Nuclear Phenomenology - Exam (final) 2019 -
(D. VERNEY) ANSWERS

Question 1

The excitation energy in CN ^{236}U formed by $^{235}U + n \rightarrow {}^{236}U$ is roughly (if we neglect mass difference between ^{236}U and ^{239}U) greater by $2 \times \delta$ than the excitation energy of CN ^{239}U - $\delta \sim 0.5$ MeV $\rightarrow$ 1 MeV difference

Question 2

(a) $x = \dfrac{\mathring{B}_C}{2\mathring{B}_S}$; $\mathring{B}_C$: Coulomb binding energy term at 0 deformation
$\mathring{B}_S$ = Surface binding energy term at 0 deformation
(both defined positive)

(b) $x < 1$: there is a fission barrier
$x > 1$: no fission barrier
larger $\mathring{B}_C \rightarrow$ larger x : Coulomb pushes toward fission
larger $\mathring{B}_S \rightarrow$ smaller x : surface tension prevents from fission.

(c) $x = 0.769 < 1$: yes there is a fission barrier

$$E_{barrier} = \underbrace{\frac{98}{15} \times \frac{(1-0.769)^3}{(1+2\times 0.769)^2}}_{0.0125} \times \mathring{B}_S$$

$\mathring{B}_S \rightarrow 18.6 \times 238^{2/3} = 714.33$

$= 8.93$ MeV $\neq 7.8$ MeV in table (because not same parametrization of LD)

(d) Saddle point

(e) $$\Delta B_{def} = \mathring{B}_S \left(\frac{2}{5}(1-x)\frac{5}{4\pi}\beta^2 - \frac{4}{105}(1+2x)\left(\frac{5}{4\pi}\right)^{3/2}\beta^3 \cos 3\gamma + \ldots \right)$$

axial $\Rightarrow \gamma = 0$
$\cos 3\gamma = 1$

$$= 714.33\left(\frac{2}{5}(1-0.769)\frac{5}{4\pi}(1.01)^2 - \frac{4}{105}(1+2\times 0.769)\left(\frac{5}{4\pi}\right)^{3/2}(1.01)^3\right)$$

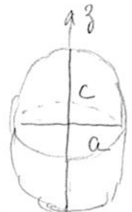
$$= 714.33 \times 0.0125 = 8.93 \text{ MeV}$$

(f) $Q_0 = \frac{3}{\sqrt{5\pi}} Z e R_c^2 \beta_2 \left(1 + \frac{2}{7}\sqrt{\frac{5}{\pi}}\beta_2 + \frac{1}{14\pi}\beta_2^2 + \dots\right)$

$R_c = 1.2\,(238)^{1/3}$ [fm] $\quad R_c = 7.44$ fm.

at first order in β_2 $\quad Q_0 = \frac{3}{\sqrt{5\pi}}\, 92 \times 1 \times (7.44)^2 \times 1.01 = 3893\ e\,\mathrm{fm}^2$

(g) $\eta = \frac{c^2 - a^2}{c^2 + a^2} \qquad Q_0 = \frac{4}{5}\,\eta R_c^2 Z e$



(h) $Q_0 \times \frac{5}{4}\,\frac{1}{R_c^2} = \eta Z$: number of protons participating to the deformation.

$3893 \times \frac{5}{4}\,\frac{1}{(7.44)^2} \simeq 88$

(i) $\eta \simeq \frac{88}{92} = 0.96 \qquad \frac{c}{a} \sim 1 + \eta = 1.96$

(j) $c/a = 2 \equiv$ super deformation.

Question 3

(a) proton core : 38 protons $1s_{1/2}\ 1p_{3/2}\ 1p_{1/2}\ 1d_{5/2}\ 1d_{3/2}\ 2s_{1/2}\ 1f_{7/2}\ 1f_{5/2}\ 2p_{3/2}$
neutron core : 50 neutrons ———— same ———— $+ 2p_{1/2} + 1g_{9/2}$

(b) $1d_{5/2}$ (for both protons and neutrons)

(c) the neutron core corresponds to a well established magic number $N=50$
but the proton core $Z=38$: there is no associated "traditional" magic number

(d) by definition of the core : all magnetic substates occupied $\Rightarrow M=0 \Rightarrow J=0$
said in other worlds the core is the ground state of ${}^{88}_{38}Sr_{50}$ – parity +
$\Rightarrow$ there are 12 neutrons more than protons $\Rightarrow T = +\frac{12}{2} = +6$

question 4

(a) ${}^{90}Zr$ is two protons more than the ${}^{88}Sr$ core : so two valence protons and zero valence neutrons

(b) $\pi(p_{1/2})^2 \quad \pi(p_{1/2}\, g_{9/2}) \quad \pi(g_{9/2})^2$

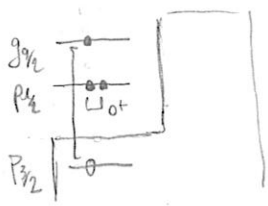
(c) $\pi(p_{1/2})^2 \rightarrow J = 0^+$

$\pi(p_{1/2}\, g_{9/2}) \rightarrow J = 4^-, 5^-$

$\pi(g_{9/2})^2 \rightarrow J = 0^+, 2^+, 4^+, 6^+, 8^+$

all these states are present in the level scheme of Fig 1(a), but there is an additional 2^+ state, and a 3^- state that cannot be represented in this valence space – this is an interesting choice for the valence space: very small but seems to be enough to describe 8/10 experimental levels.

(d)



one hole in $p_{3/2}\ (p_{1/2})^2\ g_{9/2}$ can provide $J = 3^-, 4^-, 5^-, 6^-$

3^- could explain this

but introduces several other negative parity states not observed.

there is no easy answer to this question, it is the reason why it is considered bonus

Question 5

(a) proton $2p_{1/2} \Rightarrow N = 3 \quad \kappa = 0.090 \quad \mu = 0.30$

proton $1g_{9/2} \Rightarrow N = 4 \quad \kappa = 0.065 \quad \mu = 0.57$

$$\varepsilon_{nlj} = \hbar\omega [N - \kappa \langle \vec{l}.\vec{s} \rangle - \kappa\mu\, l(l+1)] - V_0$$

$$\langle j | \vec{l}.\vec{s} | j \rangle = \tfrac{1}{2}[j(j+1) - l(l+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)]$$

$$\langle 1g_{9/2} | \vec{l}.\vec{s} | 1g_{9/2} \rangle = \tfrac{1}{2}[\tfrac{9}{2} \times \tfrac{11}{2} - 4 \times 5 - \tfrac{3}{4}] = 2$$

$$\langle 2p_{1/2} | \vec{l}.\vec{s} | 2p_{1/2} \rangle = \tfrac{1}{2}[\tfrac{1}{2} \times \tfrac{3}{2} - 1 \times 2 - \tfrac{3}{4}] = -1$$

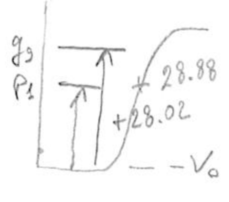
$\hbar\omega = 41 A^{-1/3}$

Core of ^{88}Sr

$\Rightarrow \hbar\omega = 88^{-1/3} \times 41 = 9.23$ MeV

$$\varepsilon_{g_{9/2}} = 9.23[4 - 0.065 \times 2 - 0.065 \times 0.57 \times 20] - V_0 = 28.88 - V_0$$

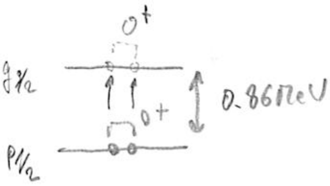
$$\varepsilon_{p_{1/2}} = 9.23[3 - 0.090 \times (-1) - 0.090 \times 0.30 \times 2] - V_0 = 28.02 - V_0$$



$$\varepsilon_{g_{9/2}} - \varepsilon_{p_{1/2}} = 28.88 - 28.02 = 0.86 \text{ MeV}.$$

(b) 0^+ ground state : $(p_{1/2})^2$

0^+ excited state : $(g_{9/2})^2$



from the pure s.p. energy point of view, the energy cost to go from

$(p_{1/2})^2 \to (g_{9/2})^2$ is $2 \times (\varepsilon_{g_{9/2}} - \varepsilon_{p_{1/2}}) = 1.720$ MeV.