Structure formation and Baryon Acoustic Oscillations

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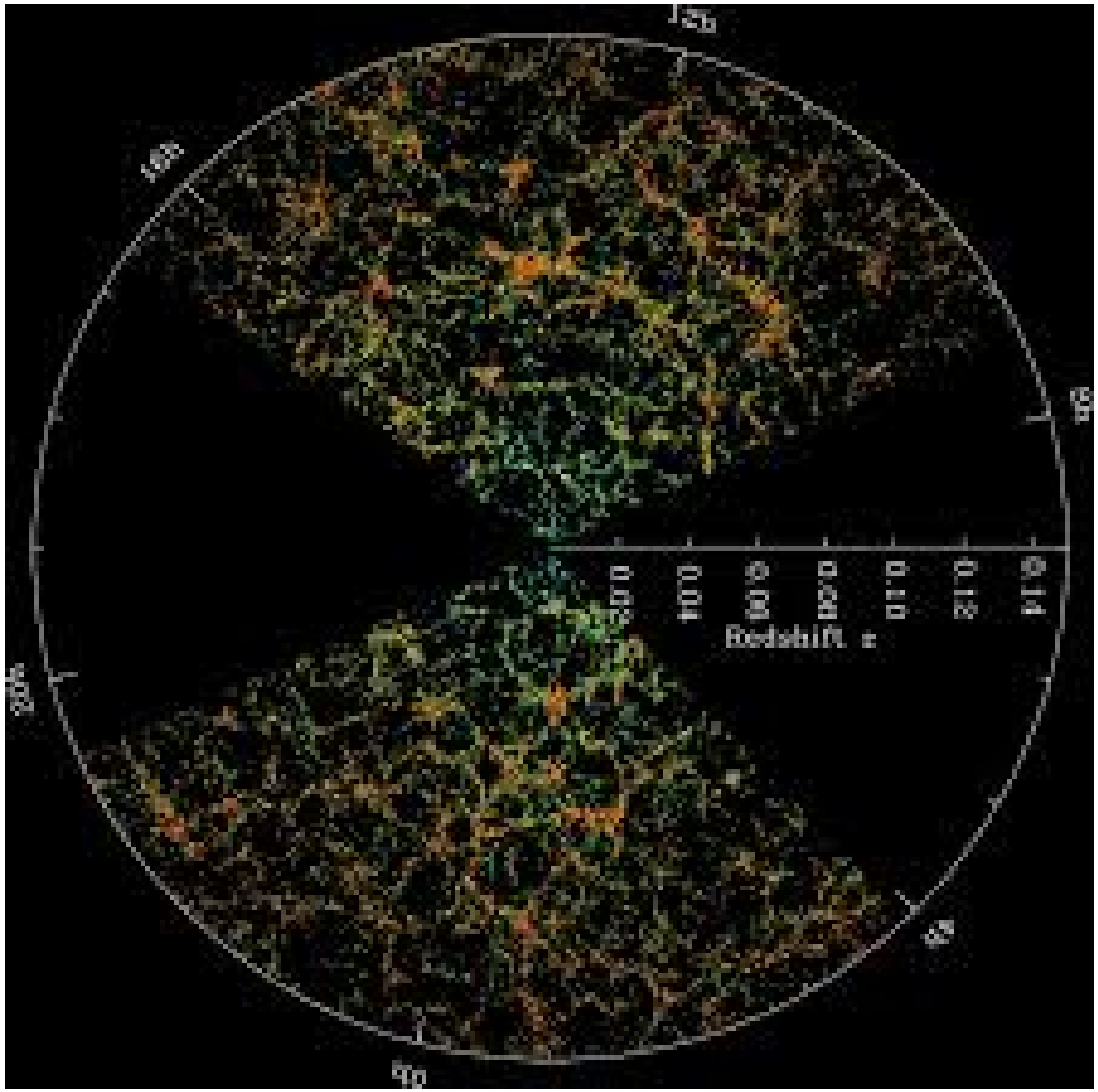
Outline

- Observed inhomogeneities
  - Large-Scale Structure (LSS)
  - Clusters of galaxies
- Spherical collapse model
  - CDM-only
  - CDM+baryons
- Determination of Ω_M and Ω_Λ with BAO
- Description of the inhomogenious universe
  - Power spectra of density and potential fluctuations
- Time evolution of Fourier modes

The universe is not homogeneous



Slice of the nearby universe ($z < 0.14$) from Sloan Digital Sky Survey (SDSS)

Universe homogeneous when averaged over distances corresponding to

$\Delta z \sim 0.05$

$\Delta r \sim 0.05 c/H_0$

$\quad \sim 60 h^{-1} Mpc$

$h = H_0/100 \mathrm{km\,s^{-1}Mpc^{-1}}$

$\quad \sim 0.7$

$d_H \equiv c/H_0 = 2998 h^{-1} \mathrm{Mpc}$

Bound Structures

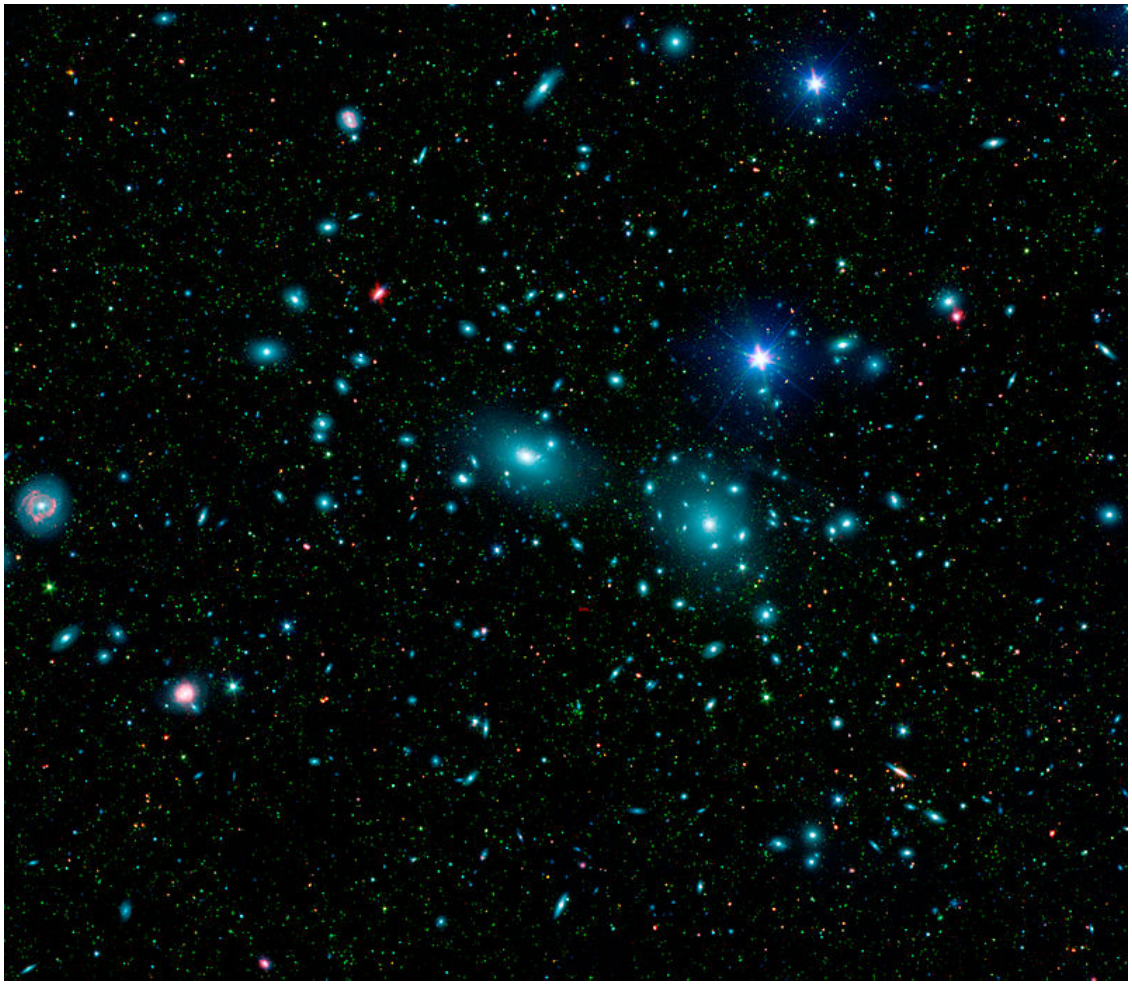
- Galaxies: $10^8 M_\odot < M \leq 10^{13} M_\odot$
  $\rho/\bar{\rho} \approx 10^5 \qquad v_{rot}^2/c^2 \sim (1/2)GM/Rc^2 \approx 10^{-6}$
- Galaxy clusters: $M \leq 10^{15} M_\odot$
  $\rho/\bar{\rho} \approx 10^3 \qquad \langle v^2 \rangle/c^2 \sim (1/2)GM/Rc^2 \approx 10^{-5}$
- Stars: $10^{-1} M_\odot < M < 20 M_\odot$
  $\rho/\bar{\rho} \approx 10^{29} \qquad GM_\odot/R_\odot c^2 = 2 \times 10^{-6}$
- Black holes: $10 M_\odot(?) < M < 10^6 M_\odot$
  $10^{46} > \rho/\bar{\rho} > 10^{28} \qquad GM/Rc^2 = 1$

The density is very inhomogeneous but
space-time is very homogeneous

metric $= (-1, 1, 1, 1) +$ order Φ

$\Rightarrow$ use of RW metric justified?

Galaxy clusters: largest bound objects



Coma Cluster:

> 1000 galaxies

$M_{coma} \sim 10^{15} M_{\odot}$

$R_{coma} \sim 1$ Mpc

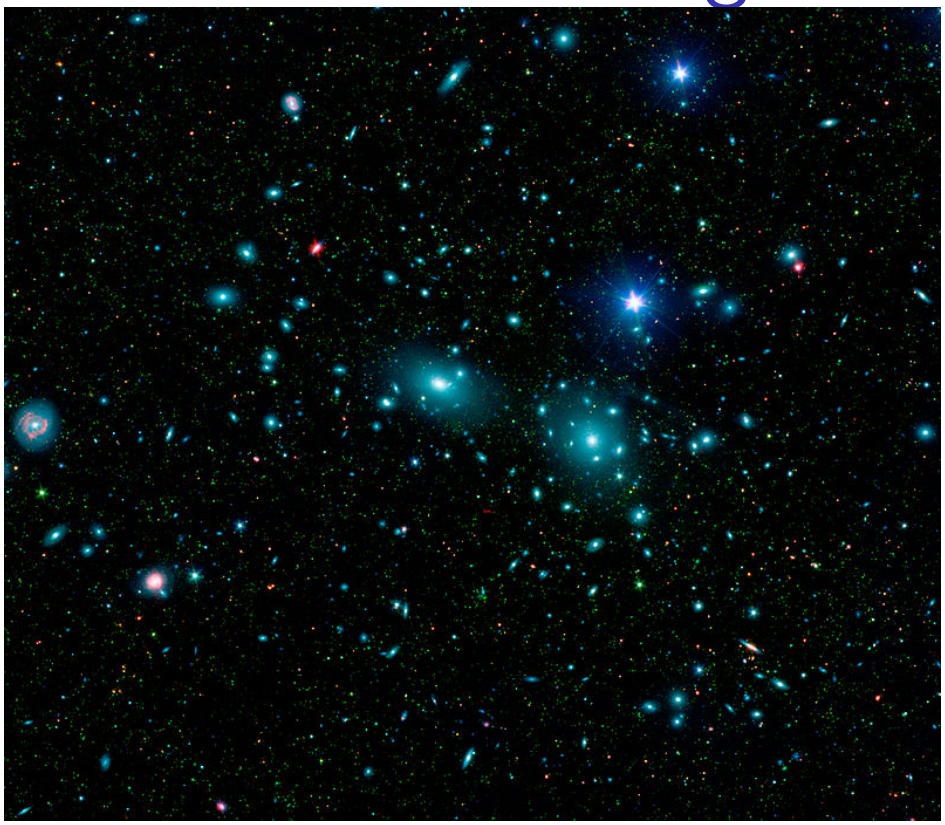
$\rho_{coma}/\rho_0 \sim 10^3$

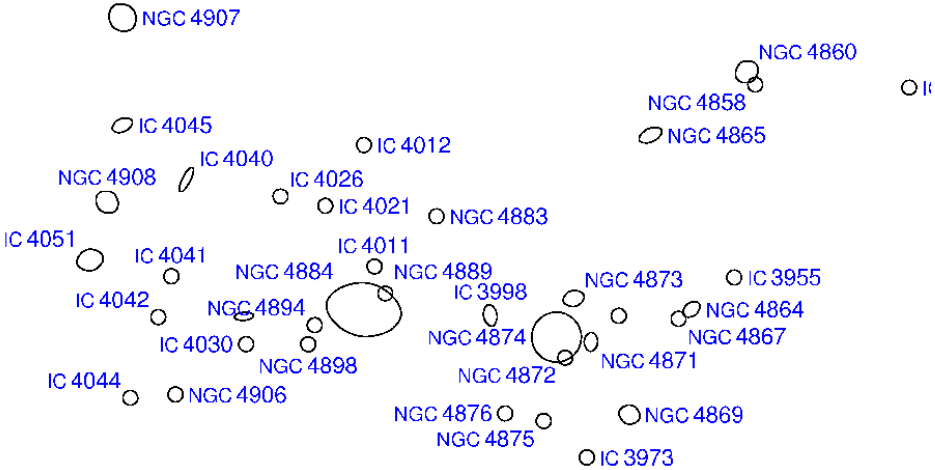
Gravitational potential:

$$\Delta\Phi_g \sim GM_{coma}/R_{coma}c^2$$
$$\sim 2\langle v^2\rangle/c^2 \sim 2 \times 10^{-5}$$

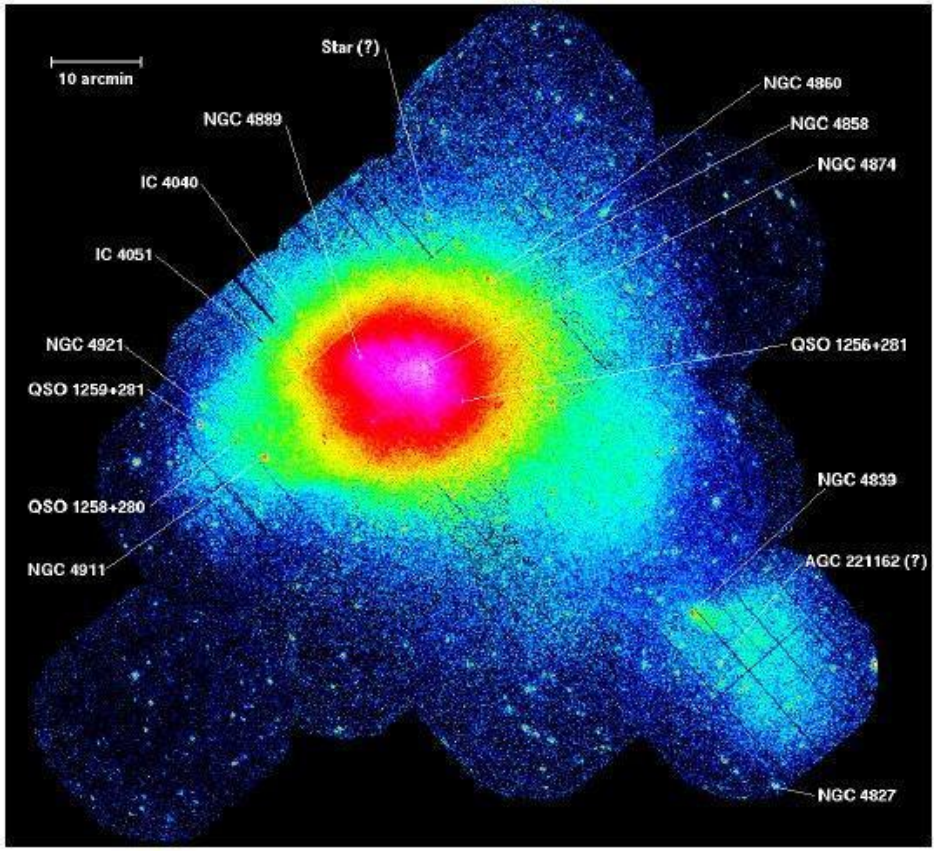
Note: $\Delta\Phi_g \leq 2 \times 10^{-5}$ for all objects in the universe except neutron stars and black holes.

Coma Cluster: the galaxies





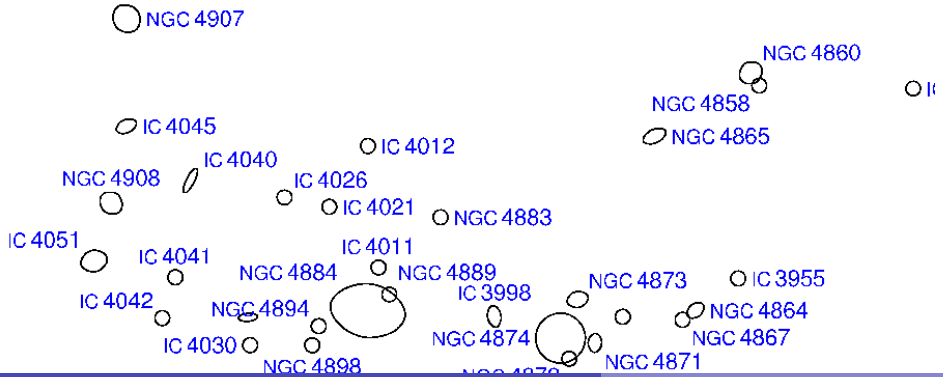
Coma Cluster: hot gas ⇒ x-rays



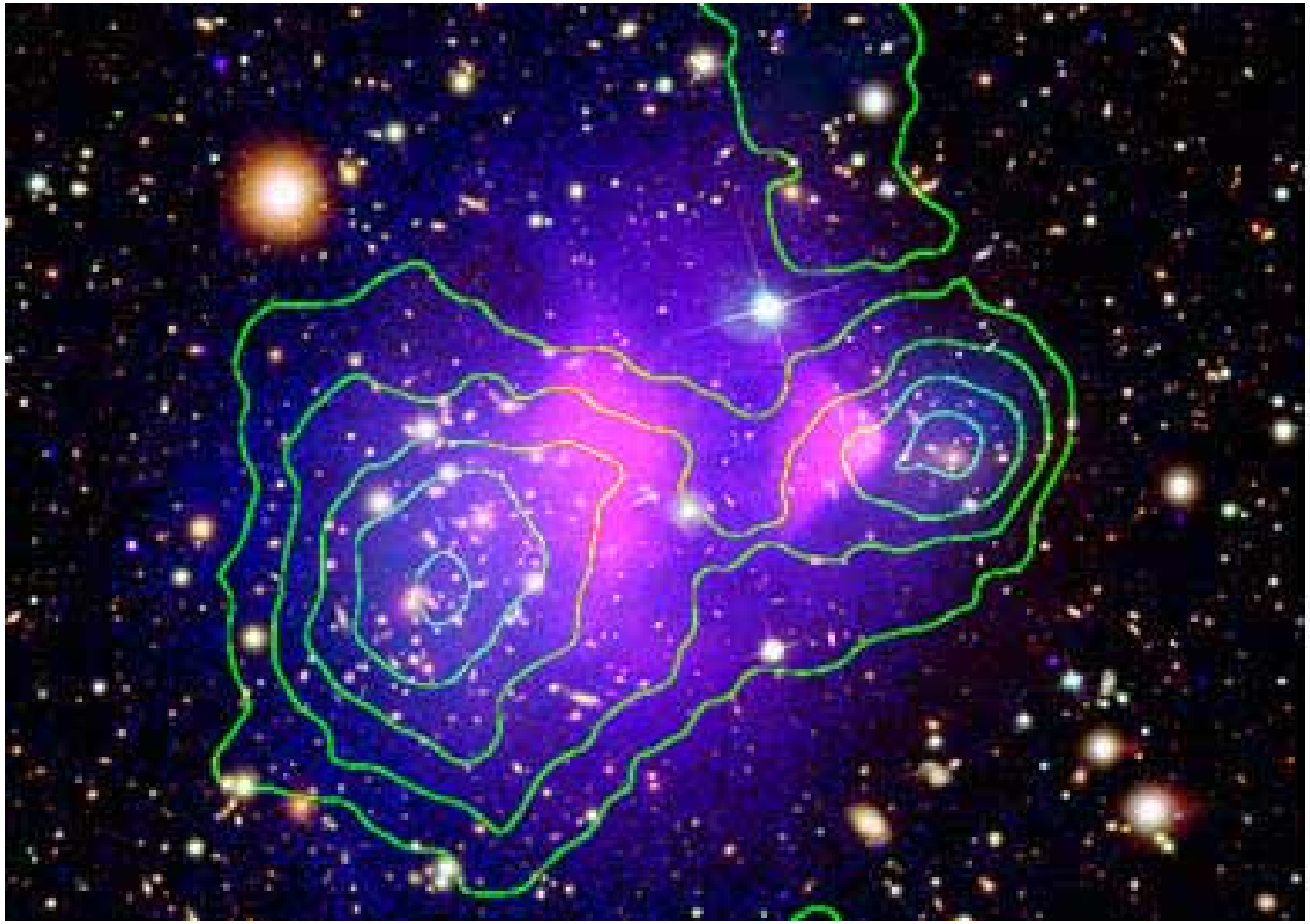
Coma Cluster of galaxies

Image courtesy of U. Briel, MPE Garching, Germany

European Space Agency

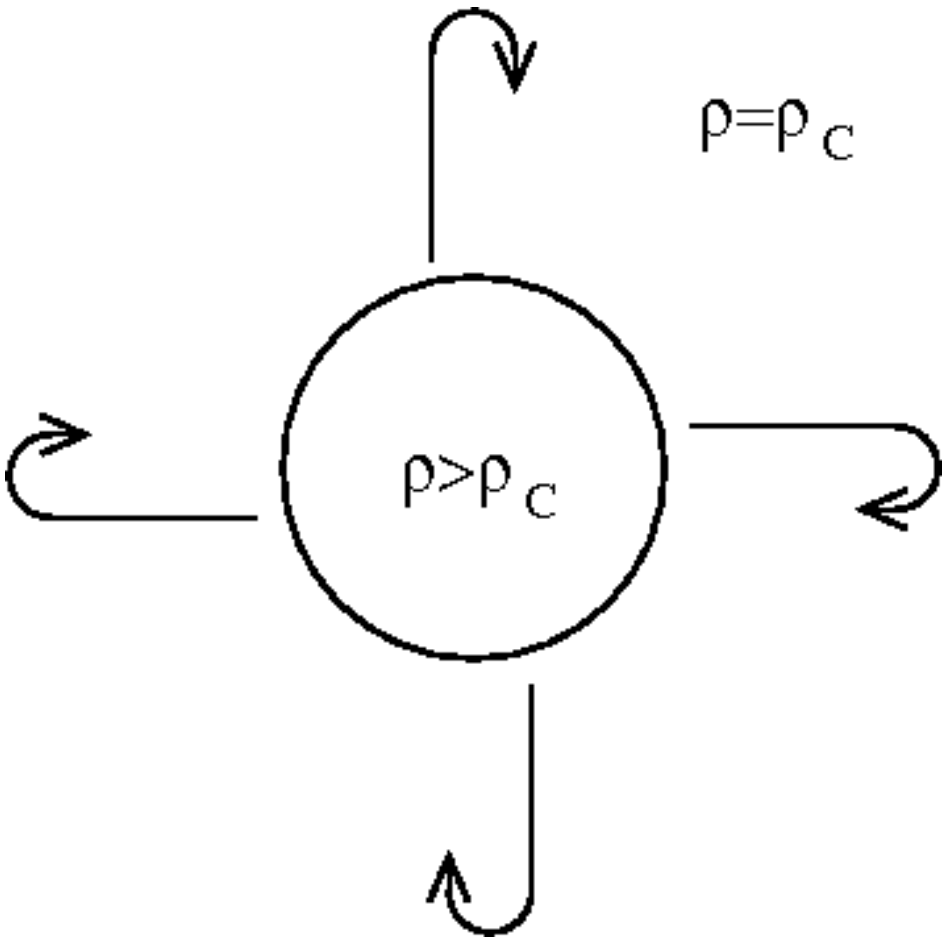


Bullet Cluster: dark matter, hot gas, galaxies



Gas separated from dark matter and galaxies in collision between two clusters.

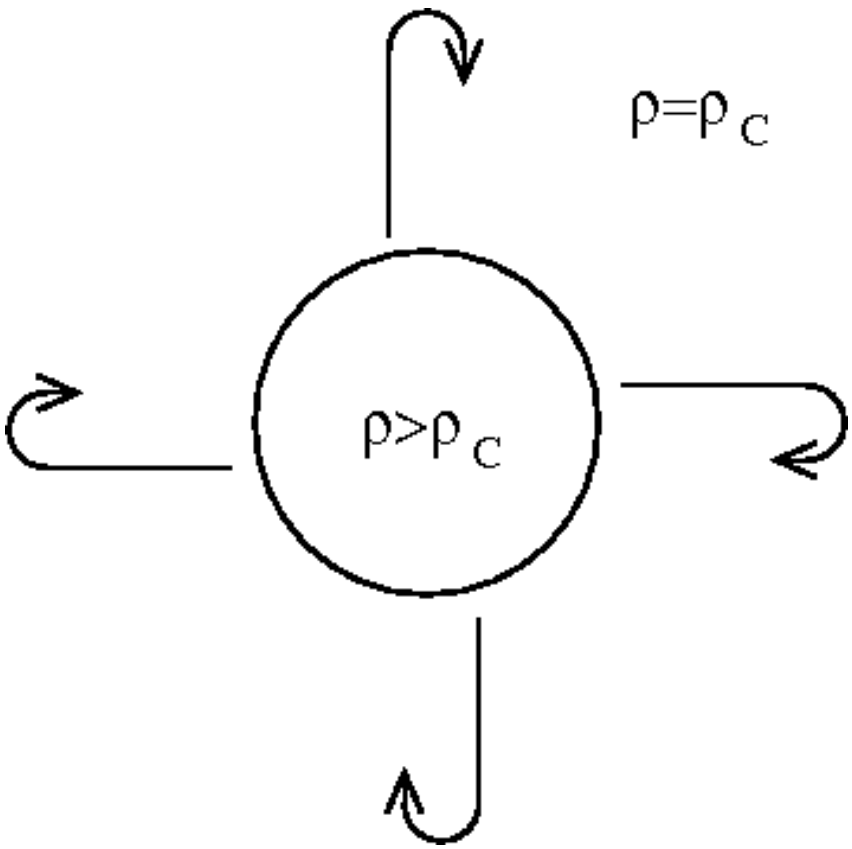
Spherical collapse model



A critical matter-only universe with a small spherical expanding region with $\rho > \rho_c$.

Overdense region acts like a mini-closed universe:
Gravity excess stops the its expansion starting a contraction phase

Spherical collapse model



Spherical symmetry $\Rightarrow$ dynamics of $R(t)$ independent of rest of universe. Conservation of energy of test particle a boundary:

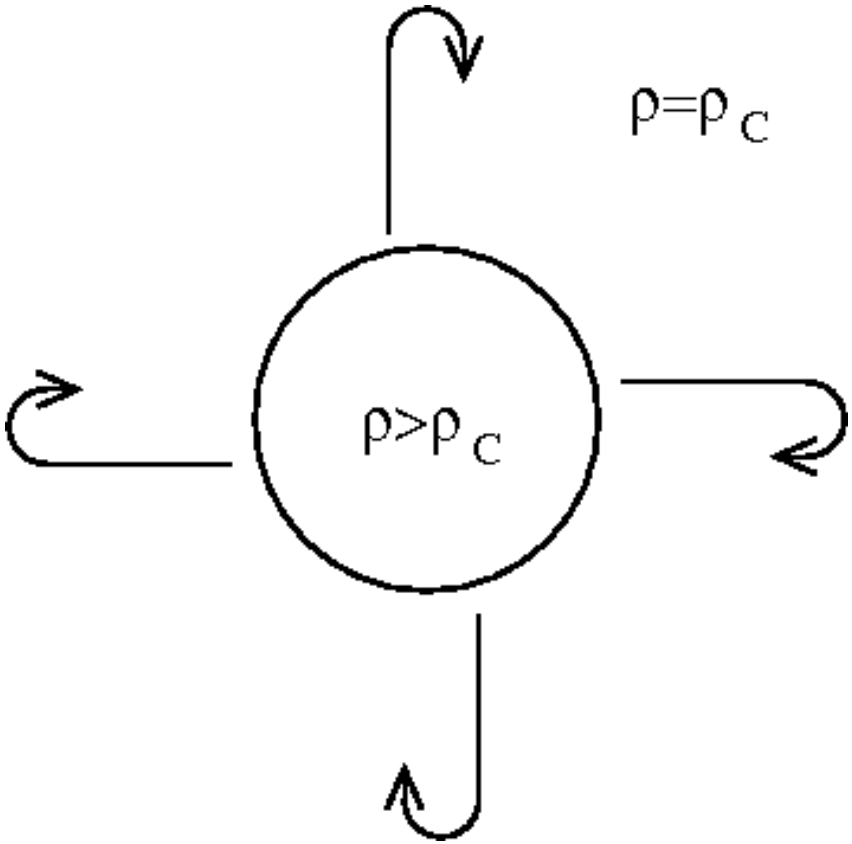
$$(1/2)\dot{R}^2 - \frac{GM}{R} = -\frac{GM}{R_{max}}$$

$M =$ mass contained in spherical region (time-independent)

$$dt = \frac{dR}{\sqrt{2\Phi_g}\sqrt{R_{max}/R - 1}}$$

Model characterized by R_{max} and $\Phi_g = GM/R_{max} <\sim 10^{-5}$)

Spherical collapse model: small time behavior



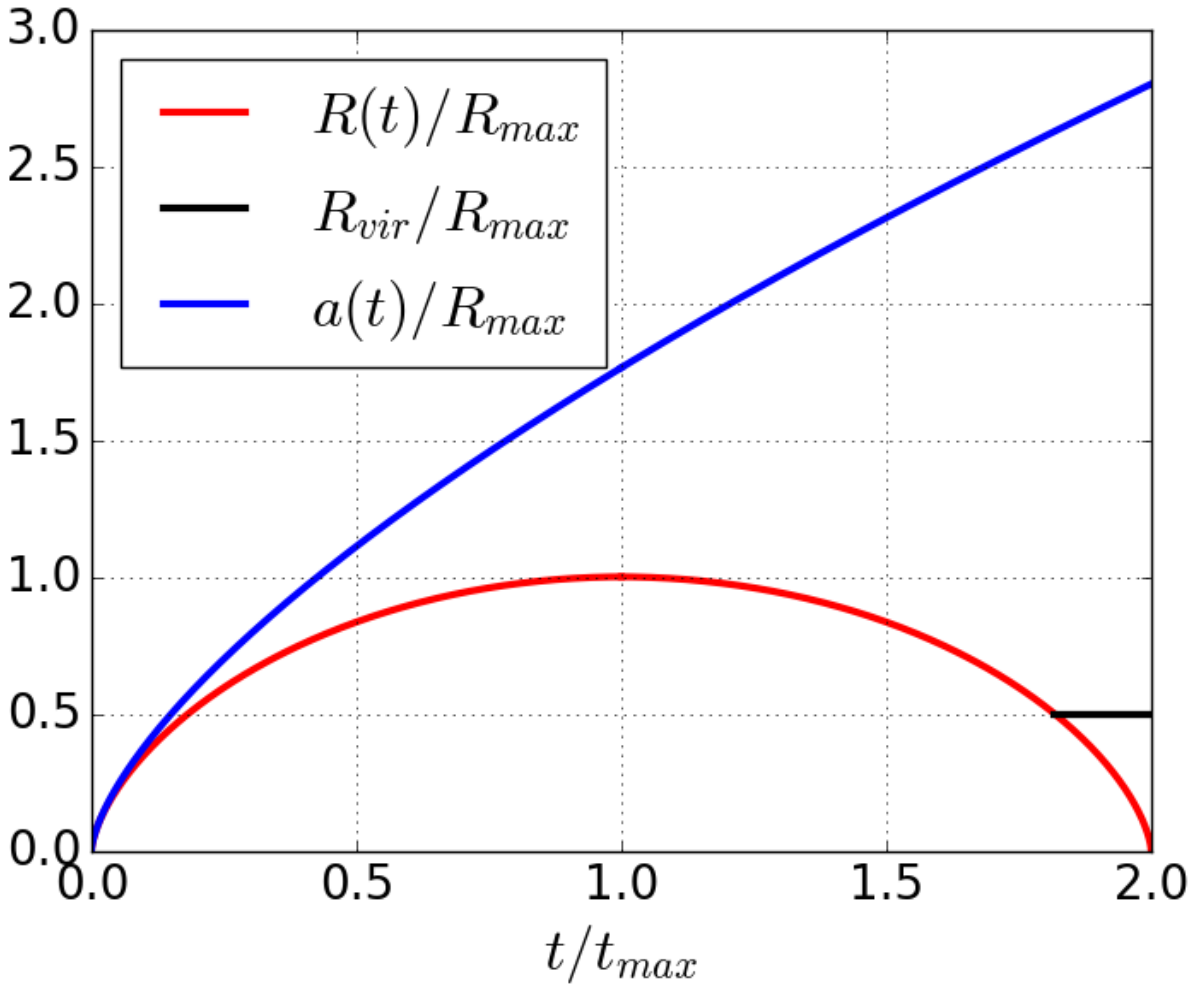
$$dt = \frac{dR}{\sqrt{2\Phi_g}\sqrt{R_{max}/R - 1}}$$

$$t \to 0 \Rightarrow R \ll R_{max}$$

$$R(t) \approx \left(\frac{9\Phi_g R_{max}}{2}\right)^{1/3} t^{2/3}$$

$a(t)$ also $\propto t^{2/3}$

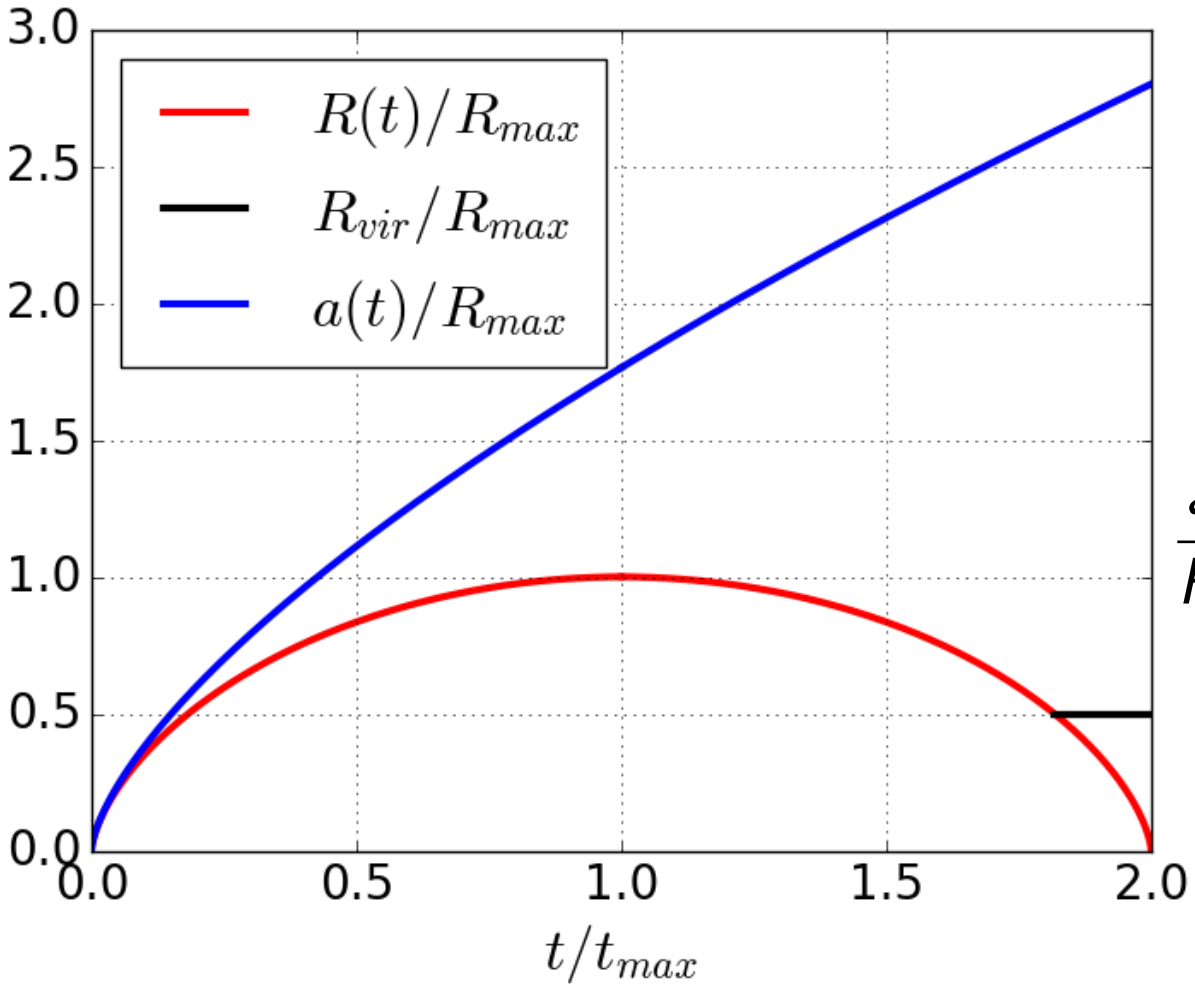
$R(t)$: expansion, collapse, virialization



$$t_{max} = \frac{R_{max}}{\sqrt{2\Phi_g}} \int_0^1 \frac{dx}{\sqrt{x^{-1} - 1}}$$

$\Rightarrow$ Small objects (small R_{max}) form before large objects.
("bottom-up" structure formation)

Density contrast at t_{max}



$a(t) \propto t^{2/3}$

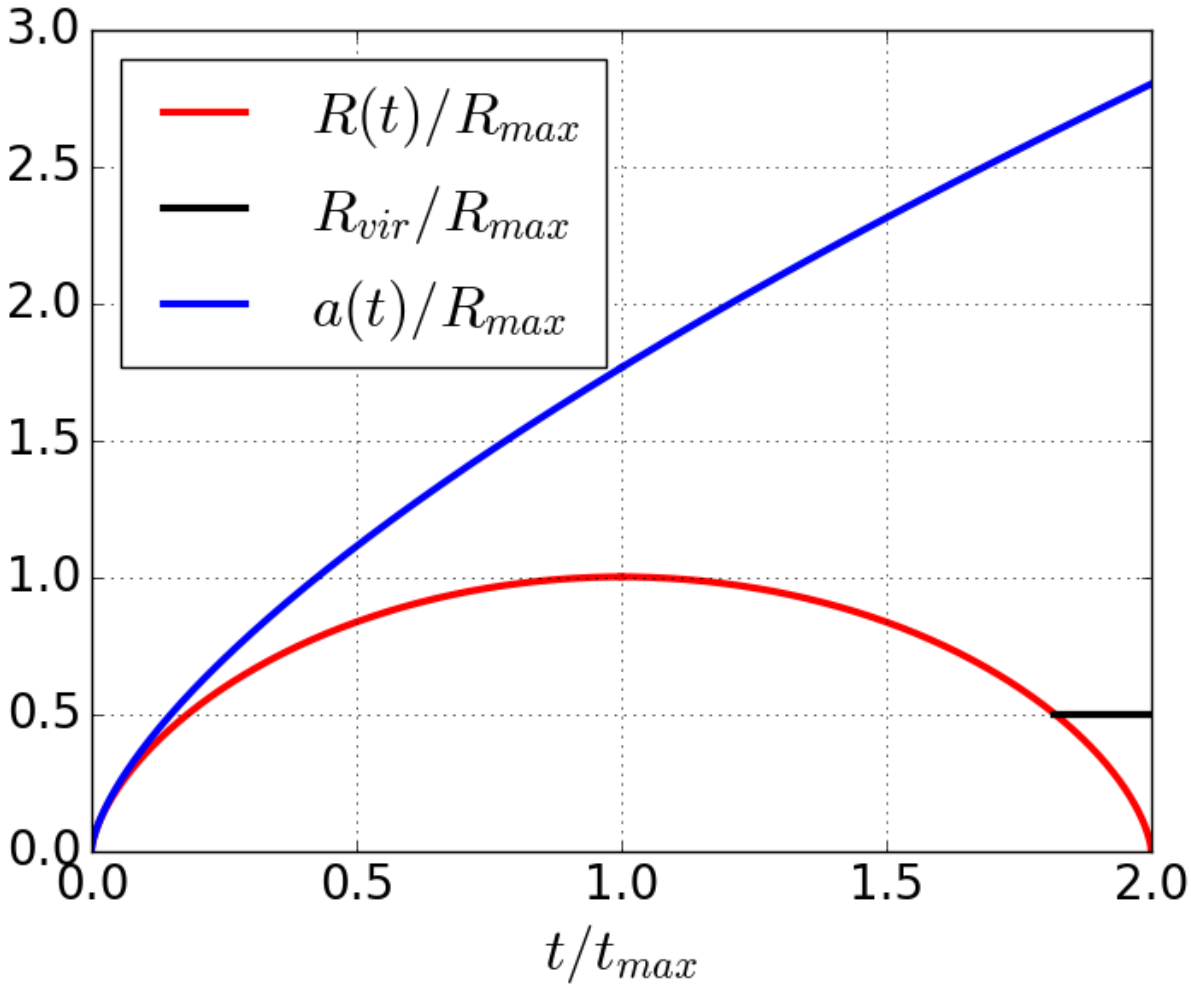
Normalization:

$a(t) = R(t)$ for $t \to 0$

$$\frac{a(t_{max})}{R(t_{max})} = \left(\frac{3\pi}{4}\right)^{2/3} \approx 1.8$$

$$\frac{\rho}{\bar{\rho}}(t_{max}) = \frac{9\pi^2}{16} \sim 5.5$$

$R(t_{max})$ is small compared to Hubble distance



$a(t) \propto t^{2/3}$

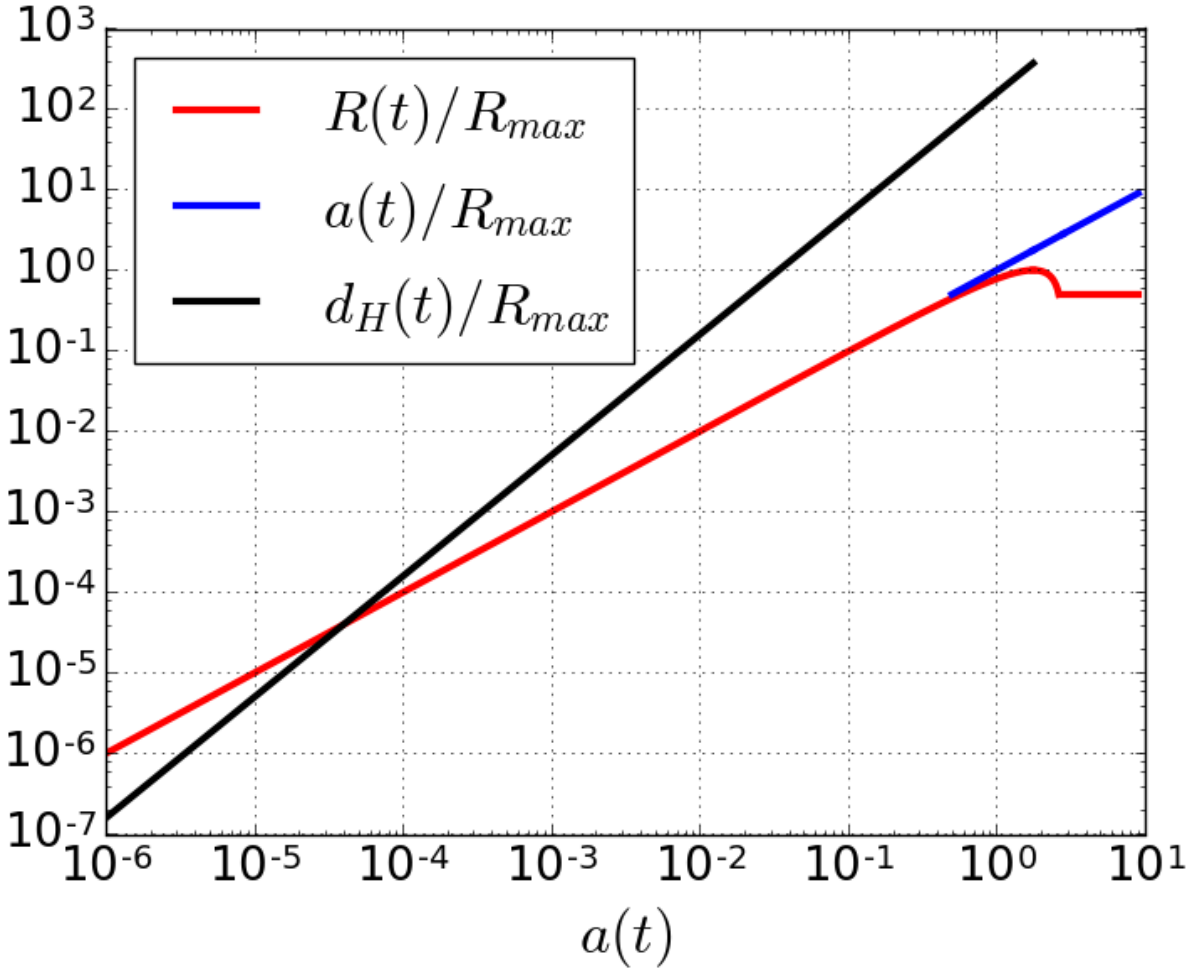
Normalization:

$a(t) = R(t)$ for $t \to 0$

$\Rightarrow \ \dot{a}/a = 2/3t$

$$\frac{c/H(t_{max})}{R(t_{max})} \approx \frac{1}{\sqrt{\Phi_g}}$$

$R(a)$: Hubble entry

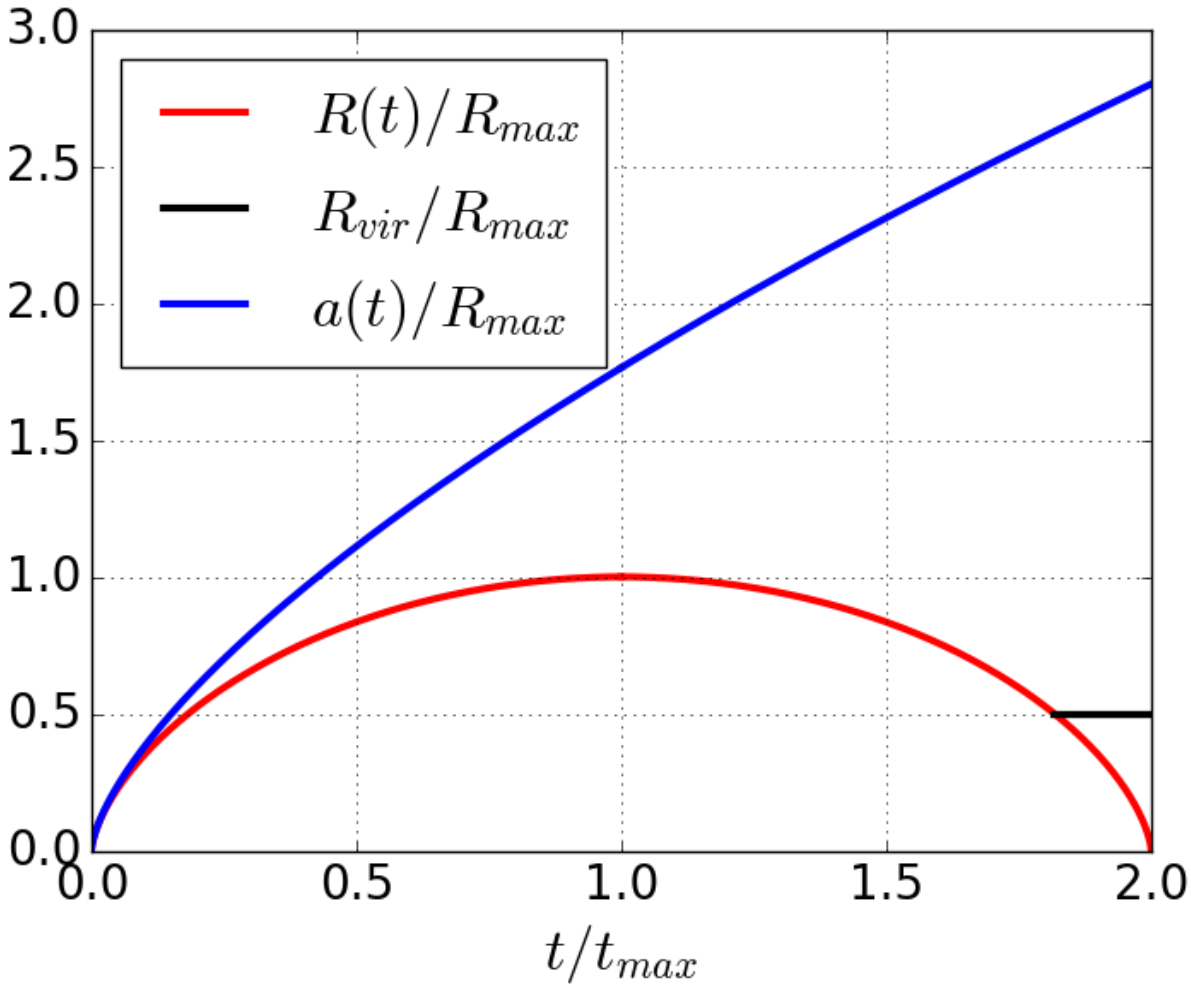


$$R(t) \propto (\Phi_g R_{max})^{1/3} t^{2/3}$$

$$d_H(t) \propto t$$

$$R_{enter} \sim \Phi_g R_{max}$$

Gravitational potential



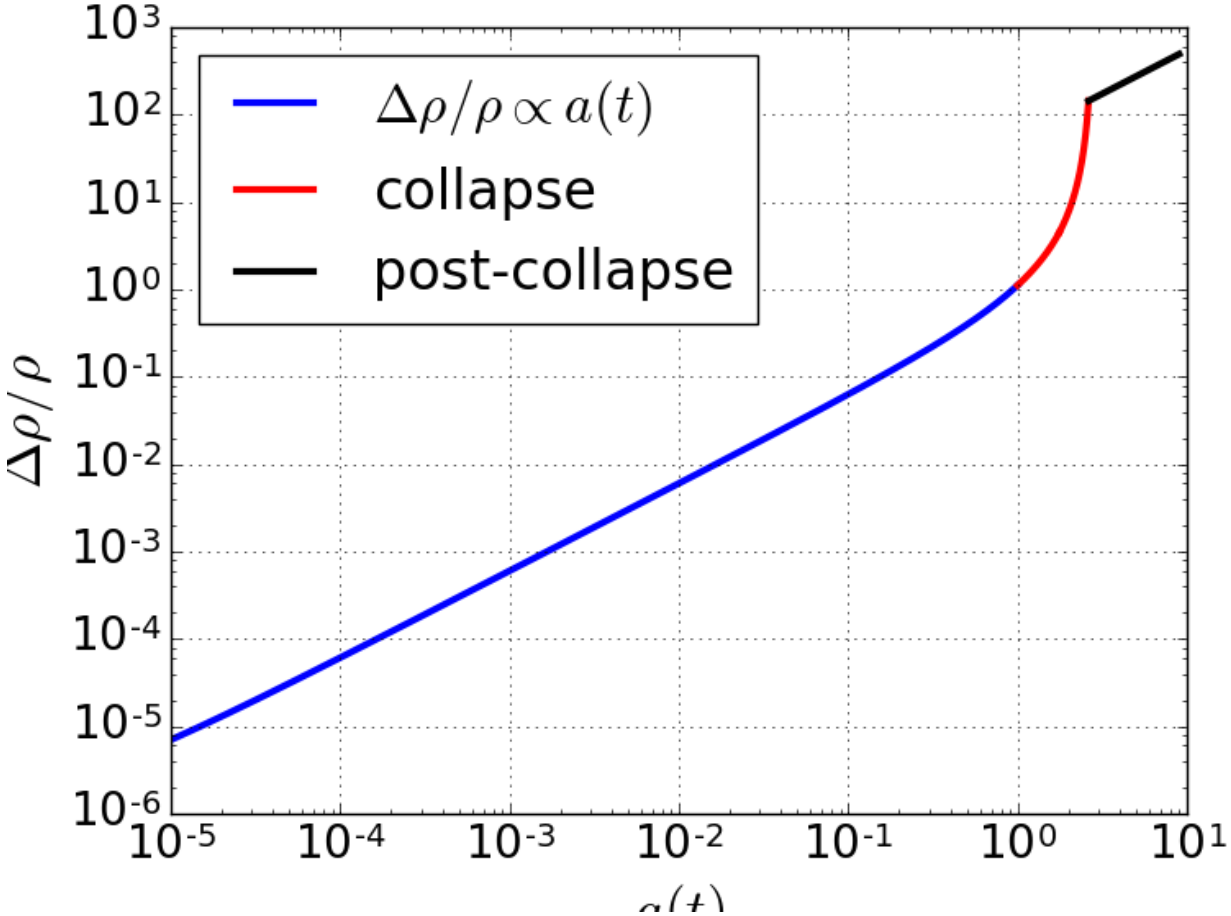
Potential fluctuation at $t \ll t_{max}$ equals potential at maximum expansion:

$$\frac{G(4\pi R^3/3)\Delta\rho}{R(t)} = \frac{GM}{R_{max}}$$

and ≈depth of virialized potential well:

$$\frac{GM}{R_{vir}} \approx 2\frac{GM}{R_{max}}$$

$\Delta\rho/\bar{\rho} = (\rho - \bar{\rho})/\bar{\rho}$



Pre-collapse:

$$\frac{\Delta\rho}{\rho} \propto a(t)$$

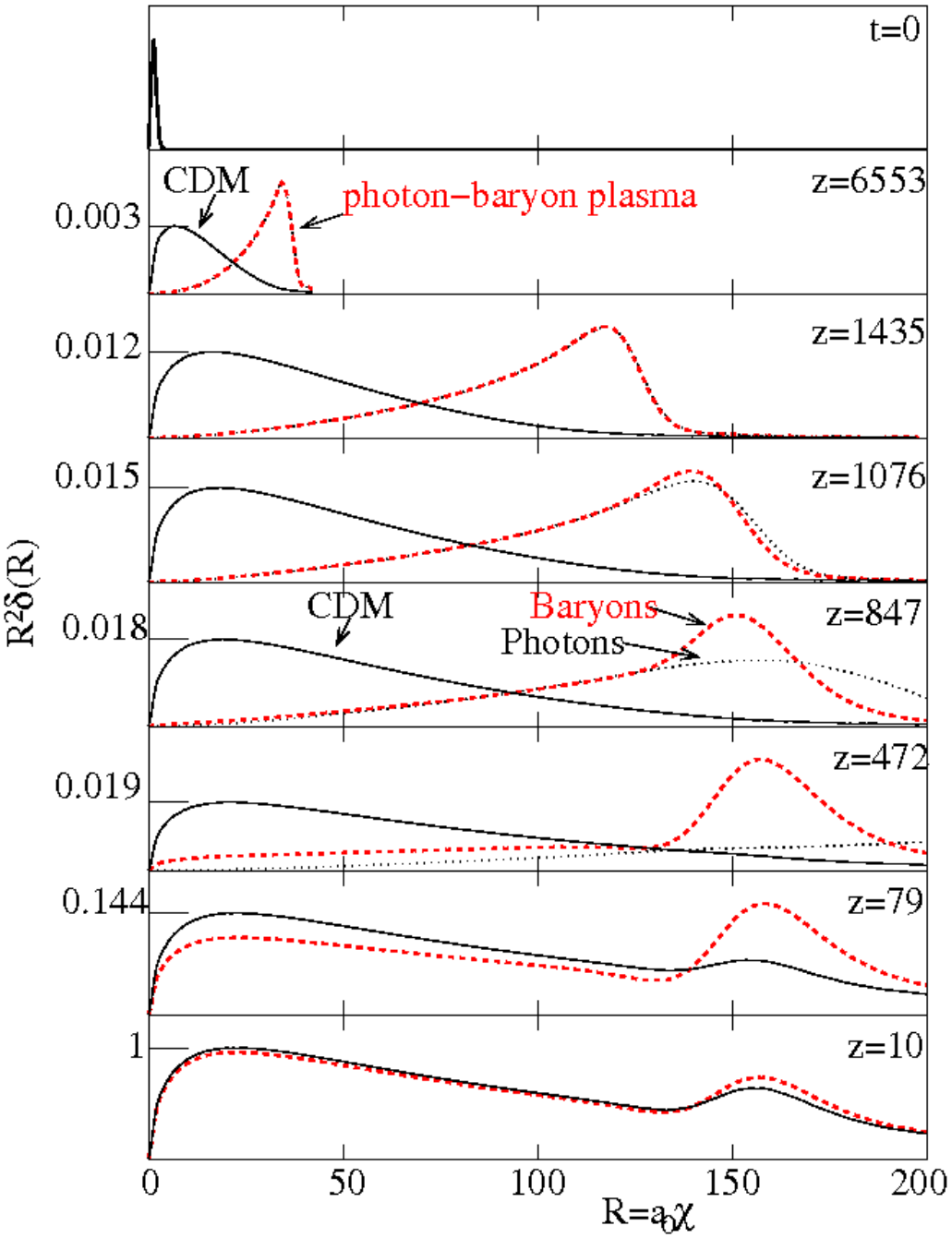
$$\Delta\Phi \quad \propto \bar{\rho} R^2 \frac{\Delta\rho}{\rho}$$

$$\propto a^{-3} \times a^{2} \times a^{1}$$

(time independent)

Summary of spherical-collapse model

- Small fluctuations lead to bound objects only if $\Omega_M \sim 1$
  If $\Omega_M < 1$ small fluctuations insufficient to give $\rho > \rho_c$
  If $\Omega_M > 1$ the whole universe collapses.
  $\Omega_M \sim 1$ for $3 \times 10^{-4} < a/a_0 < 0.5$ (our universe).
- During matter epoch ($\Omega_M \sim 1$):
  $\Delta\rho/\rho \propto a(t)$
  $\Delta\Phi_g \propto \bar{\rho} \times (\Delta\rho/\rho) \times R^2$ is time independent
- Baryon perturbations escape over-dense region with sound wave.
  $\Rightarrow$Baryon perturbations do not grow until recombination.



An initial over-density:

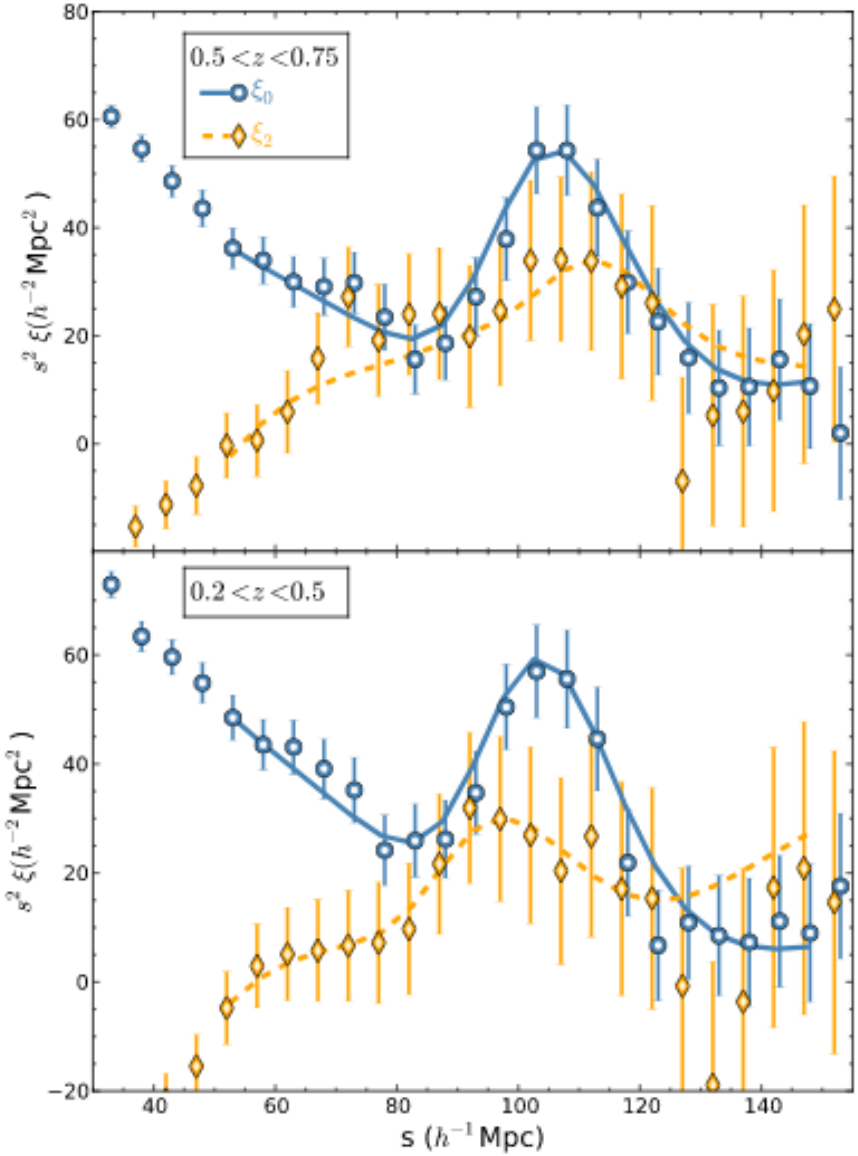
$t = 0$

$c_s \sim c/\sqrt(3)$
(γ,p,e plasma)

$c_s \to 0$ at recombination
($r \sim 150\mathrm{kpc}$)

Today: Enhanced correlation
at $r = 147.5$Mpc

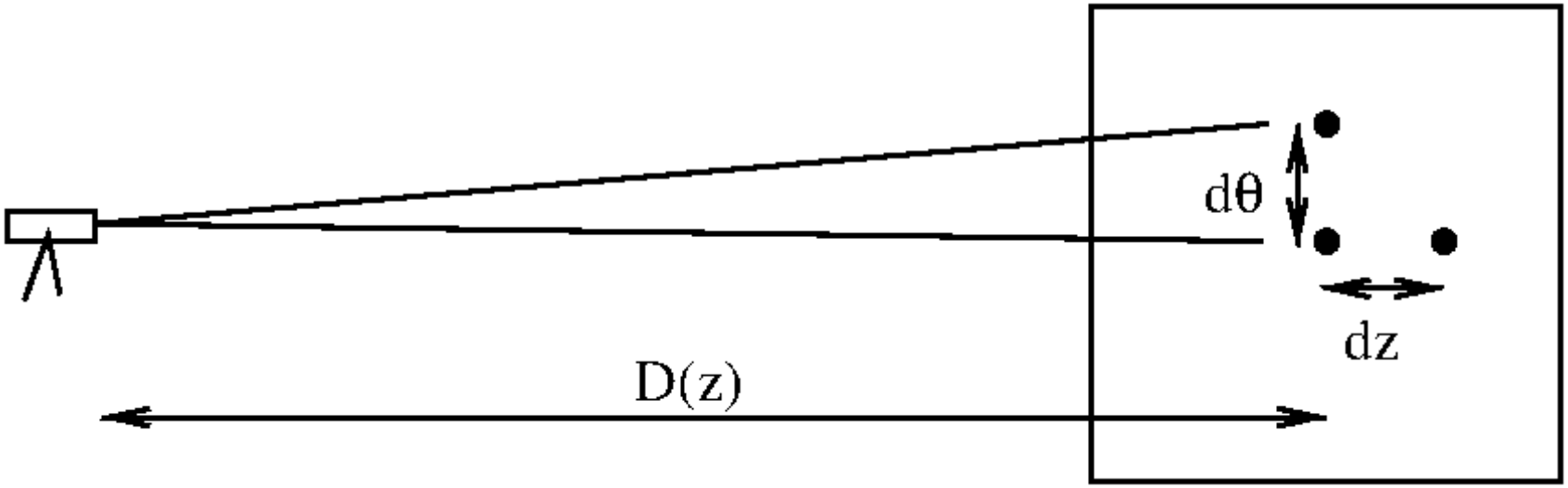
BAO Peak in galaxy-galaxy correlation function



Galaxy-galaxy correlation function at two redshifts

Baryon Oscillation Spectroscopy Survey

BAO Peak $\Rightarrow D_M(z)/r_d$ and $(c/H(z))/r_d$



Galaxy positions are found in (z, θ, ϕ) space, For an ensemble of galaxies near redshift z, the BAO peak in the correlation function in the radial and transverse directions are

$$\Delta z_{BAO} = \frac{r_d}{c/H(z)} \qquad |\Delta\vec{\theta}_{BAO}| = \frac{r_d}{D_M(z)}$$

The measured values of Δz_{BAO} and $|\Delta\vec{\theta}_{BAO}|$ determines $D_M(z)/r_d$ and $(c/H(z))/r_d$.

ΛCDM parameters from BAO

Expansion rate and Hubble distance:

$$H(z) \; = c/d_H(z) \; = H_0 \left[\Omega_\Lambda \; + \Omega_M(1+z)^3 \; + \Omega_k(1+z)^2 + ...\right]^{1/2}$$

where $\Omega_k = 1 - \Omega_M - \Omega_\Lambda$.

Distance to z:

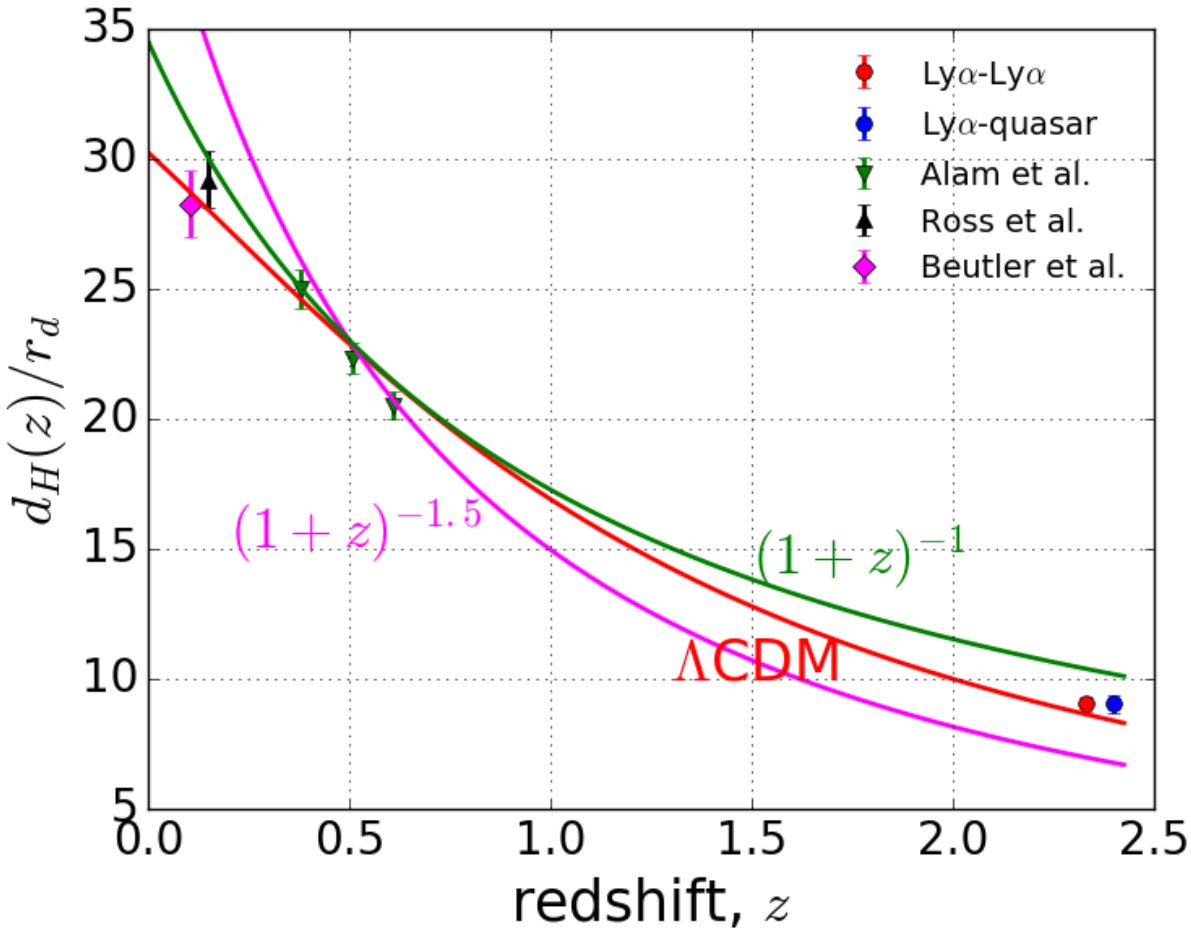
$$d(z) \; = \; \int_0^z d_H(z)dz$$

Angular diameter distance to z:

$$d_M(z) = d_c S(d(z)/d_c) \qquad d_c = \frac{c/H_0}{\sqrt{|\Omega_k|}} \qquad S = \begin{matrix} \sin \\ \sinh \end{matrix} \text{ for } \begin{matrix} \Omega_k < 0 \\ \Omega_k > 0 \end{matrix}$$

$\Rightarrow d_M(z)/r_d$ and $d_H(z)/r_d$ are functions of z and $(\Omega_M, \Omega_\Lambda, r_d H_0)$

$d_H(z)/r_d$ vs. z



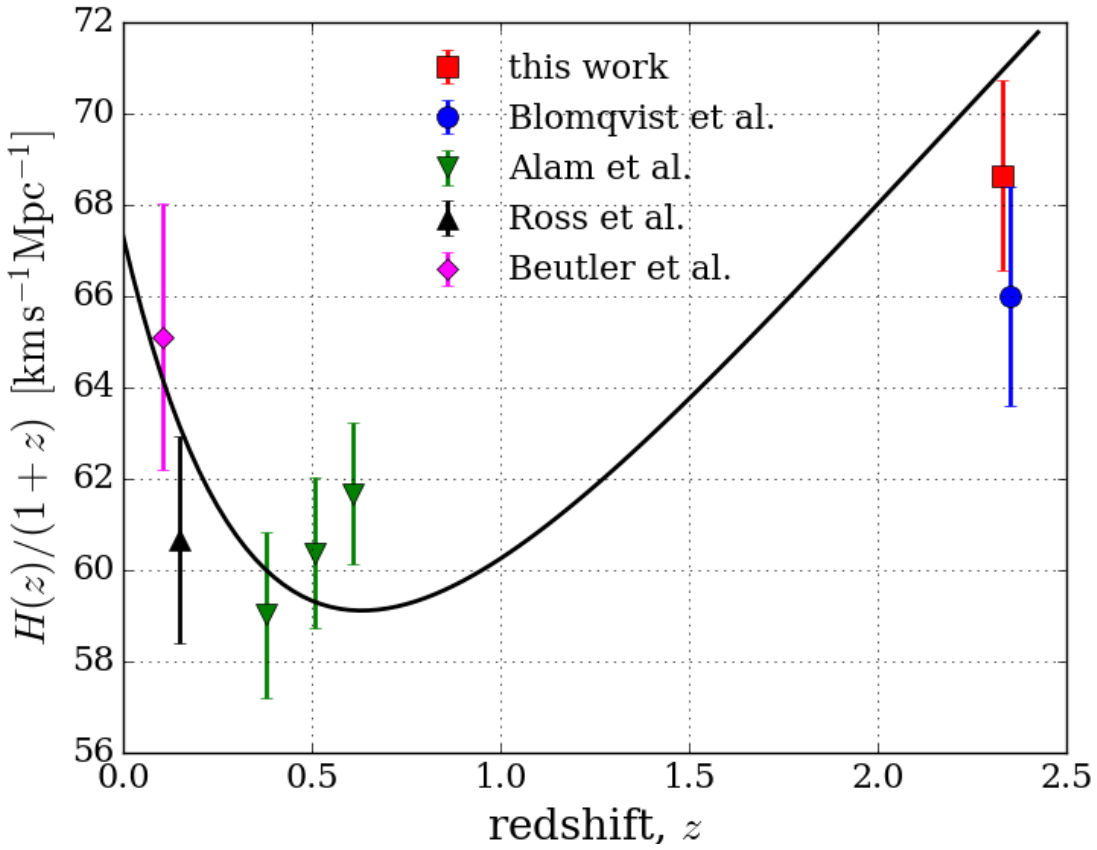
Models:

standard ΛCDM

$(\Omega_M, \Omega_\Lambda) = (1, 0)$

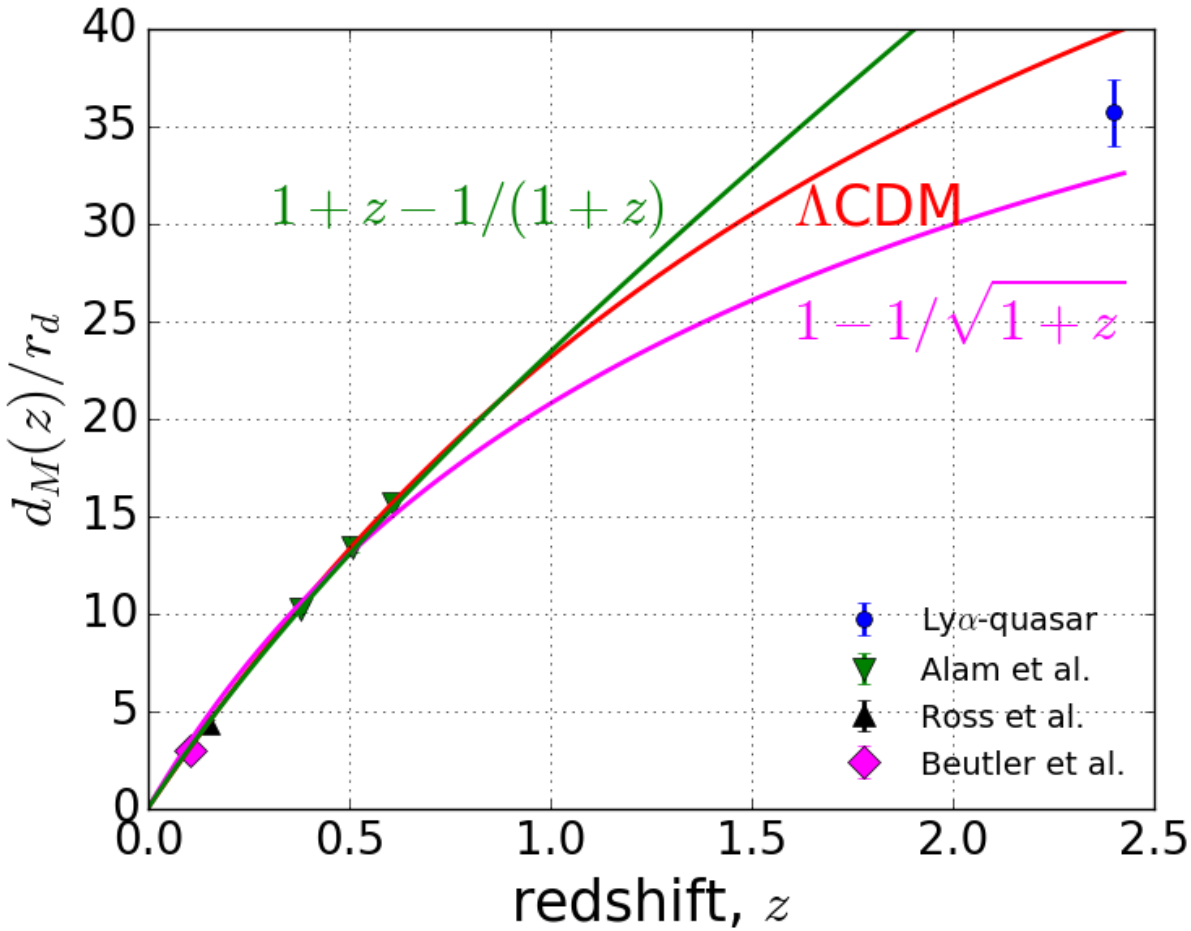
$(\Omega_M, \Omega_\Lambda) = (0, 0)$

$\dot{a} \propto H(z)/(1+z)$ vs. z



deceleration: $z > 0.6$
acceleration: $z < 0.6$

$d_M(z)/r_d$ vs. z



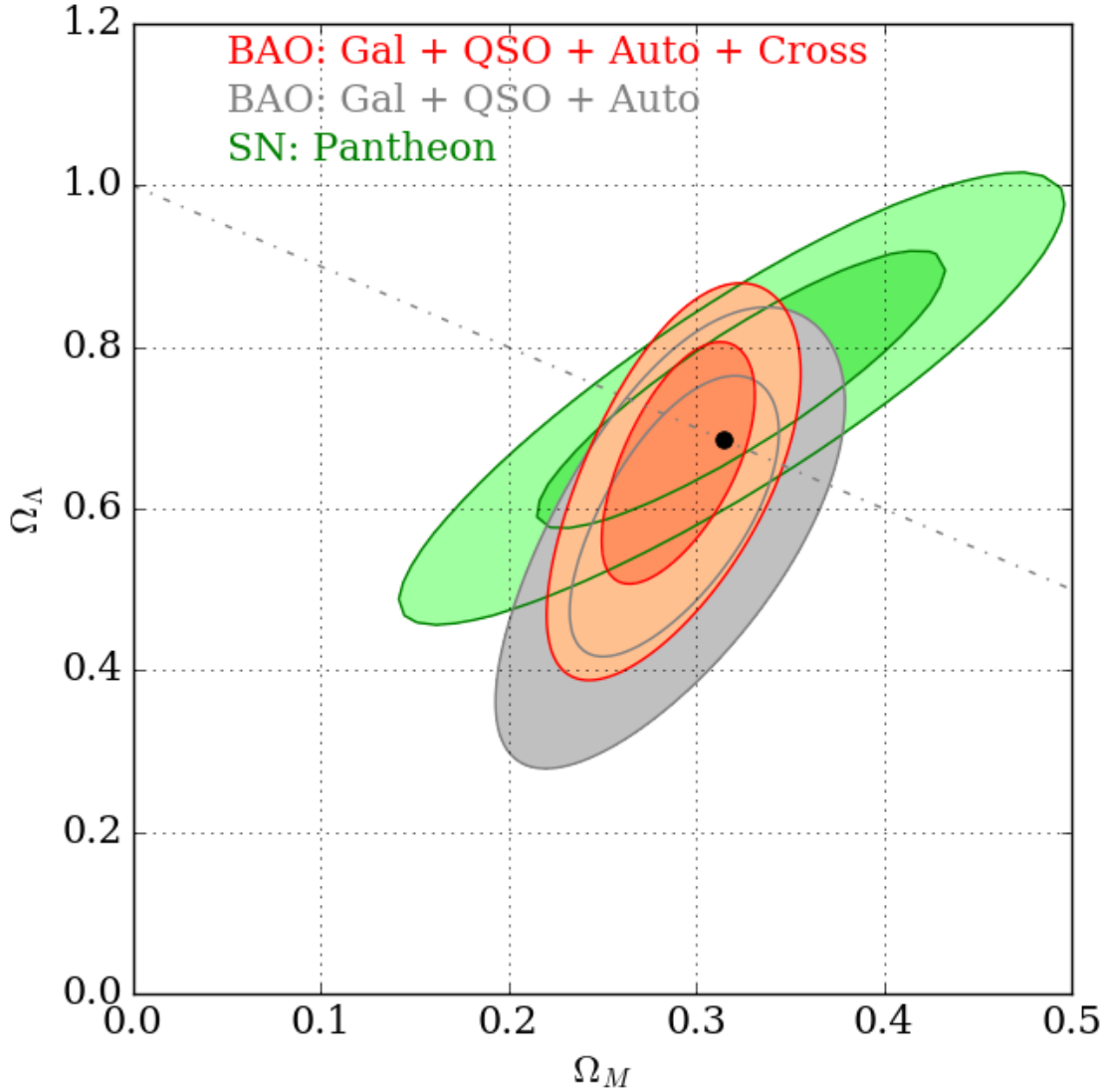
Models:

standard ΛCDM

$(\Omega_M, \Omega_\Lambda) = (1, 0)$

$(\Omega_M, \Omega_\Lambda) = (0, 0)$

BAO and SNIa constraints



BAO results:

$\Omega_M = 0.288 \pm 0.033$

$\Omega_\Lambda = 0.695 \pm 0.115$

$\Omega_k = 0.02 \pm 0.14$

$H_0 r_d = 147.33\mathrm{Mpc} \times (68.5 \pm 1.5)\ \mathrm{km\,s^{-1}Mpc^{-1}}$

(de Sainte Agathe et al, 2019)

Fourier expansion of density in box, $V = L^3$

$$\rho(\vec{r}) = \bar{\rho}\left[1 + \sum_{\vec{k}} \delta_{\vec{k}} \exp(i\vec{k}\cdot\vec{r})\right] \qquad \vec{k} = \frac{2\pi\vec{n}}{L} \qquad \delta_{-\vec{k}} = \delta^*_{\vec{k}}$$

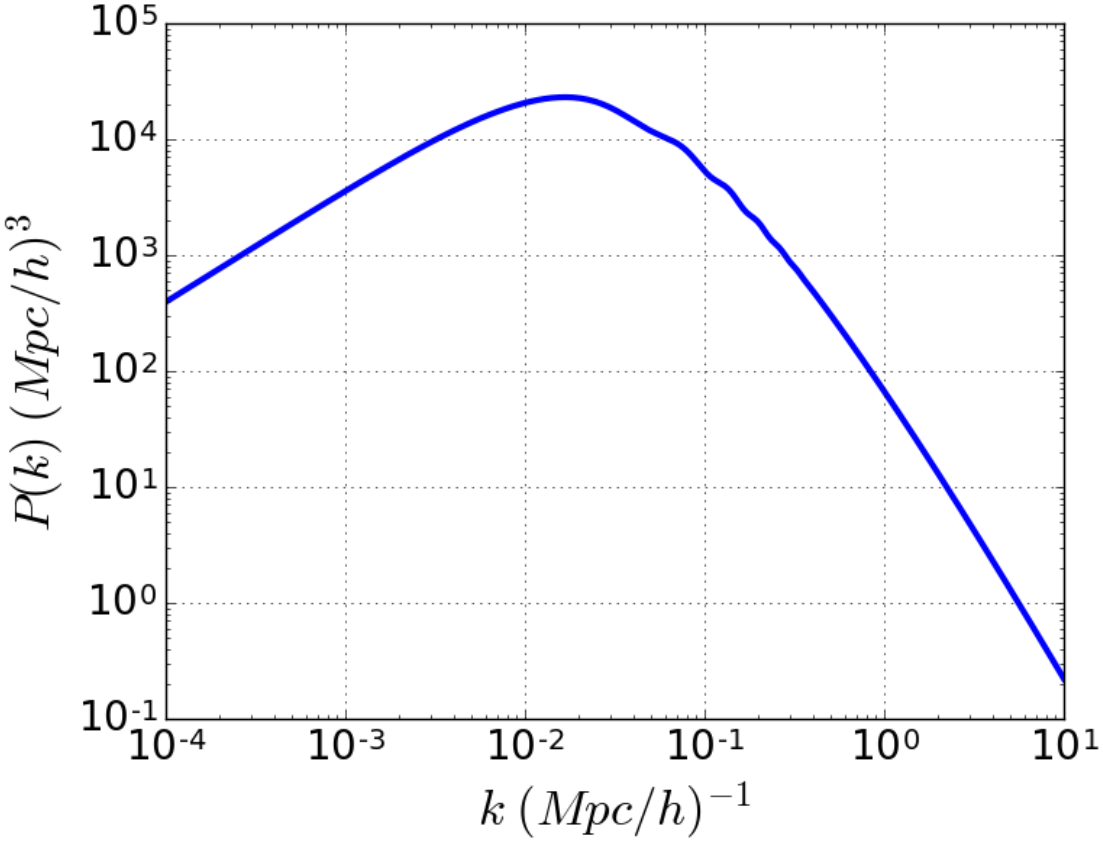
Density dispersion:

$$\frac{\overline{\rho^2} - \bar{\rho}^2}{\bar{\rho}^2} = \sum_{\vec{k}} |\delta_{\vec{k}}|^2 = \int_0^\infty \frac{dk}{k}\frac{k^3 P(k)}{2\pi^2} \qquad P(k) = V\langle|\delta_{\vec{k}}|^2\rangle$$

For now, the expansion is done at a fixed time, t. Later we will add the time dependence of the amplitude of co-moving modes:

$$\delta_{\vec{k}}(t) \qquad \lambda_k(t) = \frac{2\pi}{k}\frac{a(t)}{a_0}$$

Power spectrum in standard ΛCDM

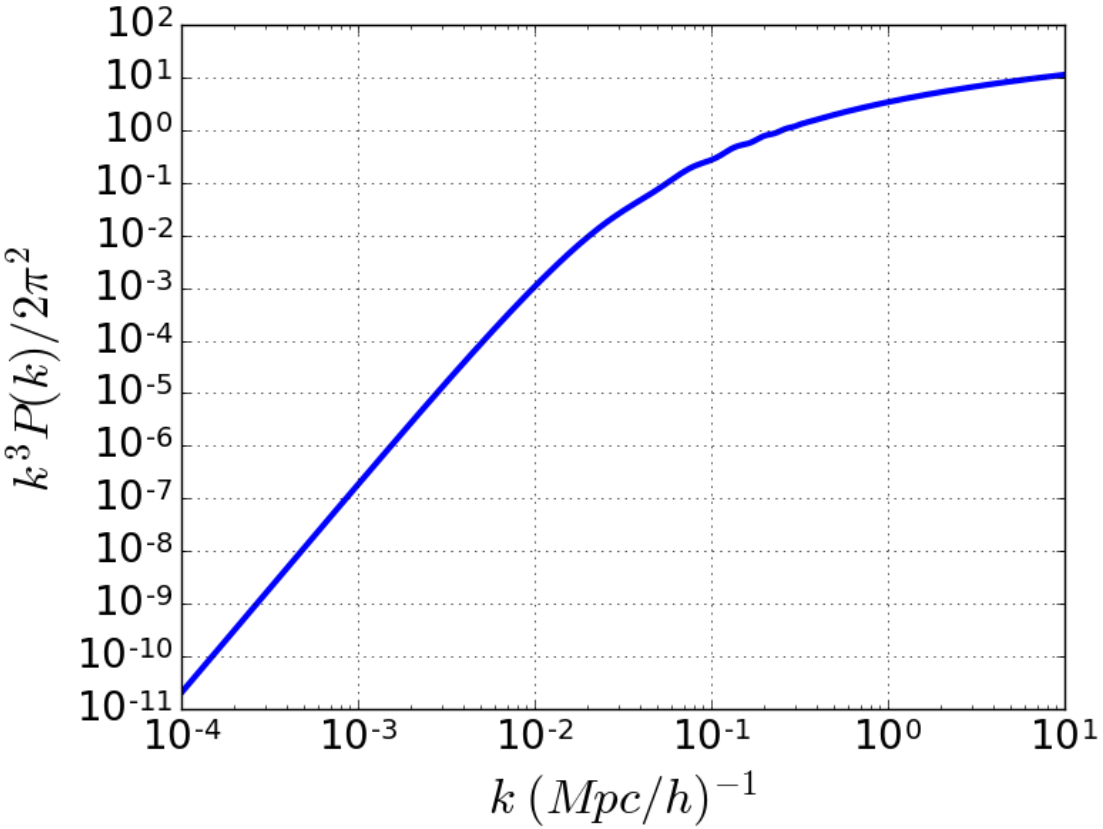


Mean square amplitude maximum at $k \sim 0.02(Mpc/h)^{-1}$

High k modes have small amplitude but there are many of them!

Standard ΛCDM : $(\Omega_{cdm}, \Omega_b) = (0.269,\ 0.0484)$,
$\Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b,\ h = 0.674,\ A_s = 2 \times 10^{-9}$

Density fluctuation vs. scale

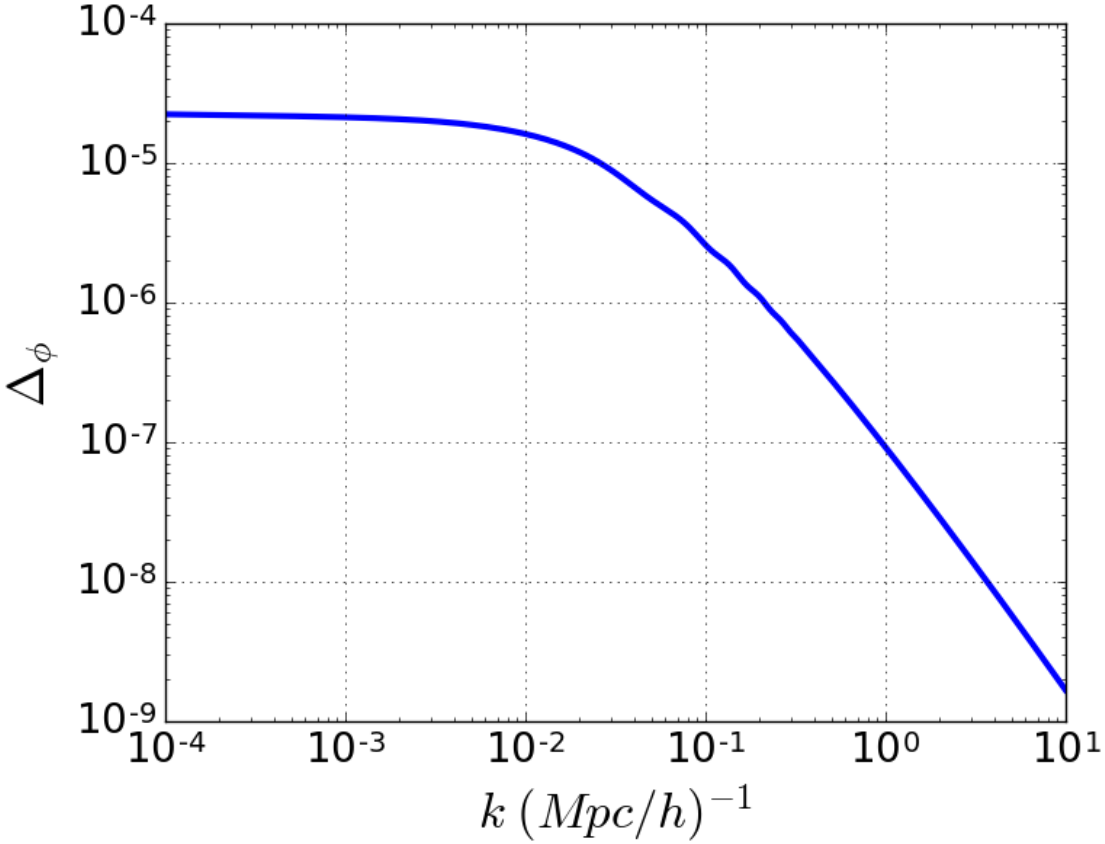


$$\left(\frac{\Delta\rho}{\rho}\right)^2 \sim \int \frac{dk}{k}\frac{k^3 P(k)}{2\pi^2}$$

Universe is $\sim$ homogeneous on scales
$k < 0.1(Mpc/h)^{-1}$
$(\lambda = 2\pi/k > 60 Mpc/h)$

Standard ΛCDM : $(\Omega_{cdm}, \Omega_b) = (0.269,\ 0.0484)$,
$\Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b,\ h = 0.674,\ A_s = 2 \times 10^{-9}$

Gravitational potential fluctuations: $\leq 2 \times 10^{-5}$



Gravitational potential fluctuation:

$$\phi_{\vec{k}} \sim \frac{4\pi G \bar{\rho}}{k^2} \delta_{\vec{k}}$$

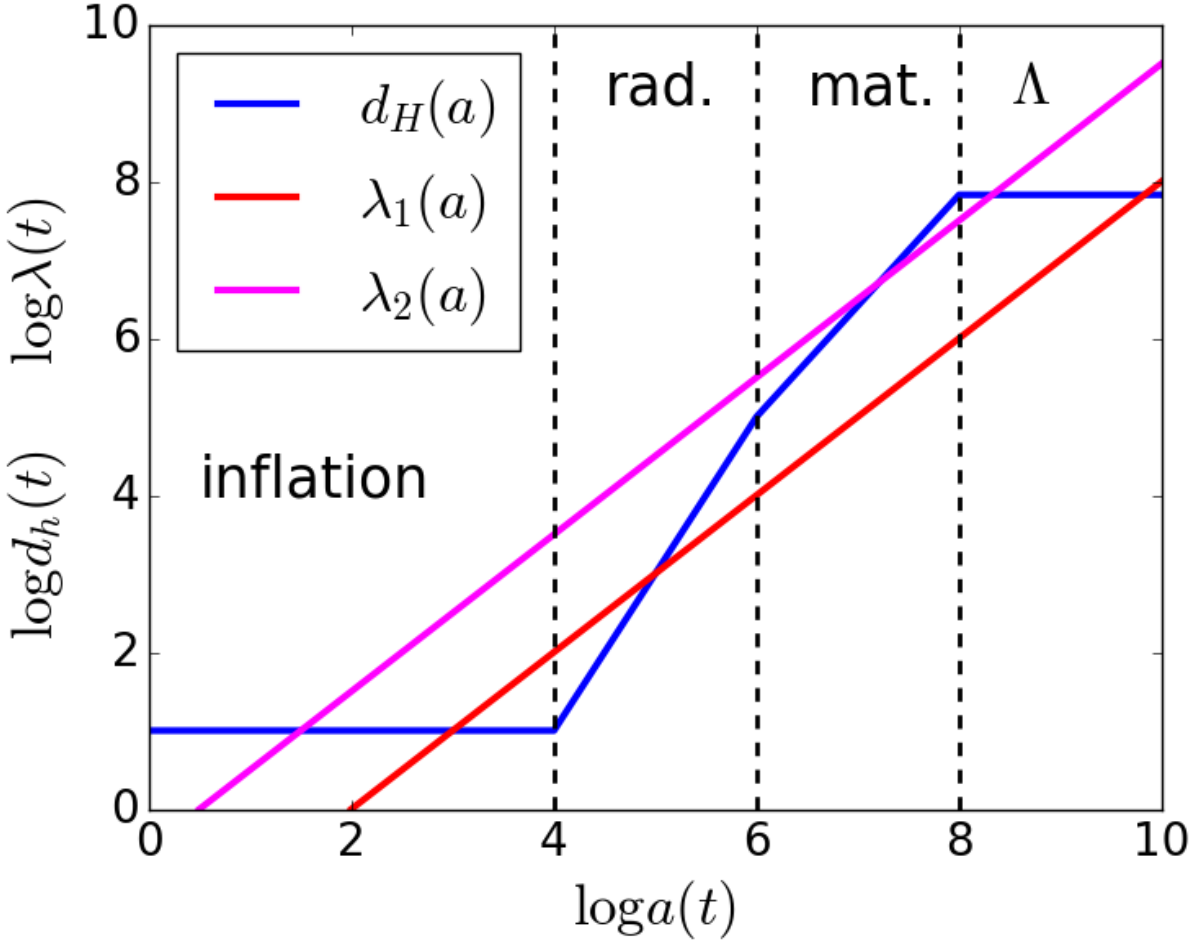
$$\sqrt{V k^3 \langle \phi_{\vec{k}}^2 \rangle} \sim 2 \times 10^{-5}$$

at small k

(Primordial scale-invariant potential fluctuations from inflation.)

Standard ΛCDM : $(\Omega_{cdm}, \Omega_b) = (0.269,\ 0.0484)$,
$\Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b,\ h = 0.674,\ A_s = 2 \times 10^{-9}$

Hubble exit, then Hubble entry



$$d_H(a) = \sqrt{3/8\pi G\rho(a)}$$

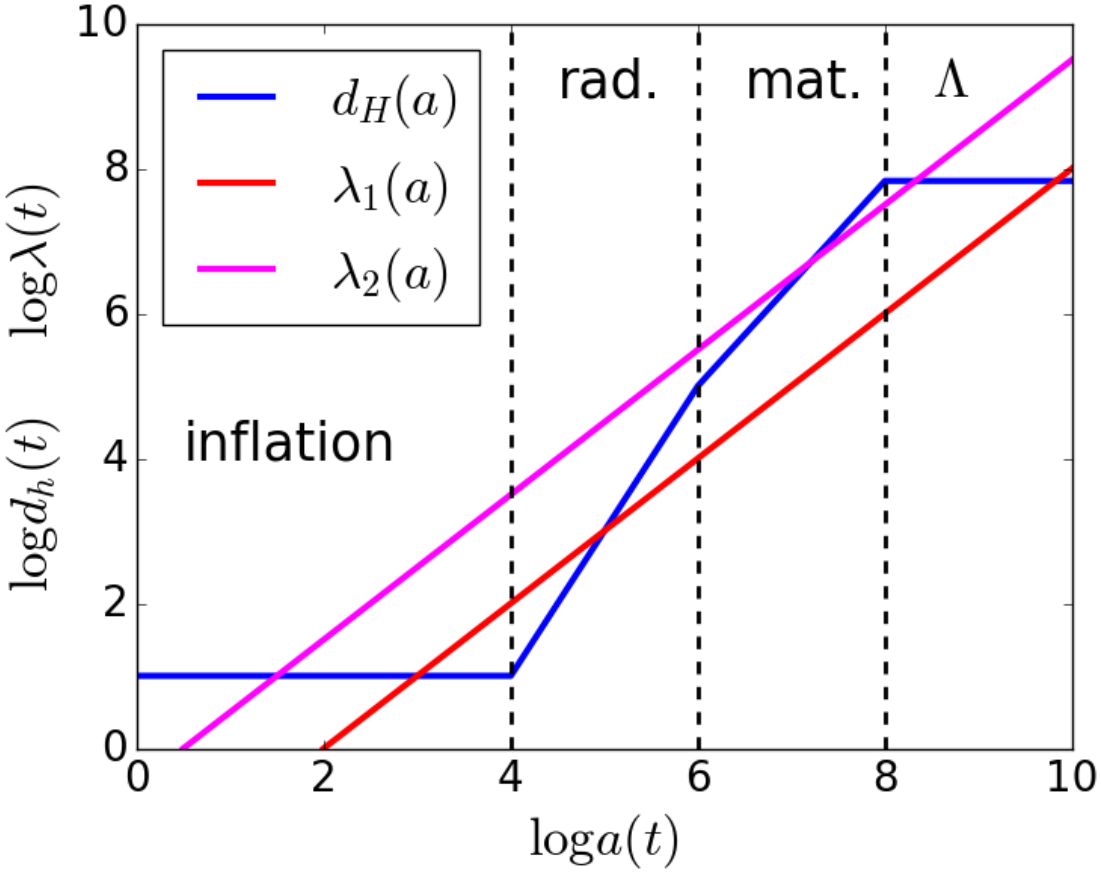
$$\lambda_k(a) = (2\pi/k)a(t)/a_0$$

λ_1 and λ_2 "leave" the Hubble volume during inflation and then "enter".

λ_1 enters during radiation epoch

λ_2 enters during matter epoch

Inflation ⇒ scale-independent fluctuations



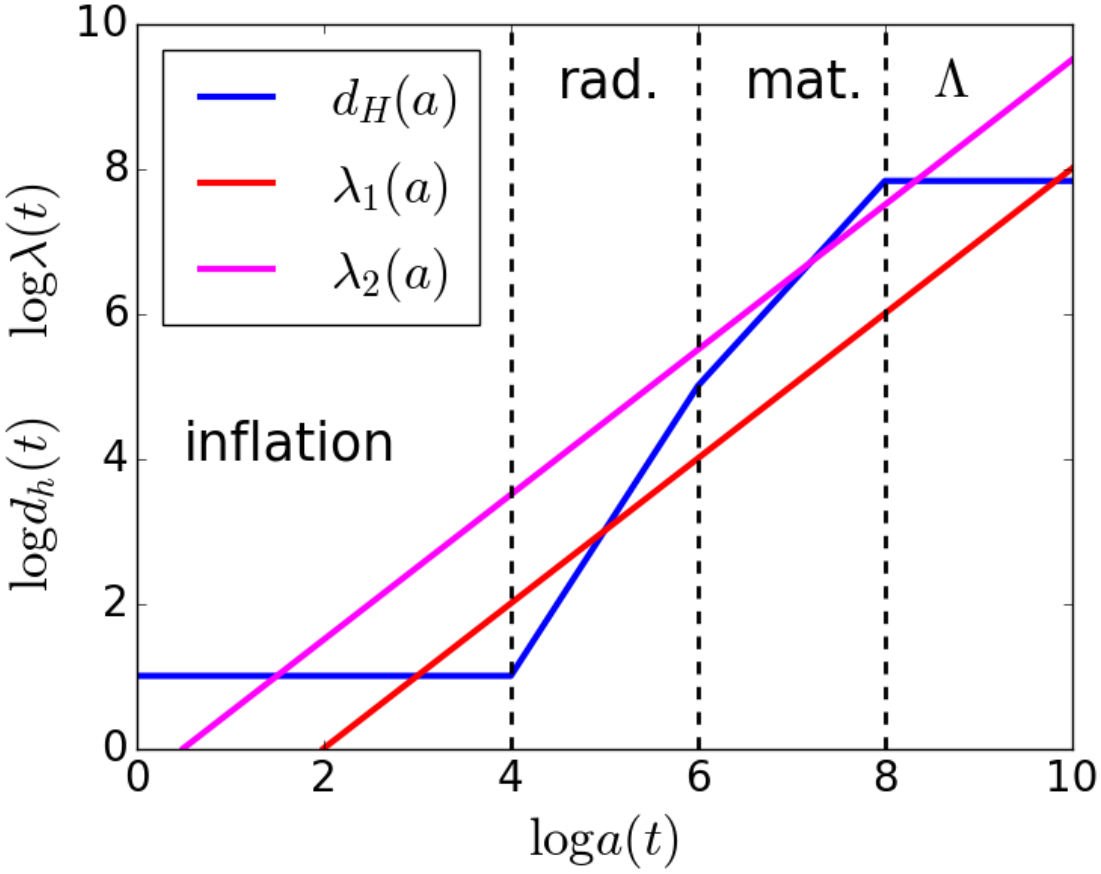
$\rho(a)$ ∼constant during inflation

⇒ fluctuation amplitude scale-independent at Hubble-exit

Super-Hubble dynamics preserves amplitude

⇒ All scales enter Hubble radius with equal amplitude.

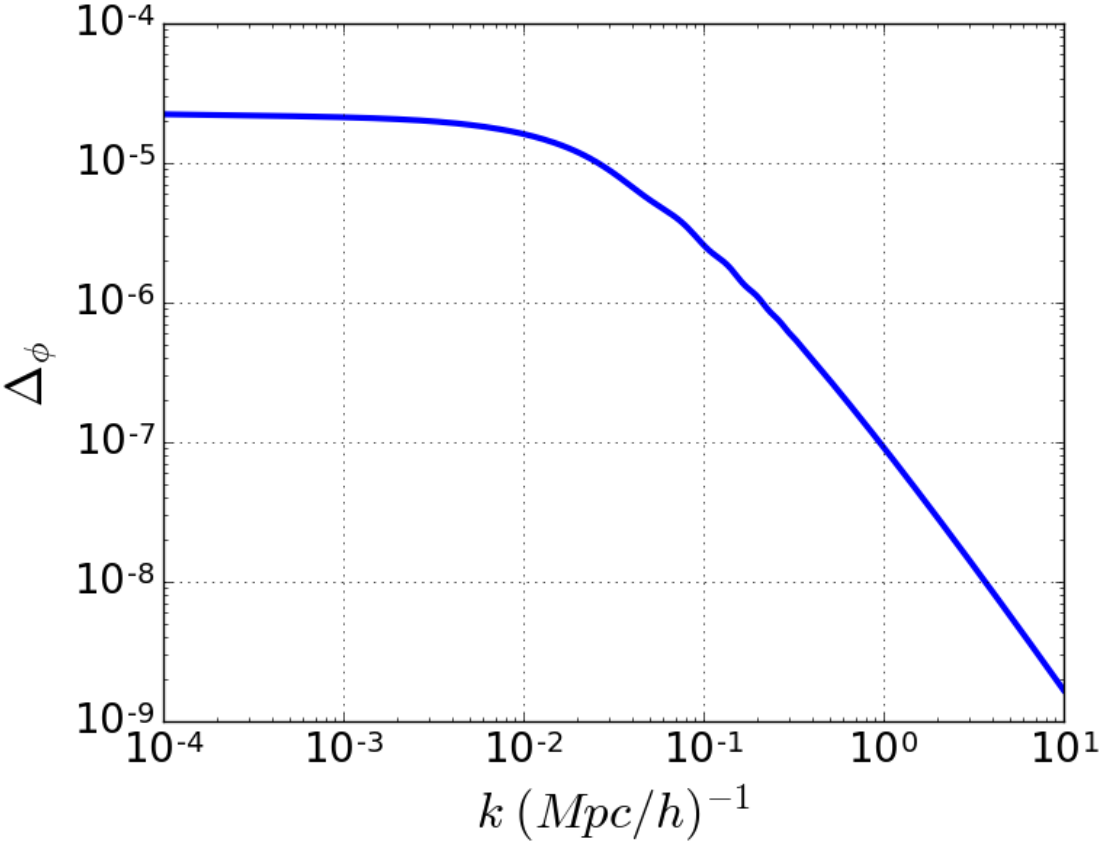
Radiation epoch: potential decay



The gravitational potential of modes that enter during the radiation epoch decays because acoustic oscillation prevents increase of $\Delta\rho/\rho$:

$$\Phi_g \sim G\bar{\rho}\,\frac{\Delta\rho}{\rho}\,\lambda^2 \sim a^{-4} \times a^2$$

Gravitational potential fluctuations: $\leq 2 \times 10^{-5}$



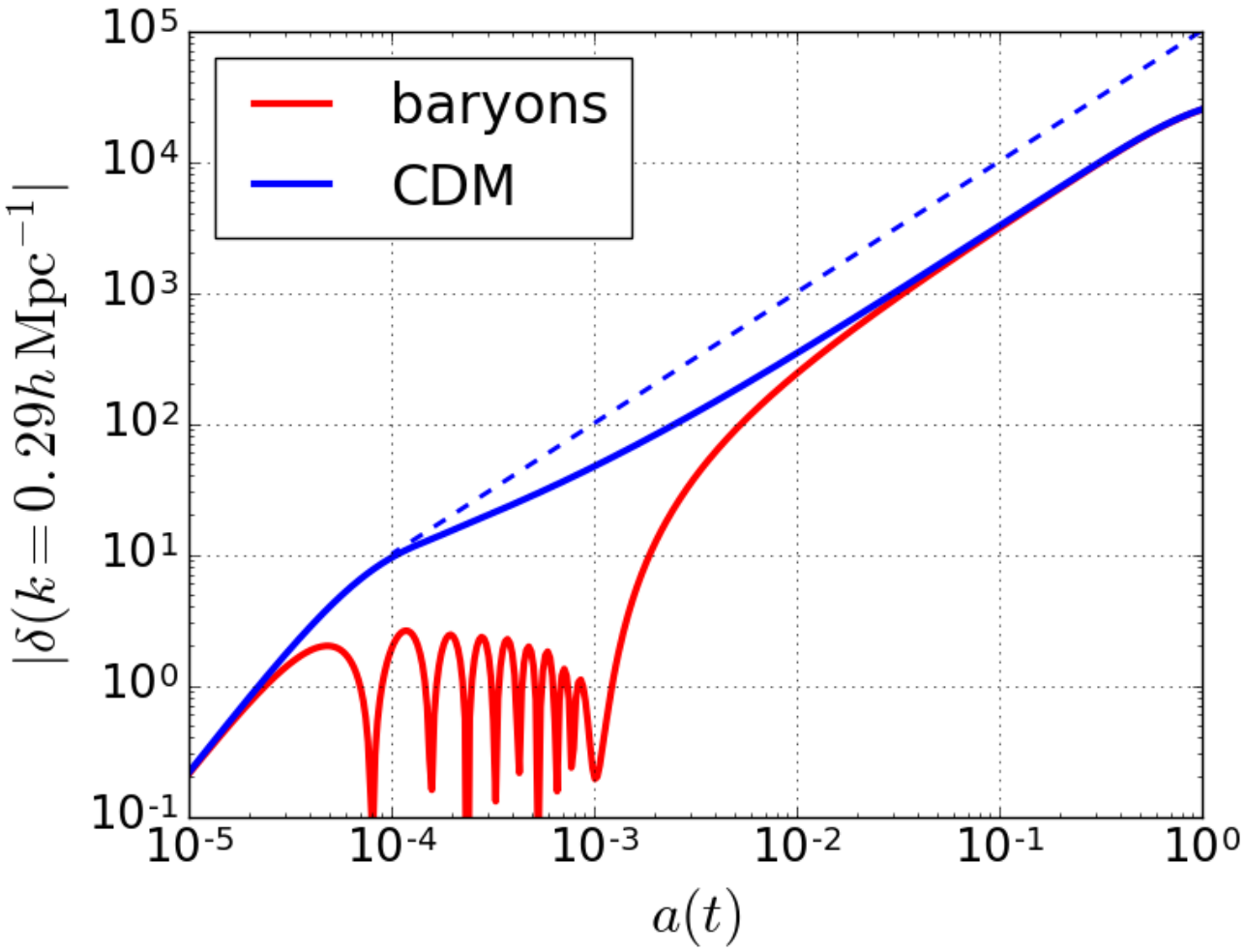
Short wavelength modes have Hubble-entry during radiation epoch resulting in decay of gravitational potential.

Long wavelength modes enter during matter epoch and therefore preserve primordial potential fluctuation from inflation.

k_{eq}: mode with Hubble entry at matter-radiation equality:

$$\frac{\lambda(a_{eq})}{2\pi} = \frac{a_{eq}/a_0}{k_{eq}} = d_H(a_{eq}) \qquad \Rightarrow k_{eq} \sim 0.01\mathrm{Mpc/h}^{-1}$$

A mode with $a_{enter} \sim 10^{-4} a_0 < a_{eq}$



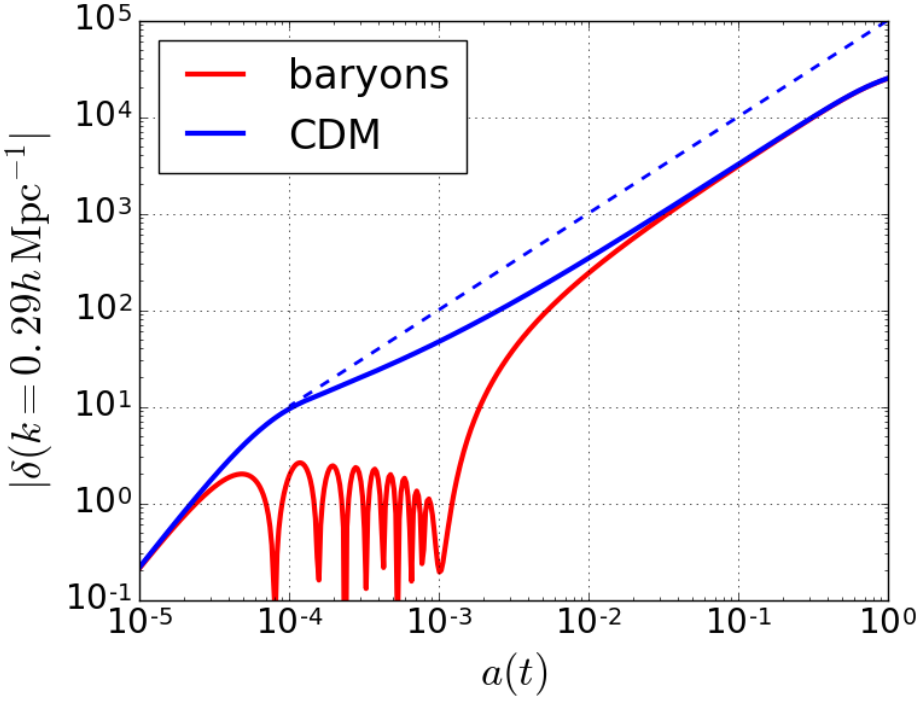
CDM growth suppressed while $a < a_{eq} \sim 3 \times 10^{-4}$

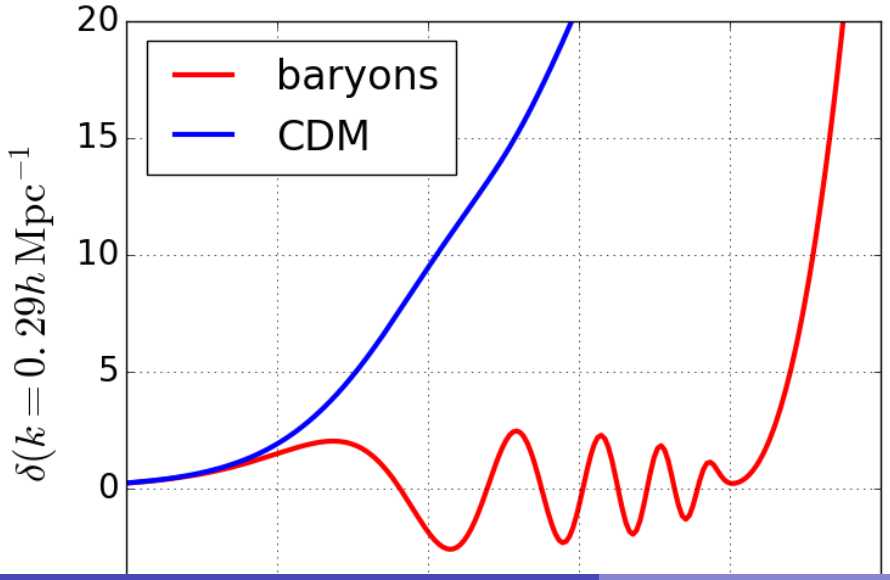
$\delta \propto a(t)$ for $a > a_{eq}$

CDM growth slows when Λ begins to dominate ($a > 0.5$)

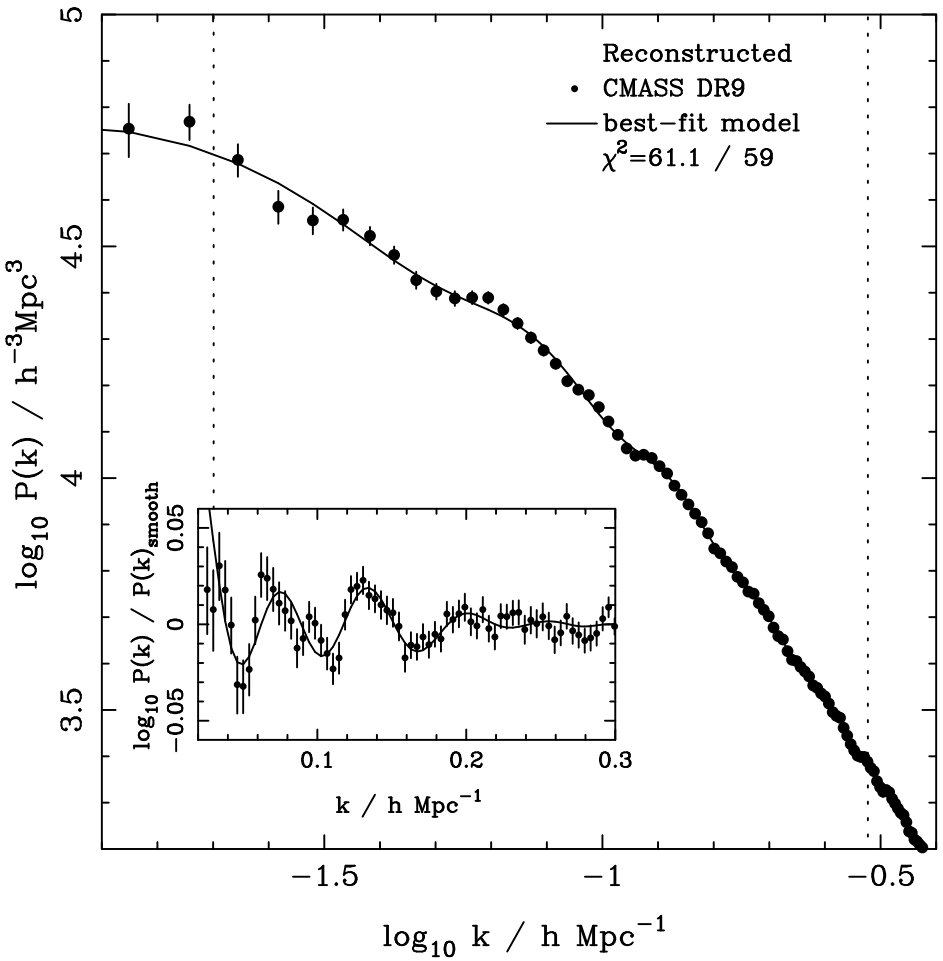
Baryons oscillate until recombination ($a \sim 10^{-3}$)

A mode with $a_{enter} \sim 10^{-4} a_0 < a_{eq}$





Observed power spectrum of galaxies



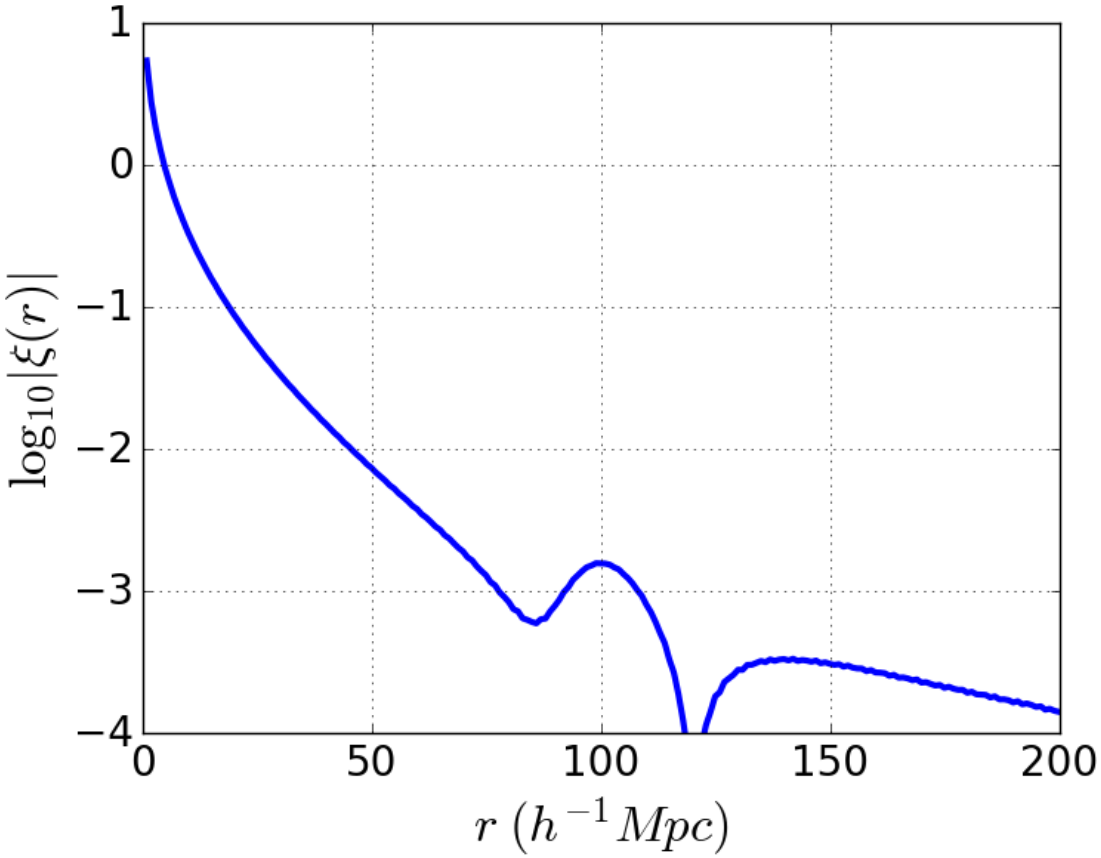
Galaxies are a "biased" tracer of matter at large scale ($k < 0.05$):

$$P(k)_{gal} = b_{gal}^2 P(k)_{matter}$$

At small scale ($k > 0.05$) non-linear growth complicates the galaxy power spectrum.

"Wiggles" due to BAO are believed to be robustly positioned, but they are more easily seen in the correlation function.

Correlation function



$$\xi(\vec{r}) = \langle \delta(\vec{r}')\delta(\vec{r}' + \vec{r}) \rangle$$

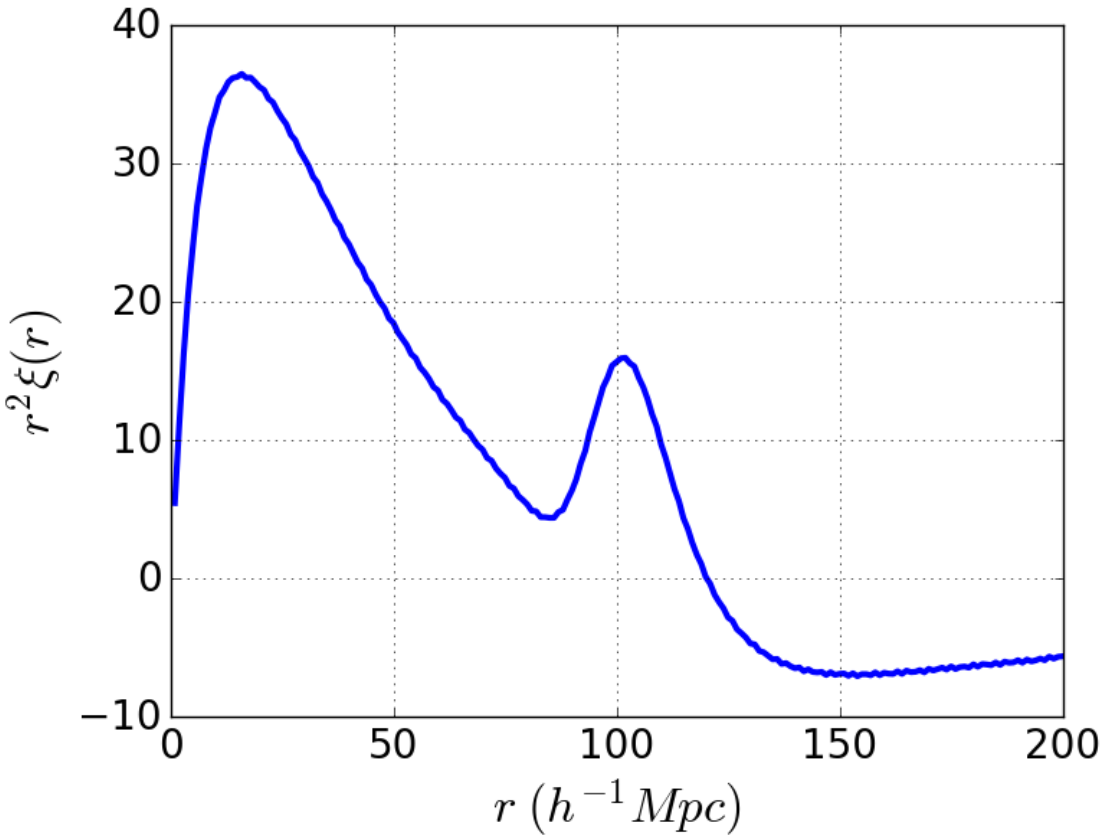
$$\sim \int e^{i\vec{k}\cdot\vec{r}} P(k)\, d^3k$$

$\xi(r)$ has a peak at $r = r_d$

(sound horizon)

Standard ΛCDM : $(\Omega_{cdm}, \Omega_b) = (0.269,\ 0.0484)$,
$\Omega_\Lambda \sim 1 - \Omega_{cdm} - \Omega_b,\ h = 0.674,\ A_s = 2 \times 10^{-9}$

Correlation function



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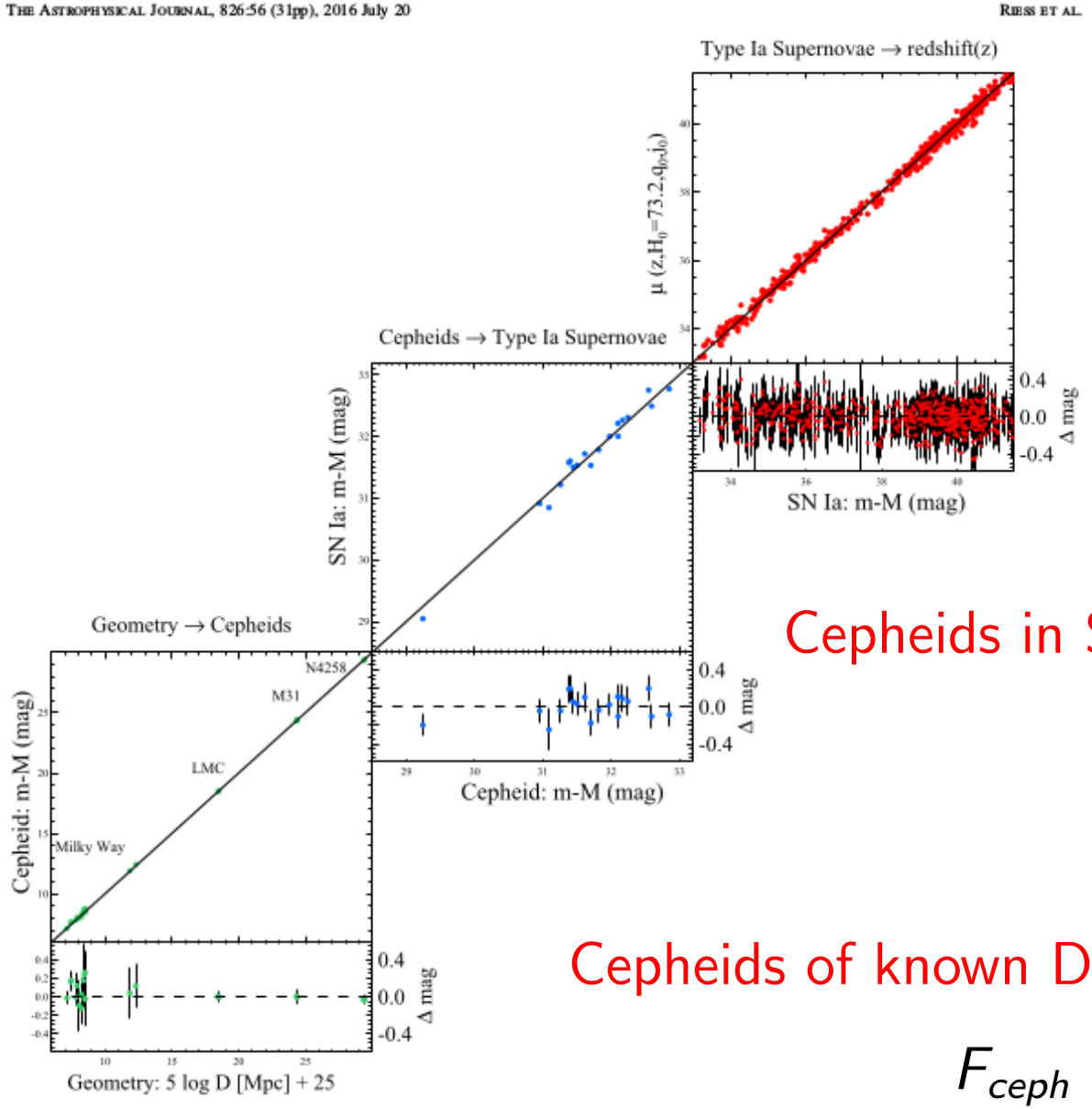
H_0 from distance ladder

$v = H_0 D$ (for $z \ll\sim 0.1$)
$v =$ recession velocity from redshift
need small "peculiar" velocity $\Rightarrow v/c = z > 0.02$
$D =$ distance from photon flux from objects of known luminosity
$F = L/4\pi D^2$

Latest result: $H_0 = 73.5 \pm 1.4 \mathrm{km\,s^{-1}Mpc^{-1}}$
A. Riess et al. arXiv:2001.03624

Objects of known luminosity:
Type Ia Supernovae (SNIa) (calibrated with cepheids)
Cepheid variable stars (calibrated with parallax/xallarap)

Distance Ladder: three steps



Hubble-flow SNI $\Rightarrow H_0 L^2_{SNIa}$:

$$F_{SNIa} = \frac{L_{SNIa}}{4\pi(zc/H_0)^2}$$

$$(0.01 < z < 0.05)$$

.

Cepheids in SNIa hosts $\Rightarrow L_{SNIa}/L_{ceph}$:

$$\frac{L_{SNIa}}{L_{ceph}} = \frac{F_{SNIa}}{F_{ceph}}$$

Cepheids of known D $\Rightarrow L_{ceph}$:

$$F_{ceph} = \frac{L_{ceph}}{4\pi D^2_{ceph}}$$

Distance Ladder: mostly small statistics

$$F_{SNIa} = \frac{L_{SNIa}}{4\pi(zc/H_0)^2}$$

Hundreds of SNIa in Hubble flow ($0.01 < z < 0.05$)

$$\frac{L_{SNIa}}{L_{ceph}} = \frac{F_{SNIa}}{F_{ceph}}$$

$$F_{ceph} = \frac{L_{ceph}}{4\pi D_{ceph}^2}$$

18 SNIa in galaxies with cepheid distances

four galaxies of known distance and observed cepheids

- NGC4258: one maser-black hole binary
- M31: two detached stellar binaries
- LMC: eight detached stellar binaries
- Milky Way: 15 cepheid parallaxes

Systematics: do the cepheids in these four galaxies have the same luminosities and photometry as the cepheids in the 18 galaxies hosting SNIa?