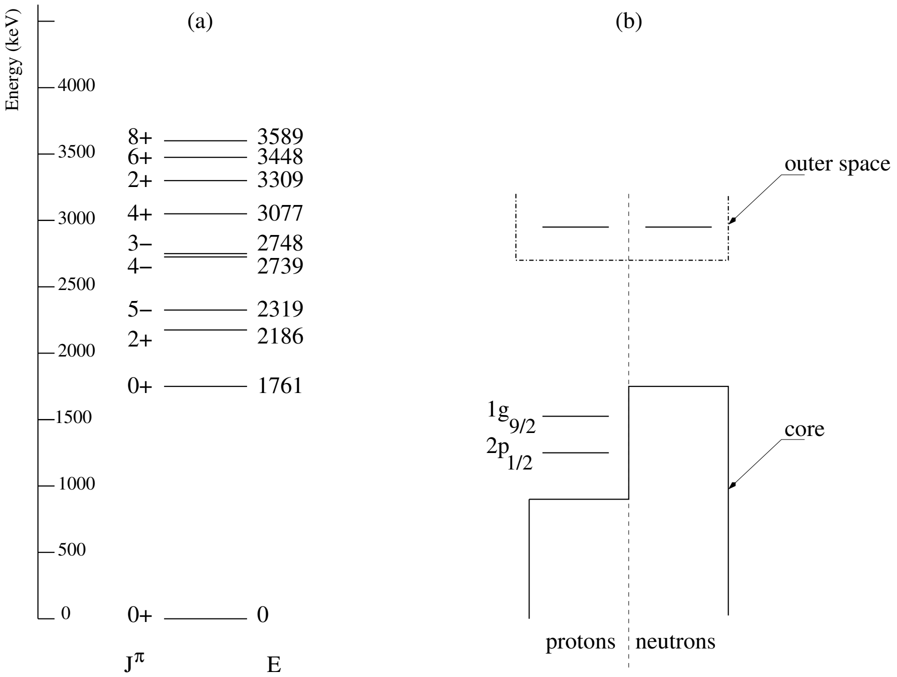

Fig. 1

Question 3

Hint: use the Woods-Saxon single-particle state energy sequence of slide "Structure 1 p. 10".

(a) List the (n, ℓ, j) labels of all single particle states which belong to the proton core and to the neutron core.

(b) Give the (n, ℓ, j) labels of the first single particle state left in the outer space.

(c) (beware, there is a trap) From which observed magic numbers such a valence space is inspired ?

(d) What are the total angular momentum J, parity π and isospin T quantum numbers of the core ?

Question 4

(a) How many particles must be active in the proton valence space and in the neutron valence space of Fig. 1(b) to describe the experimentally known level scheme shown in Fig. 1(a) ?

(b) Give all the configurations that can be formed from the various arrangements of these particles in the valence space.

(c) Considering your answer to (b), what are the experimental energy levels that cannot be described in this valence space ? Justify carefully and comment on this specific choice of a very limited valence space.

(d) (bonus) Suggest a core breaking configuration that could improve the completeness of the experimental level-scheme description.

Question 5

(a) Using Table 2 in slide "Structure 1 p. 12" calculate the energy difference between the single particle states $1g_{9/2}$ and $2p_{1/2}$.

(b) Deduce, from the pure single-particle energy point of view, what must be the configuration associated to the 0^+ ground state of ^{90}Zr and to the first excited 0^+ state of ^{90}Zr. Calculate the pure single particle energy difference between these two 0^+ states.

❗ **Warning:** From now on you enter in the beyond-pure-spherical-mean-field zone. Super-jackpot questions ahead. If you answer you win the N2P3AC : Nuclear Phenomenology Prize of NPAC for recognition in pushing beyond the pure spherical mean field (AND additional points).

Question 6

(a) You want now to evaluate the diagonal configuration energy $\langle\pi(1g_{9/2})^2_{J=0}|V(1,2)|\pi(1g_{9/2})^2_{J=0}\rangle$ and I give you the following tabulated experimental values of proton separation energies for two isotones of ^{90}Zr: $S_\text{p}(^{92}\text{Mo}) = 7.458$ MeV and $S_\text{p}(^{91}\text{Nb}) = 5.154$ MeV (Z=42 and 41 respectively). Explain **why** and **how** you can combine these two values in a certain way so that the result amounts to $\langle\pi(1g_{9/2})^2_{J=0}|V(1,2)|\pi(1g_{9/2})^2_{J=0}\rangle$.

(b) We have seen that a schematic expression for pairing energy $\langle(j_1\ j_1)_{J=0}|V(1,2)|(j_3\ j_3)_{J=0}\rangle$ would be proportional to $(j_1 + 1/2)\cdot(j_3 + 1/2)$. Additional course question: give this expression. Deduce from this expression and from (a) the value for $\langle\pi(2p_{1/2})^2_{J=0}|V(1,2)|\pi(2p_{1/2})^2_{J=0}\rangle$.

(c) Calculate from (a) and (b) the energy difference between the 0^+ ground state and the first excited 0^+ state of ^{90}Zr.

(d) Compare your result from (c) to Fig. 1(a), what is the last ingredient missing for a complete calculation of the energy difference between the two 0^+ states ?

(e) <u>Without detailed calculation</u> predict the configuration mixing ratio of the two 0^+ states of ^{90}Zr.