NPAC Astroparticles Exam 2018

1 Shock waves in magnetised plasmas

In ideal MHD (MagnetoHydroDynamics) the theorem of magnetic flux freezing states that: *the flux of the magnetic field is conserved across a surface moving with the plasma*. It follows that the strength of the magnetic field may change if the plasma is compressed. In this exercise we will investigate the effects of the magnetic flux freezing at astrophysical shocks.

Consider a plane, infinite, and strong shock moving at a constant speed u_1 through an homogeneous medium of density ϱ_1. The medium is weakly magnetised, and the presence of the magnetic field does not affect the shock jump conditions. It is convenient to move to a rest frame where the shock is at rest. In such rest frame, the upstream plasma moves towards the shock surface with a velocity u_1, while the downstream plasma moves away from the shock surface with velocity $u_2 = u_1/4$. The plasma density is ϱ_1 upstream of the shock and is compressed to a density $\varrho_2 = 4\varrho_1$ downstream of it. A uniform magnetic field $\vec{B}_1$ is present in the upstream plasma.

1. Compute the strength of the magnetic field downstream of the shock (B_2) in the particular cases of a parallel and a perpendicular shock. In a parallel shock the magnetic field $\vec{B}_1$ is parallel to the shock normal (and to the shock velocity), while in a perpendicular one the magnetic field $\vec{B}_1$ is orthogonal to it.

2. Consider now a situation where the angle between the magnetic field upstream of the shock $\vec{B}_1$ and the shock normal is ϑ_1 (see Fig. 1). Determine the strength of the magnetic field downstream of the shock B_2, and the angle ϑ_2 between the magnetic field downstream of the shock and the shock normal. Express these quantities as a function of B_1 and θ_1.

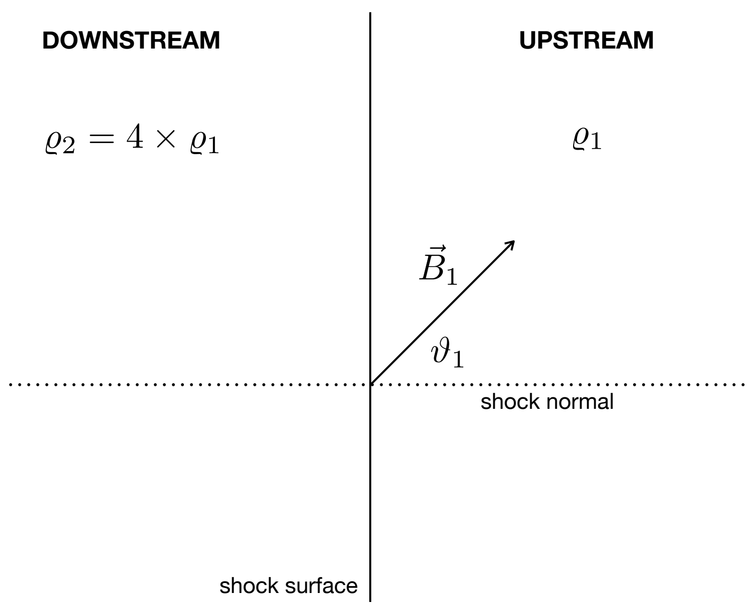

Figure 1: Shock configuration for question 2 of Exercise 1.

2 High energy cosmic ray electrons

The cosmic ray electron spectrum is measured up to a particle energy of 20 TeV. The fact that we observe at the Earth electrons of such energy has been interpreted by astrophysicists as an evidence for the presence of a nearby accelerator of cosmic ray electrons. The goal of this exercise is to show why, following the steps below.

1. In the interstellar medium, high energy electrons lose energy through synchrotron and inverse Compton mechanisms, at a rate:
$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{4}{3}\sigma_T c \gamma^2 \left(\omega_B + \omega_{CMB}\right)$$
where $\sigma_T = 6.7 \times 10^{-25}$ cm^2 is the Thomson cross section, $c = 3 \times 10^{10}$ cm/s is the speed of light, ω_B is the energy density of the interstellar magnetic field (of strength $B = 3$ μG), $\omega_{CMB} = 0.25$ eV/cm^3 the energy density of the Cosmic Microwave Background radiation. E is the energy of the electron and γ its Lorentz factor. Compute the energy loss time τ_L (in years) of 20 TeV electrons due to synchrotron and inverse Compton emission. [Constants you may need: the electron mass is $m_e = 9.1 \times 10^{-28}$ g and $m_e c^2 = 0.51$ MeV. Remember also that 1 yr $= 3.1 \times 10^7$ s and that 1 eV $= 1.6 \times 10^{-12}$ erg.]

2. Cosmic rays diffuse in space in the interstellar medium. The diffusive motion happens along magnetic field lines and is characterized by a typical diffusion coefficient equal to $D = 10^{28}(E/\mathrm{GeV})^{0.3}$ cm^2/s. Compute the typical distance that cosmic ray electrons of 20 TeV propagate along magnetic field lines in a time τ_L.

3. Assume that the 20 TeV electrons that we observe at the Earth have been produced by a single source located at a distance of d parsecs (computed along magnetic filed lines) and that the source released the electrons t_a years ago in an impulsive event (see Fig. 2). For which values of d and t_a can the 20 TeV electrons reach the Earth without losing their energy? Can you constrain the fraction of the Galactic disk from which we can receive 20 TeV electrons? [Remember that 1 pc $= 3 \times 10^{18}$ cm. To answer the last question, assume the disk to be an infinitesimally thin cylinder of radius 15 kpc.]

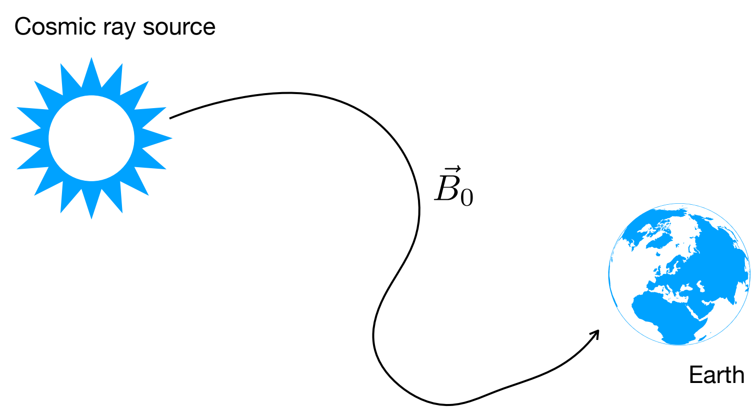

Figure 2: Configuration for question 3 of Exercise 2.

3 The gamma-ray emission from the Galactic centre clouds

The typical density of the gas in the Galactic disk is $n_{ISM} \sim 1$ cm^{-3}. However, a very small volume of the disk is occupied by clouds of gas where the gas density can be much larger than n_{ISM}. The most massive of such clouds is located at the Galactic centre. This cloud can be roughly described as a cylinder of radius $R_{cl} \sim 120$ pc and thickness $h_{cl} \sim 80$ pc, and astronomers were able to estimate its mass, which is equal to $\sim 3 \times 10^7 M_\odot$. This massive cloud is also a gamma-ray source, and observations performed by the HESS array of telescopes revealed a gamma-ray spectrum equal to:

$$F_\gamma(E_\gamma) = 5 \times 10^{-12} \left(\frac{E_\gamma}{\text{TeV}}\right)^{-2.3} \text{TeV}^{-1}\text{cm}^{-2}\text{s}^{-1}$$

in the range $E_\gamma > 0.3$ TeV. Here E_γ is the photon energy. The most plausible interpretation of this observations is that cosmic ray protons are present within the cloud, and that the gamma-ray emission is due to proton-proton interactions between the cosmic rays and the gas of the cloud. The goal of this exercise is to investigate the possibility that a supernova remnant located at the Galactic centre could provide the energy in form of cosmic rays needed to explain the gamma-ray observations.

1. Compute the average gas density of the cloud n_{cl} (in cm^{-3}) and verify that $n_{cl} \gg n_{ISM}$. [Assume that the cloud is made entirely by hydrogen atoms of mass $m_p \sim 1.7 \times 10^{-24}$ g. The mass of the sun is $1\ M_\odot = 2 \times 10^{33}$ g. 1 pc $= 3.1 \times 10^{18}$ cm.]

2. Cosmic ray protons lose energy due to proton-proton interactions with the gas. Knowing that, in first approximation, the cross section of this process does not depend on particle energy and is equal to $\sigma_{pp} \sim 4 \times 10^{-26}$ cm^2, compute the energy loss time τ_{pp} (in years) for cosmic ray protons for a density equal to n_{cl}. Remember that the inelasticity of the process is $\kappa \sim 0.5$. [Remember that cosmic rays move at the speed of light $c = 3 \times 10^{10}$ cm/s. 1 yr $= 3.1 \times 10^7$ s.].

3. Assume that a uniform magnetic field is present, and that it is aligned orthogonally to the axis of the cylinder (see Fig. 3). Assume also that the cosmic ray protons responsible for the gamma-ray emission are injected by a source located at the centre of the cloud. After injection, the cosmic rays will diffuse along the magnetic field lines and will eventually escape from the cloud. If the diffusion coefficient of cosmic rays is the standard Galactic one ($D = 10^{28}(E/\text{GeV})^{0.3}$ cm^2/s) can you estimate the typical residence time τ_{res} of cosmic rays in the cloud? Is this shorter or longer than the energy loss time computed in point 2 above? (Consider only the protons responsible for the gamma-ray emission above $E_\gamma = 0.3$ TeV).

4. Assume that the source of cosmic ray protons located in the centre of the cloud injects continuously protons with a power $Q_p(E_p)E_p^2$ (a power has dimensions of an energy per unit time). The expected gamma-ray luminosity from proton-proton interactions in the cloud is then:

   $$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\eta_\pi}{3} Q_p(E_p)E_p^2$$

   where η_π is the fraction of the cosmic ray power converted into pions:

   $$\eta_\pi = 1 - \exp\left[-\left(\frac{\tau_{res}}{\tau_{pp}}\right)\right] .$$

   Show that $Q_p(E_p) \propto E_p^{-\delta}$ and compute the value of δ. [Hint: can you simplify the expression for η_π?].

5. Consider now a different scenario where the source of cosmic rays in the centre of the clouds injects particles not continuously, but in a single impulsive event that happened t_a years ago. Assume that the spectrum of cosmic rays injected by the source is $N_p(E_p)$ (in units of number of particles per unit energy), with $E_p > 1$ GeV. Assume also that, for the protons responsible for the gamma-ray emission we have $\tau_{pp} \gg \tau_{res}(E_p) \gg t_a$. Show that $N_p(E_p) \propto E_p^{-s}$ and compute the value of s. Compute also the total energy of cosmic ray protons injected by the source as $W_p = \int_{1\text{GeV}}^{\infty} \mathrm{d}E_p E_p N_p(E_p)$. Can a single supernova explosion account for the cosmic ray energy W_p? [Remember that the distance to the Galactic centre is 8 kpc. The total kinetic energy released in a supernova explosion is 10^{51} erg.]

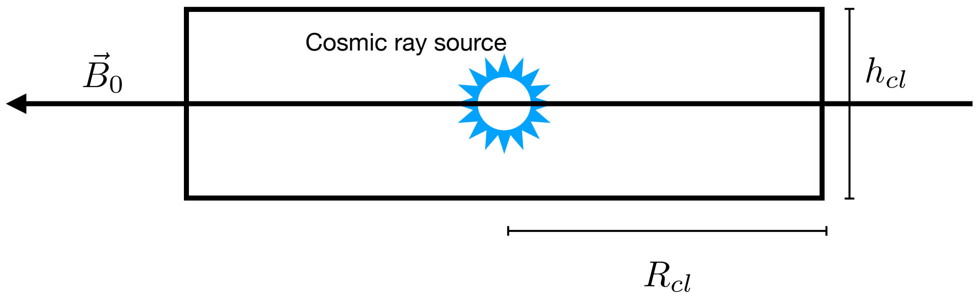

Figure 3: Configuration for question 3 of Exercise 3.

4 Detonation waves

Detonation shock waves are explosive phenomena characterised by an increase of the energy of the system with time. This can happen, for example, if the passage of the shock trough the ambient medium induces chemical reactions in the medium itself (due for example to the increase of temperature and density downstream of the shock). If the reactions are esothermic, they will release energy, increasing the total energy of the system.

This phenomenon can be described in a simplified way as follows. Suppose to have an initial point-like explosion releasing an energy E_0 at a specific location in space. An expanding, spherical, and strong shock wave will form, characterized by a radius R_s and a velocity u_s. Assume that the ambient medium is uniform and of density ϱ, and that each gram of the medium swept by the shock releases an energy ϵ (ϵ has then the units of an energy per unit mass). Assume that this energy is released instantaneously at the passage of the shock. In other words, each elementary volume of the ambient medium will release an energy $\epsilon\varrho$ immediately after the passage of the shock. The system is in the adiabatic phase, which means that no energy is radiated away in form of photons.

If the total energy of the system E is much larger than the initial energy E_0, a scale free solution of the problem can be found, in the form $R_s = At^\eta$.

1. Find η and an approximate value for A using dimensional analysis.

2. Find η and an approximate value for A using the alternative method described in the following: find how the energy of the system E scales with the shock radius R_s, and then find η and A by using the well known expression $R_s \sim (E/\varrho)^{1/5} t^{2/5}$.

3. Can you compute η if the medium is not homogeneous, but characterized by a density that decreases with the distance R from the initial explosion as $\varrho \propto R^{-\alpha}$?