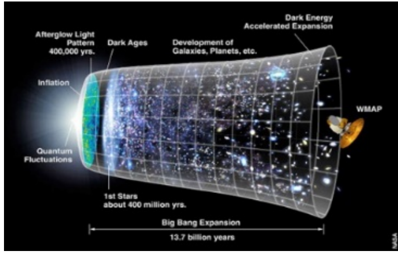

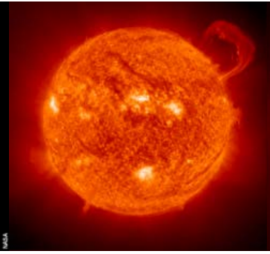
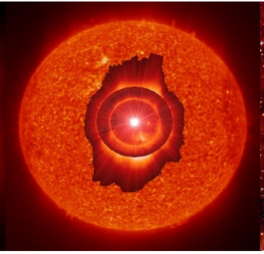

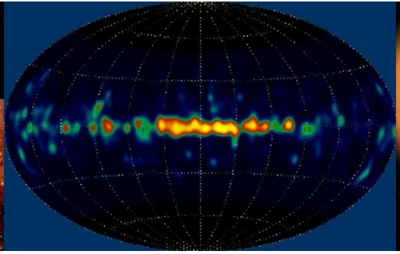
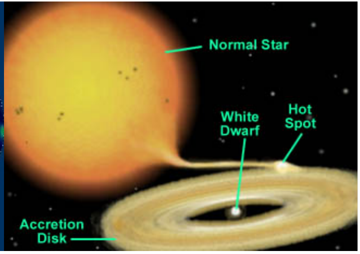

From Nuclei to stars: Introduction to nuclear astrophysics

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M2-NPAC 2018-2019

Plan of lecture I

I. Introduction : Nuclei in the Cosmos

II. Principles of stellar structure & evolution
- The equations of stellar structure
- Virial theorem
- Energy generation in stars
- Equation of state
- Star formation and evolution

III. The observed properties of stars
- luminosity, effective temperature & colours, chemical composition, metallicity, Age, mass & radius
- The Hertzprung Russel (HR) diagram & stellar evolution

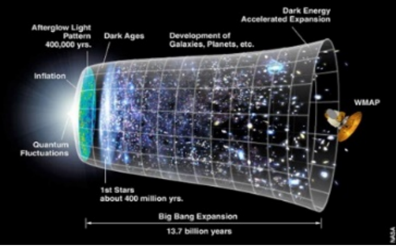
IV. Chemical abundances
- Abundance from stellar spectra
- Meteorites

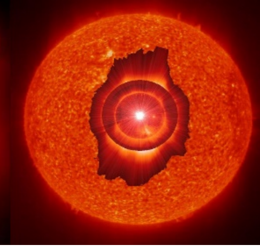
Text books

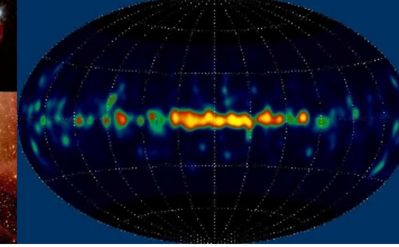
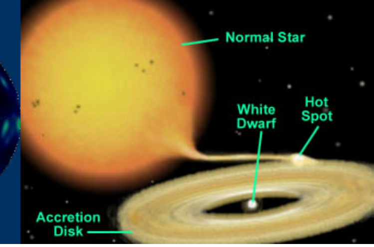
- **Cauldrons in the Cosmos**, Nuclear Astrophysics ,
  Claus E. Rolfs and William S.Rodney
  The University of Chicago Press, 1988
  ISBN 0-226-72456-5

- **Principles of Stellar Evolution and Nucleosynthesis**,
  Donald D. Clayton, The University of Chicago Press ,1968
  ISBN 0-226-10953-4

- **Stellar Interiors, Physical Principles, Structure, and Evolution**
  Carl J. Hansen, Steven D. Kawaler, Virginia Trimble
  Second Edition, 2004, 1994 Springer-Verlag New York, Inc
  ISBN 0-387-20089-4



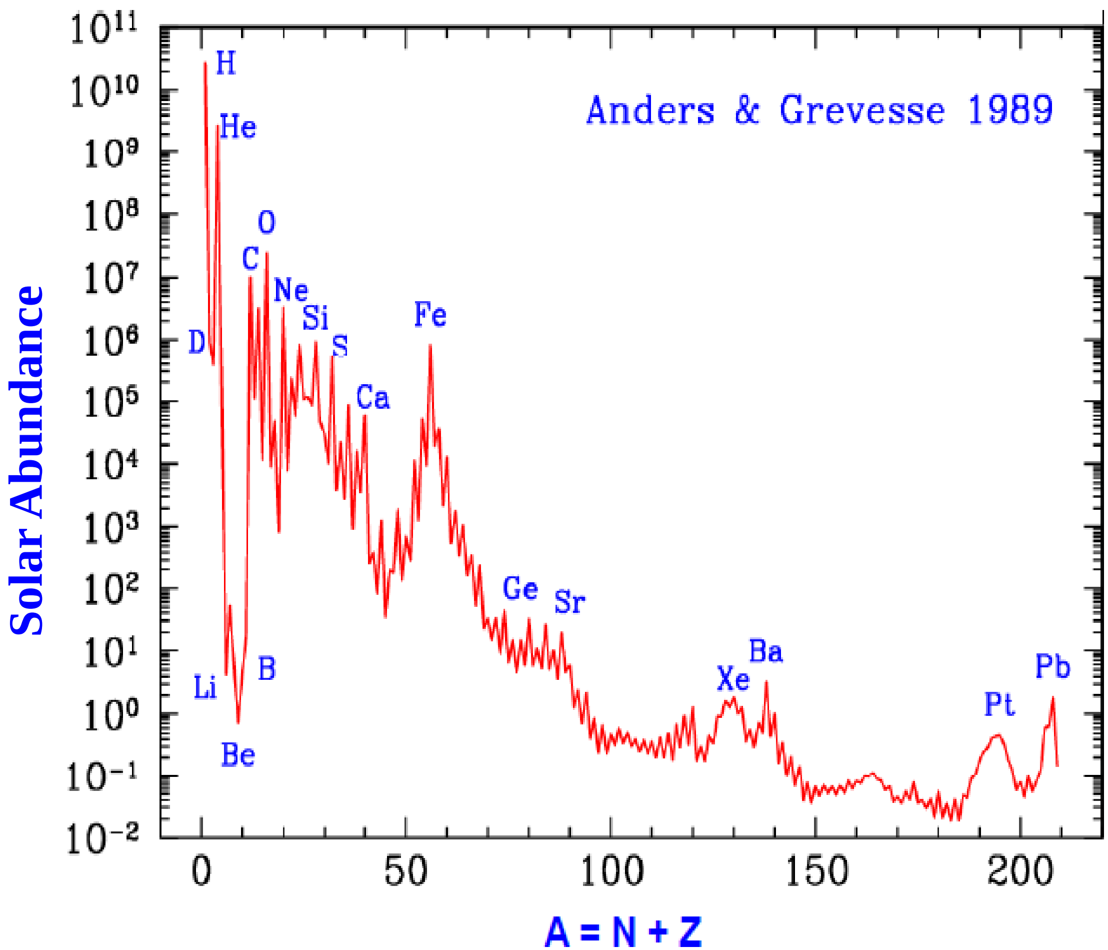



NUCLEAR ASTROPHYSICS

Nuclear astrophysics is the science which addresses some of the most compelling questions in nature:

- How do stars form and evolve?
- What powers the stars?
- What is the origin of the chemical elements present in our Universe?
- Which nucleosynthesis processes are responsible of the observed solar abundances?

Abundance curve of the elements:



Data sources:

Earth, Moon, meteorites, solar & stellar spectra, cosmic rays...

Characteristics:

- 12 orders-of-magnitude span
- H ~ 75%
- He ~ 23%
- C → U ~ 2% ("metals")
- D, Li, Be, B under-abundant
- O the 3rd most abundant
- C the 4th most abundant
- exponential decrease up to Fe
- peak near Fe
- nearly flat distribution beyond Fe with some peaks

Introduction: Nuclei in the Cosmos

The answer to all the questions concerning the stars and the origin of the nuclei in the cosmos is given by the interaction of three fields:

Observations (astronomy & geology)

Nuclear Physics
Synthesis of nuclei
(Main field at NPAC)

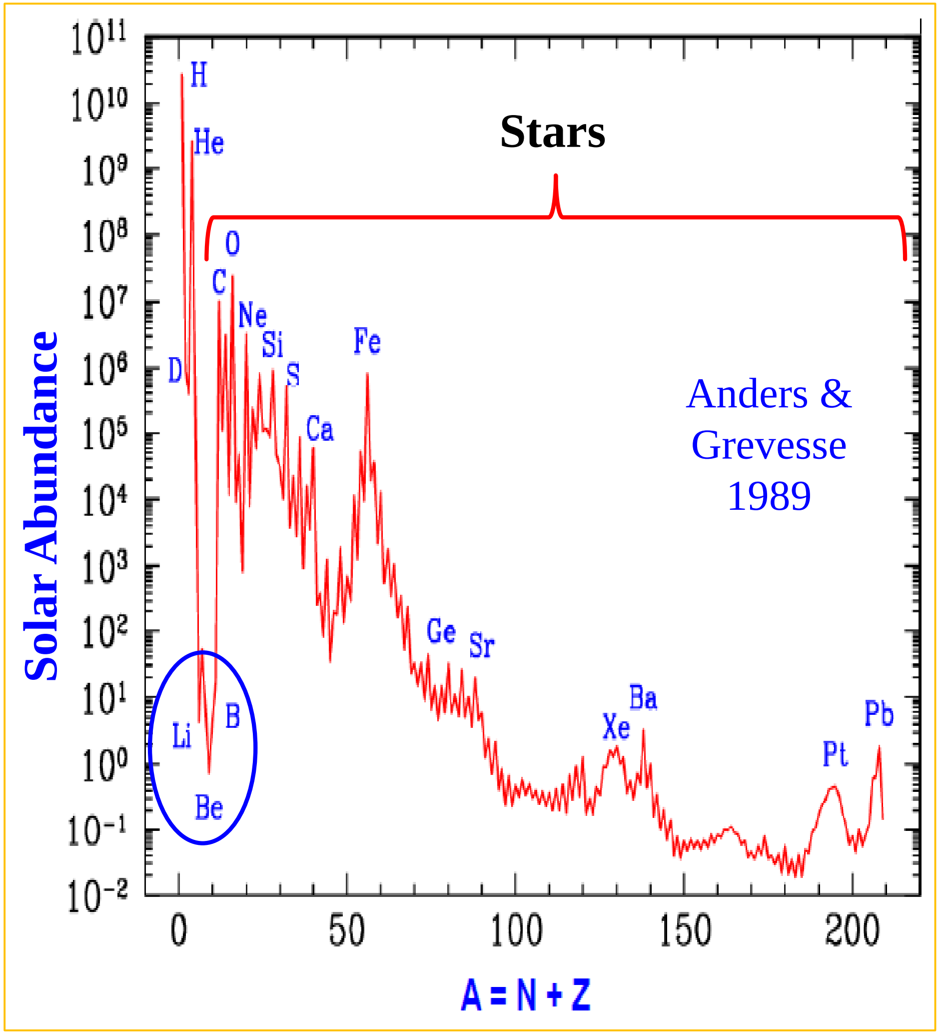
Astrophysics modelling (Big-Bang & stellar)

Nucleosynthesis: When and where?

➢**H, D, He, ^{7}Li$^{\#}$**
→ primordial nucleosynthesis
(**Big-Bang**) (Lecture II)

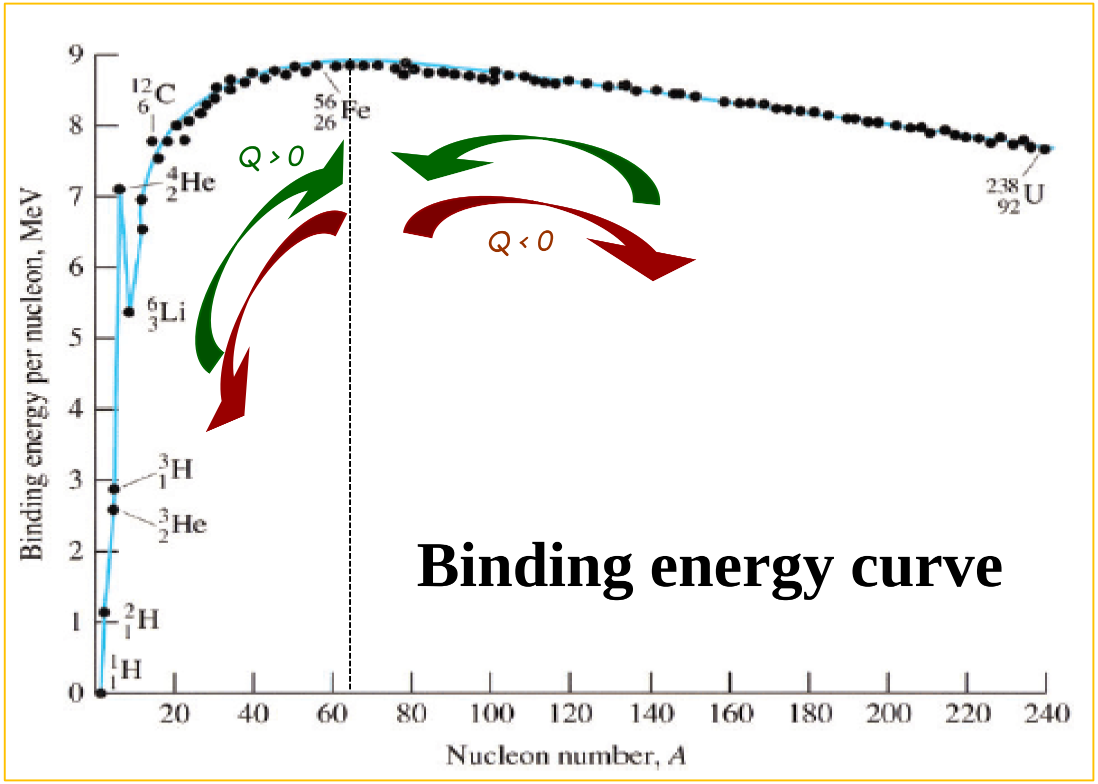
➢**Li$^{\#}$, Be, B**
→ Cosmic ray spallation in Inter-Stellar Medium (ISM) : heavier and abondant nuclei (CNO) broken by interaction with p or α particle (lecture II)

➢**C, N, O ..., Fe, ... Pb,…**
→in star (calm & explosive)
(lecture II)



Introduction: Nuclei in the cosmos & nuclear physics

➢ Nuclear reactions in stars play a key role in understanding energy production & nucleosynthesis of the elements in stars



➢Stability of a nucleus is related to the **binding energy per nucleon** (energy needed to separate a nucleon from the nucleus).

➢Increasing binding energy as a function of atomic mass A until ^{56}Fe

➢Very low decreasing binding energy for heavier elements

➢ Q > 0 → fusion up to Fe region, fission of heavy nuclei

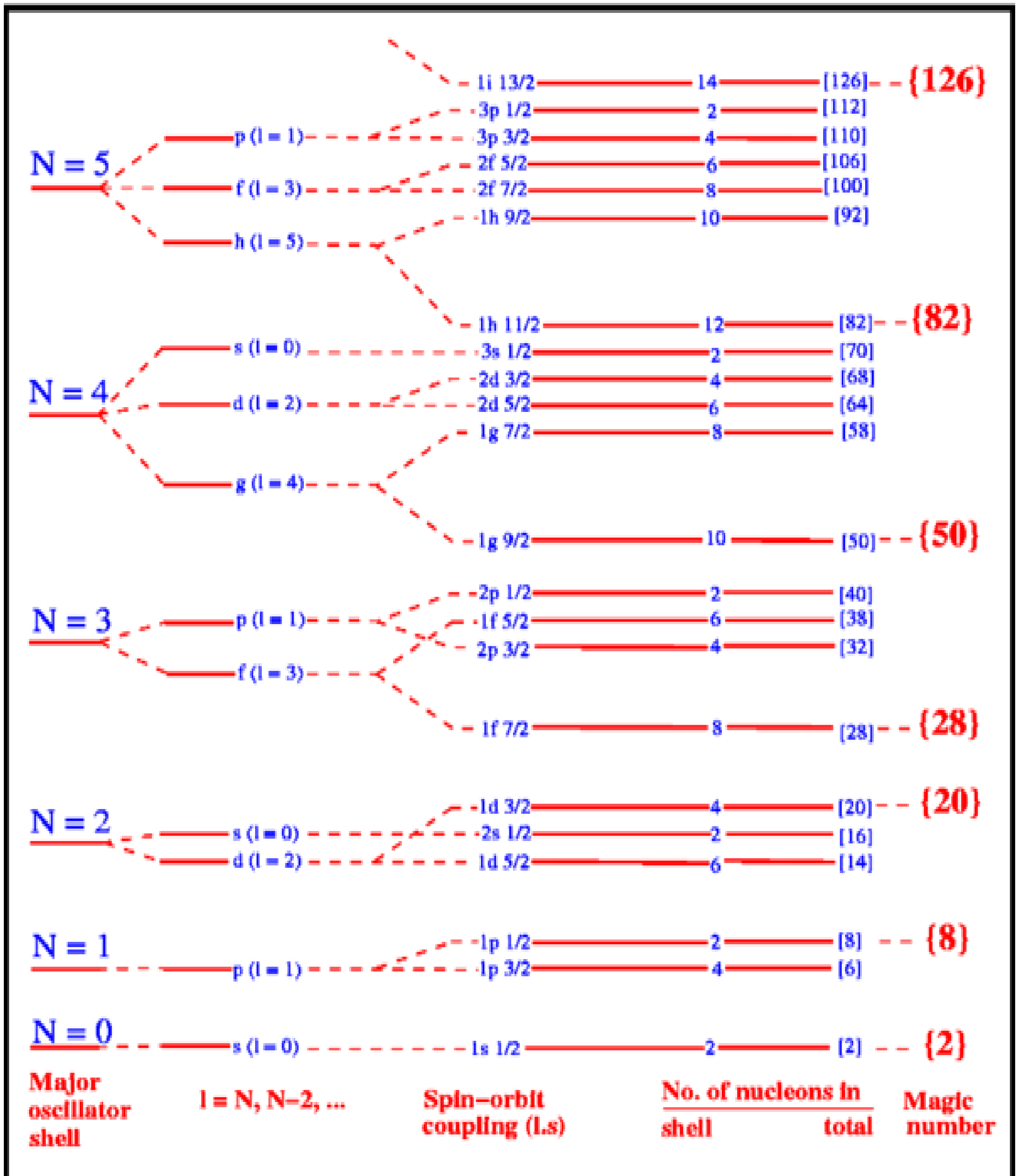
➢ $\Delta E/A$ is maximum (8.8 MeV) near ^{56}Fe => "iron peak"

➢ LiBeB : relatively fragile

Reactions in stars are mainly FUSION

Introduction: Nuclei in the cosmos & nuclear physics

Shell model



➢Nuclear stability is related to shell closure and pairing

➢Z and N odd or even→ oscillation of the abundance

➢Nuclei with Z or N equal to a magic number → abundances peak

➢ Double magicity Z=82 & N=126 → ^{208}Pb peak

Principles of stellar structure & evolution

What are the main physical processes which determine the structure of stars ?

- Stars are held together by gravitation – attraction exerted on each part of the star by all other parts
- Collapse is resisted by internal thermal pressure.
- These two forces play the principal role in determining stellar structure – they must be (at least almost) in balance
- Thermal properties of stars – continually radiating into space. If thermal properties are constant, continual energy source must exist
- Theory must describe - origin of energy and transport to surface

We make two fundamental assumptions :

1) Neglect the rate of change of properties – assume constant with time
2) All stars are spherical and symmetric about their centers

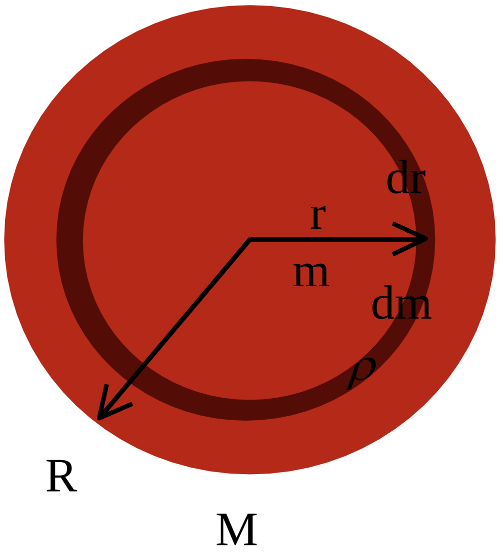
For an isolated, static, spherically symmetric star, four basic laws / equations needed to describe structure:

- **Conservation of mass**
- **Equation of hydrostatic equilibrium** (at each radius, forces due to pressure differences balance gravity)
- **Conservation of energy** (at each radius, the change in the energy flux equals the local rate of energy release)
- **Equation of energy transport** (relation between the energy flux and the local gradient of temperature)

Basic equations are supplemented by:

- **Equation of state** (pressure of a gas as a function of its density and temperature)
- **Opacity** (how transparent it is to radiation)
- **Nuclear energy generation rate** as $\varepsilon(\rho,T)$

Equation of mass conservation



Let r be the distance from the center
$\rho(r)$ the density as function of radius

Let m be the mass contained inside the sphere of radius r, then conservation of mass implies that:

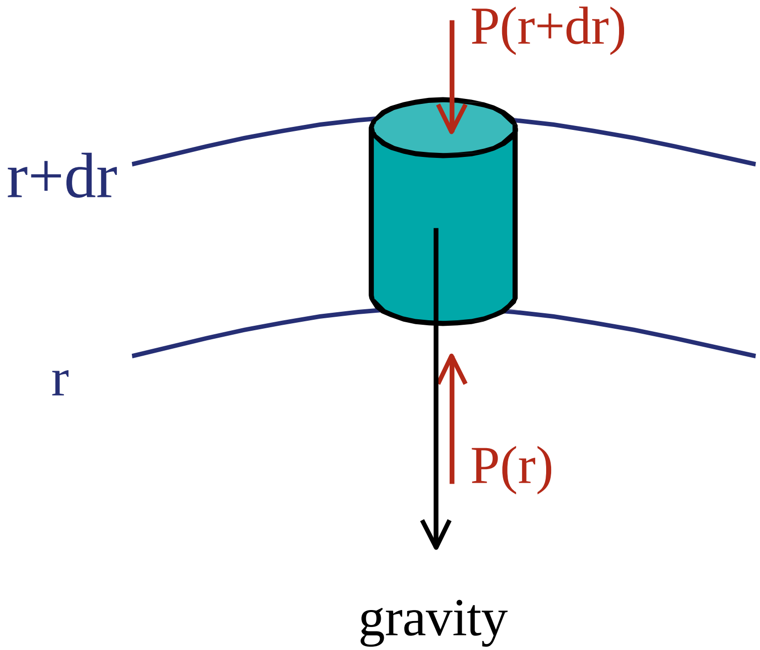
$$dm = 4\pi r^2 \rho dr$$

$$\Rightarrow \quad \boxed{\frac{dm}{dr} = 4\pi r^2 \rho}$$

1st stellar structure equation

Hydrostatic equilibrium

➢ Balance between gravity and internal pressure is known as ***hydrostatic equilibrium***



- Consider small cylindrical element between radius r and radius r + dr in the star.
- Surface area = dS ; Mass = Δm
- Mass of gas in the star at smaller radii = m = m(r)

Radial forces acting on the element:

Gravity (inward): $$F_g = -\frac{Gm\Delta m}{r^2}$$

gravitational constant $G = 6.67 \times 10^{-8}$ dyne cm^2 g^{-2}

Pressure (net force due to difference in pressure between upper and lower faces):

$$F_p = P(r)dS - P(r+dr)dS$$

$$= P(r)dS - \left[P(r) + \frac{dP}{dr} \times dr \right] dS$$

$$= -\frac{dP}{dr} dr dS$$

Mass of element: $\Delta m = \rho \, dr \, dS$

Applying Newton's second law (F=ma) to the cylinder:

$$\Delta m \ddot{r} = F_g + F_p = -\frac{G m \Delta m}{r^2} - \frac{dP}{dr} dr dS$$

acceleration = 0 everywhere if star static

By setting acceleration to zero, and substituting for Δm, one gets:

$$0 = -\frac{Gm\rho drdS}{r^2} - \frac{dP}{dr}drdS$$

<u>Equation of hydrostatic equilibrium</u>:

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

2nd stellar structure equation

If we use enclosed mass as the dependent variable, we can combine the 1st and 2nd stellar structure equations into one:

$$\frac{dP}{dm} = \frac{dP}{dr} \times \frac{dr}{dm} = -\frac{Gm}{r^2}\rho \times \frac{1}{4\pi r^2 \rho}$$

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$ ← alternate form of hydrostatic equilibrium equation

Properties of hydrostatic equilibrium equation:

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

1) Pressure always **decreases** outward

2) Pressure gradient vanishes at r = 0

3) Condition at surface of star: P = 0

(2) and (3) are **boundary conditions** for the hydrostatic equilibrium equation

- Fundamental theorem describing the properties of auto-gravitating systems at hydrostatic equilibrium (e.g.stars)

Start with: $$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

Multiply both sides by volume: $V = (4/3)\pi r^3$

$$VdP = (4/3)\pi r^3 \times \left(-\frac{Gm}{4\pi r^4}\right)dm = -\frac{1}{3}\frac{Gm}{r}dm \qquad (1)$$

Now integrate over the whole star. Left Hand Side gives:

$$\int VdP = \left[PV\right]_0^R - \int PdV$$

But P = 0 at r = R, and V = 0 at r = 0, so this term vanishes

If we have a small mass dm at radius r, the gravitational potential energy is given by:

$$d\Omega = -\frac{Gm}{r}\,dm$$

Hence, integrating Right Hand Side of equation (1) over the star:

$$\int -\frac{1}{3}\frac{Gm}{r}\,dm = \frac{1}{3}\int d\Omega = \frac{1}{3}\Omega$$

Where Ω is the **gravitational potential energy of the star -** i.e. the energy required to assemble the star by bringing gas from infinity (very large radius).

Putting the pieces together: $-\int P dV = \frac{1}{3}\Omega$

$$0 = \Omega + 3\int_0^{V(r=R)} P dV \longleftarrow$$

One version of the Virial theorem

With some assumptions about the pressure, one can progress further. Often, one can write the pressure in the form:

$$P = (\gamma - 1)\rho u$$

- ρ is the density
- u is the internal energy per unit mass (per gram of gas)
- γ is a constant

Substitute this equation of state into the virial theorem:

$$0 = \Omega + 3 \int_0^{V(r=R)} (\gamma - 1)\rho u \, dV$$

$\rho\, u$ has units of (g cm^{-3}) x (erg g^{-1}) = erg cm^{-3}
it is the internal energy per unit **volume**

Integral of internal energy per unit volume over all volume in the star is just the total internal energy of the star, U.

$$0 = \Omega + 3(\gamma - 1)U$$

For an ideal mono-atomic gas $\gamma = 5/3 \rightarrow$ this is the ratio of the specific heat at constant pressure for a constant volume.

$$\boxed{0 = \Omega + 2U}$$

The Virial theorem

gravitational potential energy of the star

total internal energy of the star

So far we have only considered the dynamical properties of the star, and the state of the stellar material. We need to consider the source of the stellar energy.

Let's consider the origin of the energy i.e. the conversion of energy from some form in which it is not immediately available into some form that it can radiate.

How much energy does the sun need to generate in order to shine with it's measured flux $L_{sun} = 4\times10^{26}$ Js^{-1}?

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Let's consider the origin of the energy i.e. the conversion of energy from some form in which it is not immediately available into some form that it can radiate.

How much energy does the sun need to generate in order to shine with it's measured flux $L_{sun} = 4\times10^{26}$ Js^{-1}?

$L_{sun}=4\times10^{26}$ $J.s^{-1}$

Sun has not changed flux in 4.6×10^{9} yr (yr=3×10^{6} s)

$\Rightarrow$Sun has radiated 5.5×10^{42} J

Source of energy generation

What is the source of this energy ? 2 possibilities :

- Cooling or contraction
- Nuclear Reactions

1.Cooling and contraction

Suppose the radiative energy of Sun is due to the Sun being much hotter when it was formed, and has since been cooling down .Or the sun slowly contracting with consequent release of gravitational potential energy, which is converted to radiation. Virial theorem gives: $0 = \Omega + 2U \Rightarrow U = -\Omega / 2$

$\Omega_{sun} \sim -4\times10^{41}$ J $\rightarrow U_{sun}=2\times10^{41}$ J

With $L_{sun}=4\times10^{26}$ Js^{-1} , the thermal time scale over which the thermal energy will cover radiative surface losses is given by: $\tau_{sun}=U_{sun}/L_{sun}=$ **1.67×10^{8} year**

$\rightarrow$ This limit of duration of sunshine is a factor of ~28 too short to account for the constraints on age of the Sun imposed by fossil and geological records

Source of energy generation

2. Nuclear Reactions

→ The only known way of producing sufficiently large amounts of energy is through nuclear reactions.

→ There are two types of nuclear reactions, fission and fusion.

Both fusion and fission could power the Sun. Which is the more likely ?

→ As light elements are much more abundant in the solar system than heavy ones, nuclear fusion is the dominant source. (Lecture II)

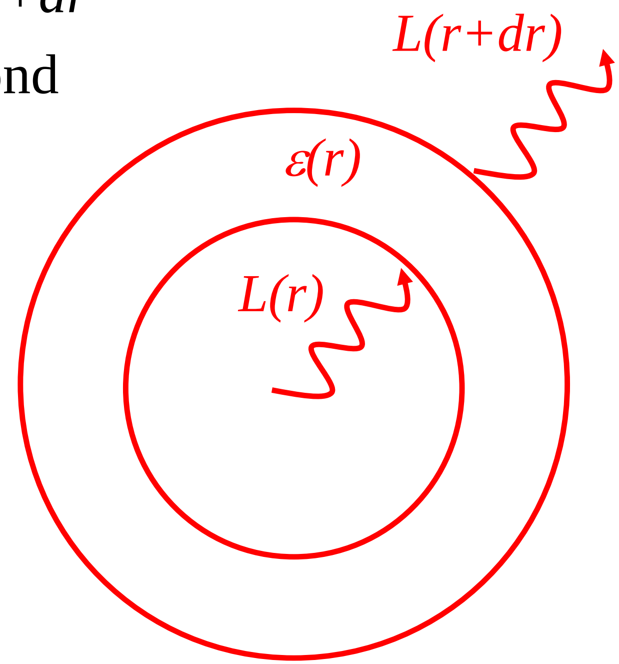
Conservation of thermal energy :

Consider a spherically symmetric star in which energy transport is radial and in which time variations are unimportant.

L(r)= rate of energy flow across sphere of radius r

L(r+dr)=*rate* of energy flow across sphere of radius *r* +*dr*

ε(r) = energy released from nuclear processes per second per unit mass (erg $s^{-1}g^{-1}$)



⇒ Energy release in shell: $4\pi r^2 \rho(r)\varepsilon(r)dr$

Conservation of energy at the thermal equilibrium:

$$L(r+dr) = L(r) + 4\pi r^2 \rho(r)\varepsilon(r)dr$$

$$\Rightarrow \quad \frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\varepsilon(r)$$

3rd stellar structure equation

Equation of energy transport :

There are three ways energy can be transported in stars

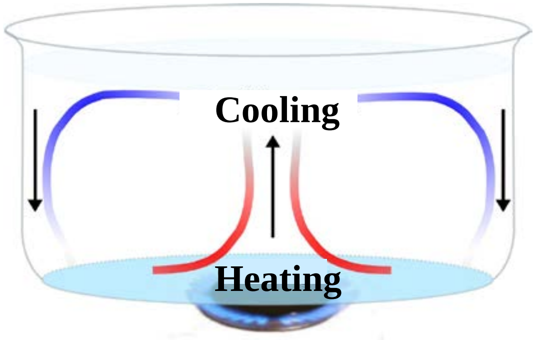
• **Conduction-** by exchange of energy during collisions of gas particles (usually e^-)

→ not important in most star interiors (the mean free path of the ions and e- in high gas densities are extremely short)

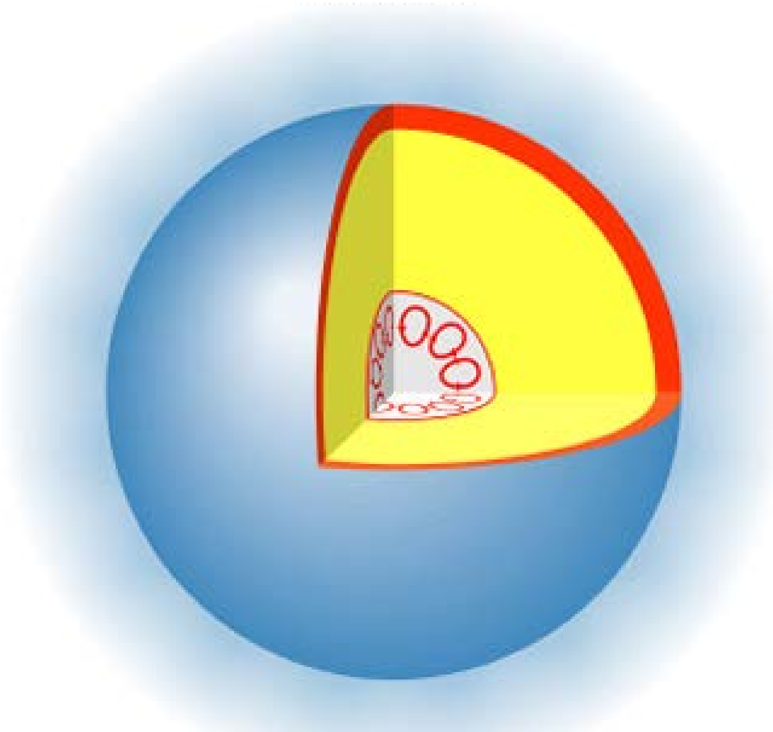
• **Radiation-** energy transport by the emission and absorption of photons : photons produced by nuclear reactions and atomic transitions can (i) scatter with electrons and ions and (ii) be absorbed and re-emitted many times before reaching the surface : random walk

→ Dominant mechanism if the T gradient dT/dr (or the opacity) is not too large

• **Convection-** energy transport by mass motions of the gas (motion due to temperature gradient). Only occurs when temperature gradient exceeds some critical value



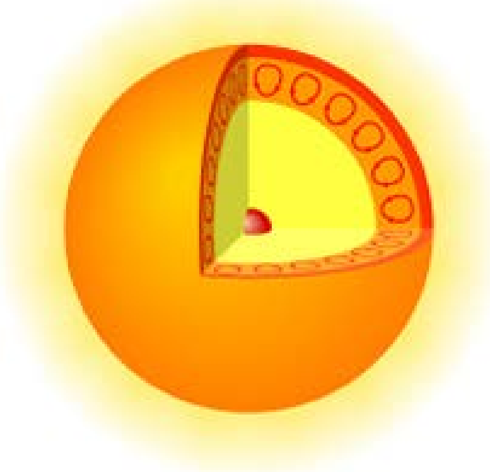
Convection & radiative transport:



Massive stars

Radiative envelop

Convective core



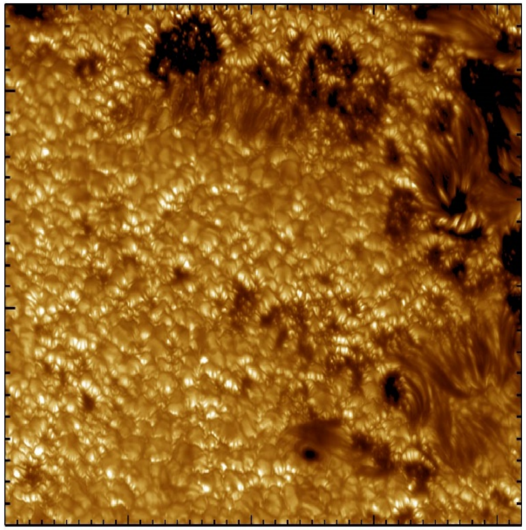
$1M_{\odot}$ star

Convective envelop

$0.7\ R_{\odot}$-Surface

Radiative core

Solar surface from
Swedish Solar Telescope



Solar granulation =
convection cells
Cell size~100 km

Equation of energy transport :

- **Radiation transport:** M. Schwarzschild, The structure and Evolution of the Stars (Princeton University Press 1958)

$$L(r) = -4\pi r^2 (4ac/3) \frac{T(r)^3}{\kappa(r)\rho(r)} (dT/dr)$$

4th stellar structure equation

Opacity: mass absorption coefficient, it depends on the composition of the gas

→ The photons emitted at high T in the center of the star are continually emitted and reabsorbed and gradually degraded to **longer λ** as they proceed outward. In case of the **sun**, they emerge from the surface as **visible light**.

- **Convection transport:** M. Harwitt, Astrophysical Concepts (New-York: Wiley, 1973)

$$dT/dr = (1-1/\gamma)(T(r)/P(r))(dP/dr)$$

where γ= 5/3 for an ideal monoatomic gas: the ratio of specific heats capacity

➢ Interior of a star contains a mixture of ions, electrons, and radiation (photons). For most stars (exception very low mass stars and stellar remnants) the ions and electrons can be treated as an ideal gas and quantum effects can be neglected.

Total pressure:

$$\begin{aligned} P &= P(\rho, T, \{X_i\}) = P_I + P_e + P_r \\ &= P_{gas} + P_r \end{aligned}$$

The equation of state of normal stars

- P_I is the pressure of the ions
- P_e is the electron pressure
- P_r is the radiation pressure

Gas Pressure

The e**q**uation of state for an ideal gas is:

$$P_{gas} = nkT$$

n is the number of particles per unit volume.
$n = n_I + n_e$, where n_I and n_e are the number densities of ions and electrons respectively

In terms of the mass density ρ:

$$P_{gas} = \frac{\rho}{\mu m_H} \times kT$$

m_H: mass of hydrogen in atomic mass unit

μ: mean molecular weight value. It depends on the composition of the gas and the state of ionization. Ex: Neutral hydrogen $\mu=1$, Fully ionized hydrogen $\mu=0.5$

$$P_{gas} = \frac{R_g}{\mu} \rho T \quad \text{where} \quad R_g = \frac{k}{m_H} \quad \text{is the ideal gas constant}$$

Radiation Pressure

The radiation pressure of a blackbody radiation is given by:

$$\boxed{P_r = \frac{1}{3} a T^4}$$

Stefan's law

a : the radiation constant

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3}$$

$= 7.565 \times 10^{-15}$ erg.cm^{-3}K^{-4}

$= 7.656 \times 10^{-16}$ J.m^{-3}K^{-4}

Comparison of gas and radiation pressure in the core of the **Sun**

$$\frac{P_r}{P_g} = \frac{aT^4}{3} \Big/ \frac{R_g}{\mu} \rho T = \frac{\mu a}{3R} \frac{T^3}{\rho}$$

With $T = 1.6 \times 10^7$ K, $\rho = 150$ g cm^{-3}, $\mu = 0.83$ and $R_g = 8.3 \times 10^7$ erg g^{-1} K^{-1}

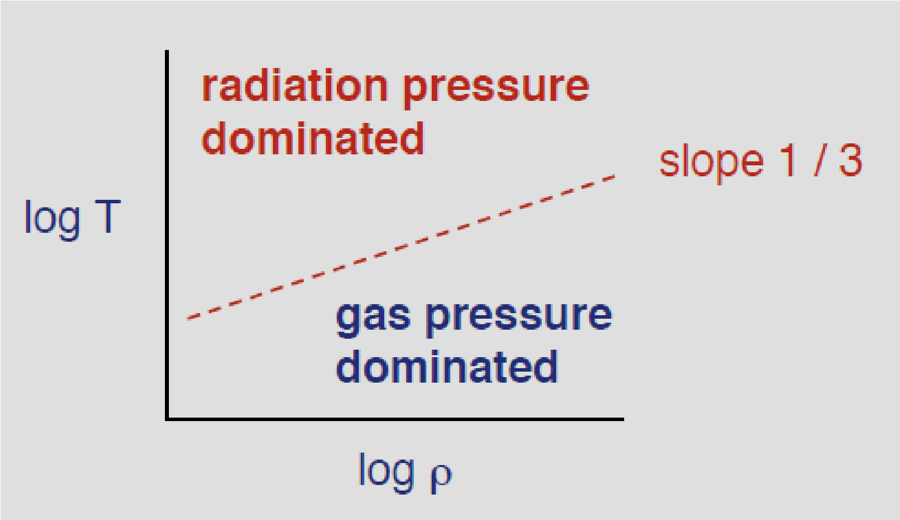
$$\rightarrow \boxed{\frac{P_r}{P_g} = 7 \times 10^{-4}}$$

$\Rightarrow$ Radiation pressure is not at all important in the center of the Sun

In which stars are gas and radiation pressure important?

P_{gas} & P_r equal when:

$$\boxed{T^3 = \frac{3R}{a\mu}\rho}$$



- The temperature of a star scales as $T \propto M/R$
- The density scales as : $\rho \propto M/R^3$

$$\boxed{\frac{P_r}{P_{gas}} \propto M^2}$$

Gas pressure is most important in **low mass stars**
Radiation pressure is most important in **high mass stars**

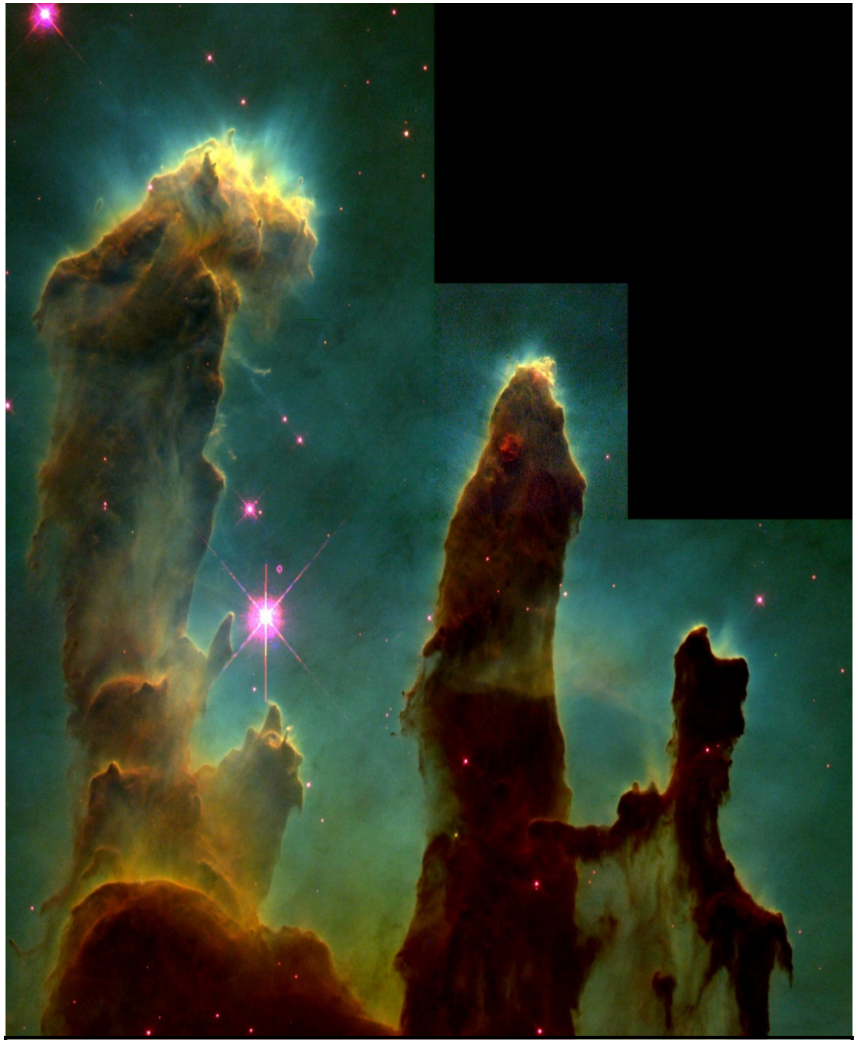
➢ Stars are formed from condensation of gas (pressure and gravitational energy are the key of the process)

➢ Basic principle : gravitational contraction of a molecular (H2) gas cloud that became unstable

→ Instability → collapse

➢ Gravitational collapse can be spontaneous or triggered by external influence

- gas pressure (≡ f(temperature, density, composition)$_{cloud}$ can't balance the gravity
- external event (nearby supernovae, collision with other cloud...)



The "Pillars of Creation" within the Eagle nebula (M16), Hubble (1995)

The Jeans Mass

The Jeans mass is the minimum mass a cloud must have if gravity is to overwhelm pressure and initiate collapse.

Borderline case where a cloud is in a hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho \qquad (1)$$

To derive an estimate of the Jeans mass, consider a cloud of mass M, radius R:

- approximate derivative dP/dr by –P/R:
- assume pressure is that of an ideal gas:

$$P = \frac{R_g}{\mu}\rho T$$

…where R_g is the gas constant

Replace in (1): $$-\frac{R_g}{\mu}\frac{\rho T}{R} = -\frac{GM}{R^2}\rho \quad \rightarrow \quad M = \frac{R_g}{\mu G} TR$$

Can replace R in favor of the density ρ using: $$M = \frac{4}{3}\pi R^3 \rho$$

$$M = \frac{R_g}{\mu G} T \times \left(\frac{3}{4\pi}\right)^{1/3} M^{1/3} \rho^{-1/3}$$

Tidy this up to get a final expression for the Jeans mass:

$$\boxed{M_J = \left(\frac{R_g}{\mu G}\right)^{3/2} \left(\frac{3}{4\pi}\right)^{1/2} T^{3/2} \rho^{-1/2}}$$

→ Basic formula for star formation.

Example:

Mol. H2 (μ=2), cold dense cloud, T=10 K, $\rho=10^{-19}$ g/cm^3 → $M_J \sim 7.6\times10^{32}$g $=0.4M_\odot$

(Sun mass: $M_\odot=2\times10^{33}$ g)

Stars are formed in nebulae interstellar clouds of dust & gas (mostly H)

★ During the contraction of a cloud, the central density increases but T ~ constant (if radiative cooling is efficient)

$\Rightarrow$ M_J ($\propto \rho^{-1/2}$) decreases

$\Rightarrow$ smaller and smaller regions of the cloud become unstable

★ Dense parts of these clouds undergo gravitational collapse

🡻

accretion of matter to the center

🡻

Protostar

To achieve life as a star → equilibrium is needed

How it works?

1- gravity pulls gas and dust inward the core

↓

2- T(core) ↗

3- ρ(core) ↗

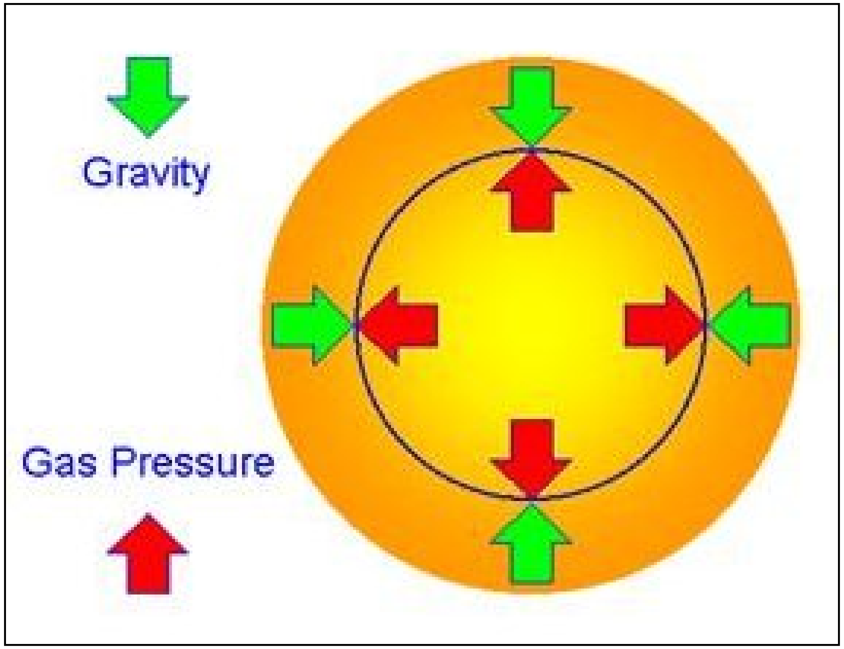
↓

4- gas pressure ↗ → resists the collapse of the nebulae

5- when gas pressure (T,ρ)= gravity

↓

equilibrium → accretion stop



Principles of stellar structure & evolution: Star in equilibrium &evolution

- Mechanical equilibrium (pressure against gravity)

=> short time to reach equilibrium (few hour for Sun)

- Thermal equilibrium :
  - nuclear fusion reaction in star core provide energy (exothermic)
  - heat transport at star's surface (radiation and convection)
  - loss of energy by radiation

    => relatively long time in equilibrium phase

- Star has several phases of equilibrium (H, He, C... burning with increasing ignition temperature) (lecture II)

- During fuel burning, composition of the star change slowly

- Once the nuclear fuel is burned, reactions stop => no energy provided to maintain pressure against gravity => collapse stage => core temperature increase until next burning stage (lecture II)

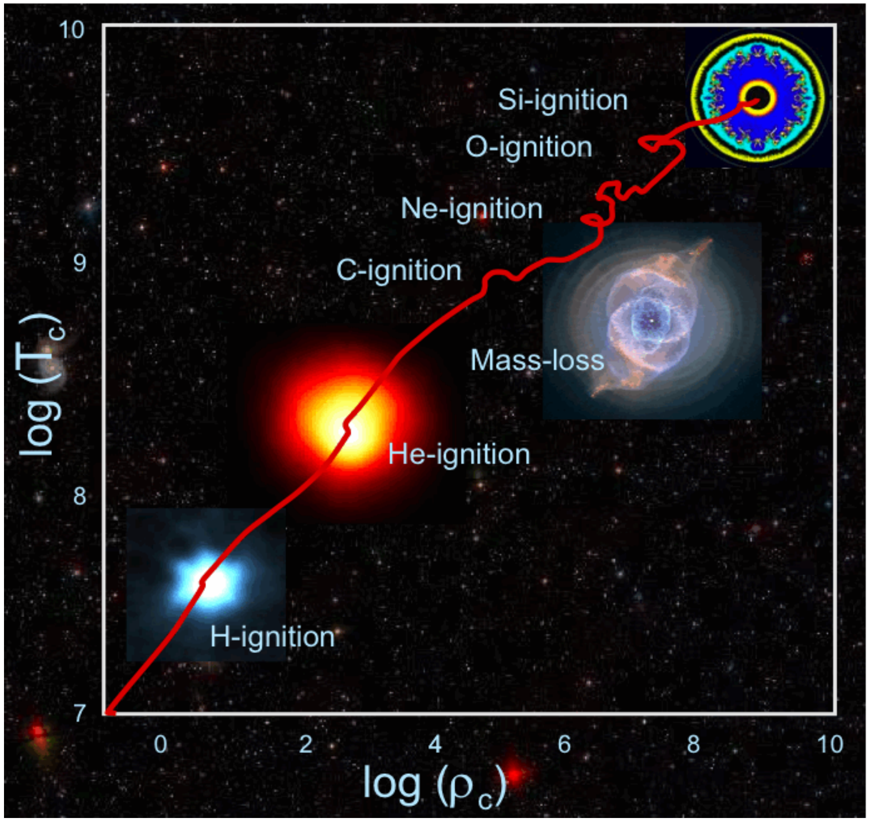
➢ Total energy of a star : $E = U + \Omega = \Omega/2 = -U$ (virial theorem)

Because a star shines, E decreases with time

=> Ω decreases ($\Omega < 0$) => R decreases => the star contracts

=> U increases => the temperature of the star increases

- Half of the gravitational energy lost by the star turns into heat ($U = -\Omega / 2$), the other half is radiated away ($E = \Omega / 2$)
- The increase of the central temperature T_c allows the ignition of successive nuclear burning phases



Mass & time scale

- ➢Pressure P increase as mass M increase (pressure balances gravitation), hence temperature T increase.

- ➢Higher rate of energy fusion is favoured by higher T.

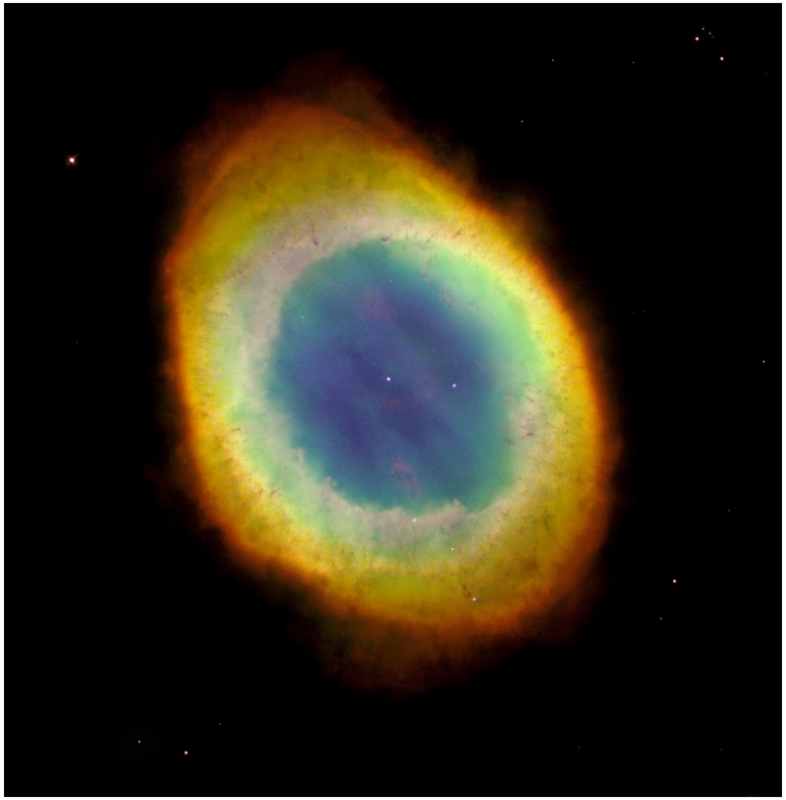
- ➢Equilibrium → fusion energy produced in the core, is balanced by energy lost by radiation at the stellar surface (luminosity L= energy radiated per unit time).

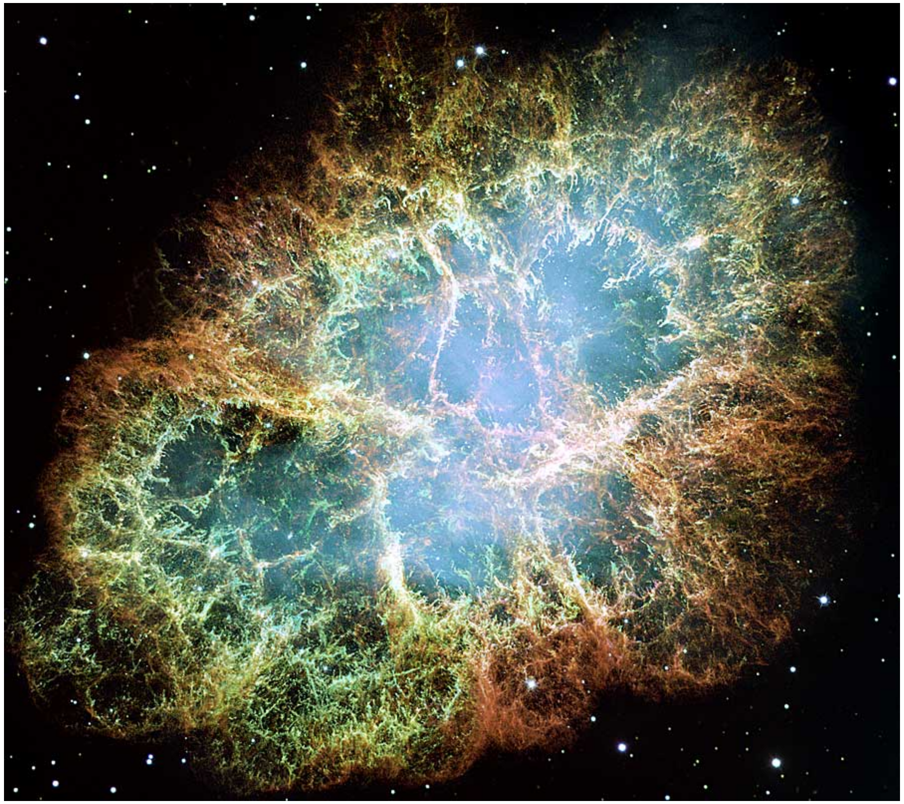
$$\tau_{nucl} = \frac{E_{fusion}}{L}$$

For H burning sequence : $\tau_{nucl} \approx 10^{10}\left(\frac{M}{M_{sun}}\right)^{-2.5}$

Nuclear life τ_{nuc} of massive star is shorter than nuclear life of lower mass.

- The end of the stars depend on their mass (lecture II)





- The Ring Nebula (M57) in the constellation of Lyra: "Planetary" nebula (look for the white dwarf)

- Remnant of the supernova of 1054 (Crab nebula, M1)

  (look for the neutron star)

Principles of stellar structure & evolution: Summary…

- **Structure equations:**

$$\boxed{\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)}$$ Mass equilibrium

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}}$$ Hydrostatistic equilibrium

$$\boxed{\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\varepsilon(r)}$$ Thermal equilibrium

$$\boxed{\left.\frac{dT(r)}{dr}\right|_{rad} = f(L(r), \kappa(r), T)} \quad \boxed{\left.\frac{dT(r)}{dr}\right|_{conv} = f(P(r), T(r))}$$ Energy transport

- **Equation of state:** $$\boxed{P = P(\rho, T, \{X_i\})}$$

- **Nuclear energy production rate:** $$\boxed{\varepsilon = \varepsilon(\rho, T, \{X_i\})}$$

The observed properties of the stars

- Solar mass $M_{\odot} \sim 2 \times 10^{30}$kg

- Angle:

•1 degree = 1° = 1 deg = 1/360 of a circle

•1 arcmin = $1'$ = 1/60 of a degree

•1 arcsec = $1''$ = 1/60 of an arcminute = 1/3600 of a degree

- Distance:

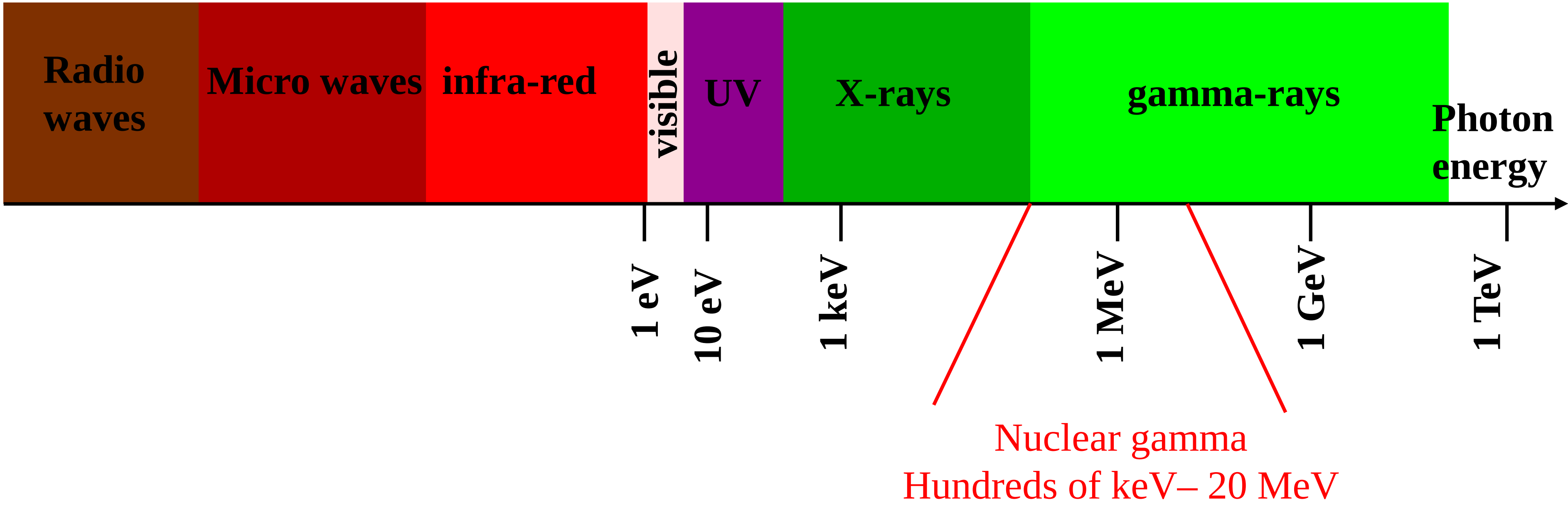
• Astronomical unit (au) $\sim$ 150 000 000 km

• Light year (ly) = 0.95×10^{16} m

• Parsec (pc) = 3.09×10^{16} m = 3.26 ly=> 1 pc = 1 au/ 1 arc sec

Energy – Wavelenght

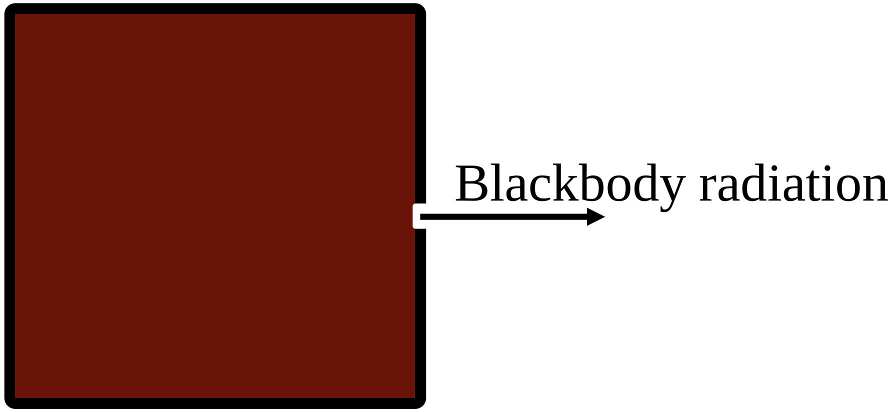
- Energy unit : 1 erg = 1 $g.cm^2.s^{-2} = 10^{-7}$ J = 624.15 GeV
- Stars are observed in different wavelenghts but most often in the optical range



The observed properties of stars: Blackbody radiation

➢ Most important type of radiation is **blackbody radiation**.

➢Lab source of blackbody radiation: hot oven with a small hole which does not disturb thermal equilibrium inside:



Important because:

- Interiors of stars are like this
- Emission from many objects is roughly of this form.

➢ The frequency dependence of blackbody radiation is given by the **Planck function**:

$$\boxed{B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}}$$

- $h = 6.63 \times 10^{-27}$ erg s is Planck's constant
- $k = 1.38 \times 10^{-16}$ erg K^{-1} is Boltzmann's constant
- T is temperature of the blackbody

Basic properties of stars one needs to know in order to compare theory against observations:

- Luminosity L
- Surface temperature – **effective temperature T_e**
- Chemical composition
- Radius R
- Mass M
- Age

For the Sun we have:

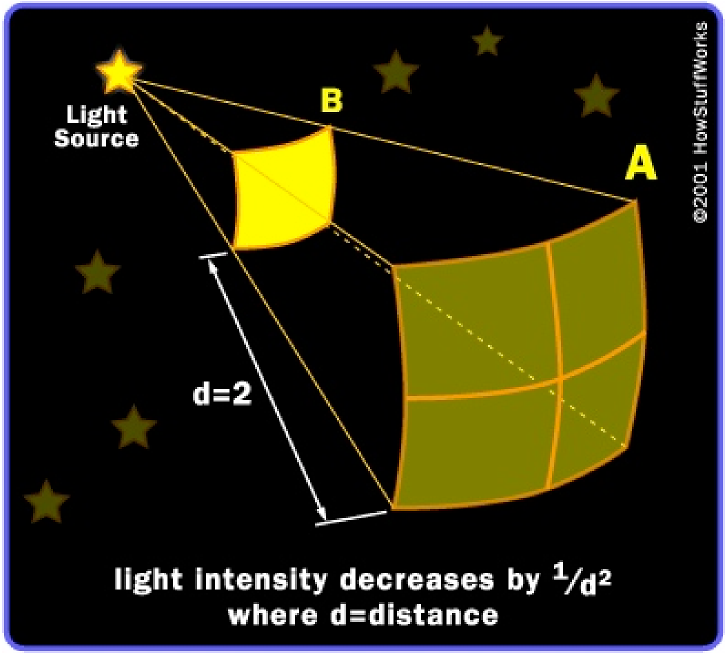
$L = 4.10^{26}$ W
$M = 2.10^{30}$ kg
$R = 7.10^{5}$ km
($T_{surface} \sim 6000$K)
(Age $\sim 4.6\times10^{9}$ y)

- **Luminosity *L* :** the energy per unit time (≡ power) emitted by the star
  - this can apply to any kind of energy but we will usually mean e/m radiation

- **Apparent brightness *F*** or the observed flux: the energy received from the star per unit time per unit area at the distance D (*D* is the distance between surface detection and object observed)
  → The area under consideration must be oriented face-on to line of-sight to the star

Suppose a star emits equally in all directions (the emission is isotropic) and is steady in time. Then, if D is the distance to the star, the **observed flux F** and the **luminosity L** are related according to:

$$L = 4\pi D^2 F \qquad F = \frac{L}{4\pi D^2}$$



The observed properties of stars: Magnitude

➢ 2000 years ago, Hipparcus ranked the apparent brightness of stars according to "magnitudes":
- 1st magnitude $\rightarrow$ brightest stars in sky
- 2nd magnitude $\rightarrow$ bright but not brightest
- …
- 6th magnitude $\rightarrow$ faintest stars visible to human eye

➢ This system is based on visual perception (which is a logarithmic system): mag 1 is factor of f brighter than mag 2 which is is factor of f brighter than mag 3...

➢ Modern definition: If two stars have fluxes F1 and F2, then their **apparent magnitudes** m1 and m2 are given by:

$$\boxed{m_2 - m_1 = -2.5 \log_{10} \frac{F_2}{F_1}}$$

Note: The star Vega was defined to have an apparent magnitude of zero! (**now:** +0.03)

This allows one to talk about the apparent magnitude of a given star rather than just differences in apparent magnitudes $\rightarrow$ $m = k - 2.5 \log_{10} F$

The value of cst k is set by reference to Vega

- The interesting physical quantity is the luminosity : $L = 4\pi D^2 F$
  where D is the distance

- To compare the luminosity of different objects, we bring them to a common distance, chosen to be 10 pc

- The **absolute magnitude (M)** of a star is its apparent magnitude if it were placed at a distance of 10pc.

$$m - M = -2.5\log_{10}\left(\frac{L}{4\pi D^2} \times \frac{4\pi(10)^2}{L}\right) = 5\log_{10}\left(\frac{D_{pc}}{10}\right)$$

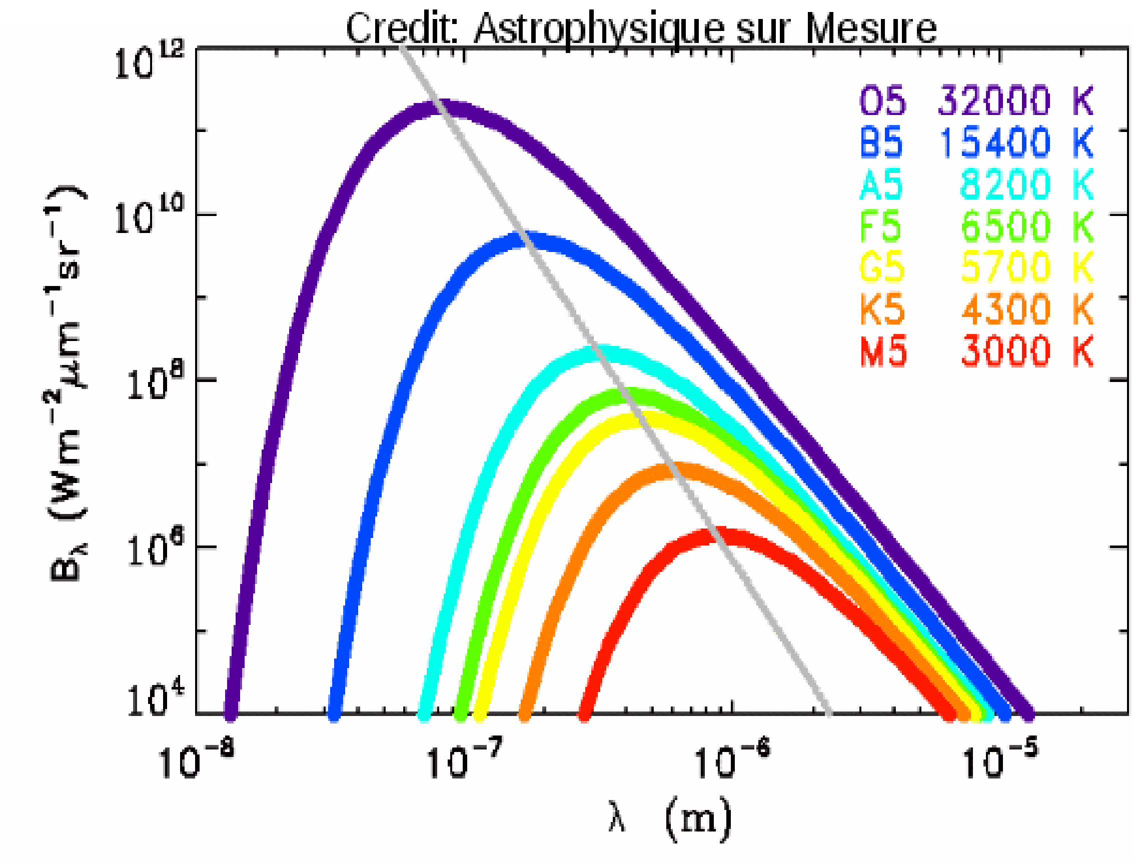
where D_{pc} is the distance in pc & **m − M** is called the **distance modulus**

All of this sounds complicated… but just keep in mind that:

- Apparent magnitude ↔ Apparent brightness or observed flux
- Absolute magnitude ↔ luminosity
- Distance modulus ↔ distance

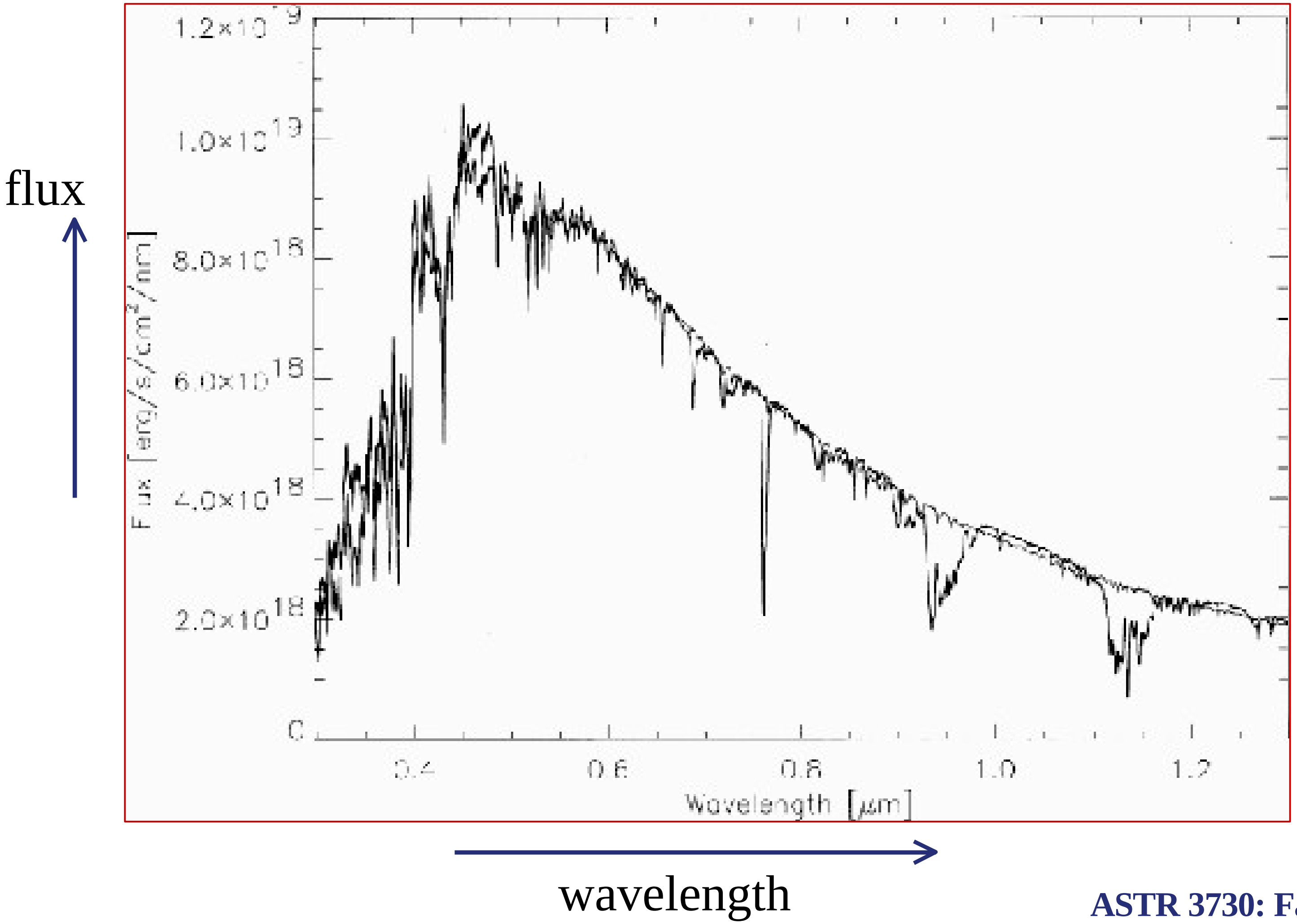
➢ A star has a "blackbody" spectrum (first approximation):

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$$



→ for many stars the blackbody approximation is not such a good one, since their spectrum contain absorption and emission lines, and might have several thermal components in their spectrum (e.g, photosphere, corona, etc.).

Solar spectrum - approximately of blackbody form. Very cool stars show larger differences from thermal spectra



The observed properties of stars: Effective temperature & colours

➢ The **effective temperature** of a star is defined as the temperature of a black body having the same radiated power per unit area. So, for a star with radius *R* (radius of the visible surface) and luminosity *L*, the effective temperature would be defined by:

$$\boxed{L = 4\pi R^2 \sigma T_{eff}^4} \qquad \text{(Stephan-Boltzmann law)}$$

Where σ is the Stefan-Boltzmann's constant $\approx 5.67\times10^{-8}\ W\ m^{-2}\ K^{-4}$

Ex: T_{eff}(sun)= 5780 K

➢ Since stars are often very faint, one cannot easily measure monochromatic fluxes. For that reason one often uses **colours** in astronomy.

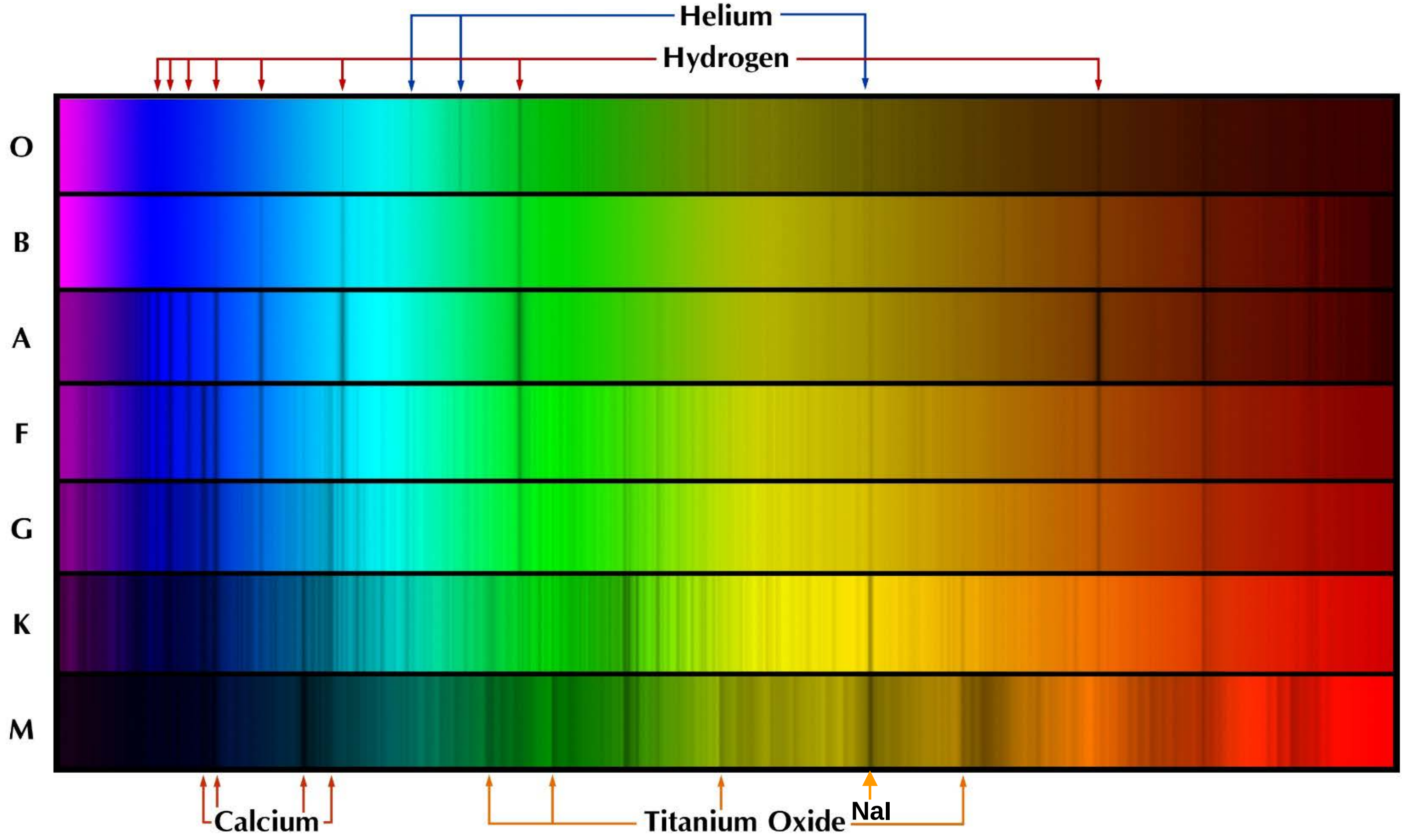
➢ **Colours** are defined by comparing power outputs over different parts of the spectrum.

➢ The most commonly used filter system to observe stars is Johnson (1966)'s UBV system → 3 filters : U (Ultraviolet) central value: λ= 360 nm, width= 70 nm
B (Blue) central value: λ= 440 nm, width= 100 nm
V (Visible) central value: λ=550 nm, width= 90 nm

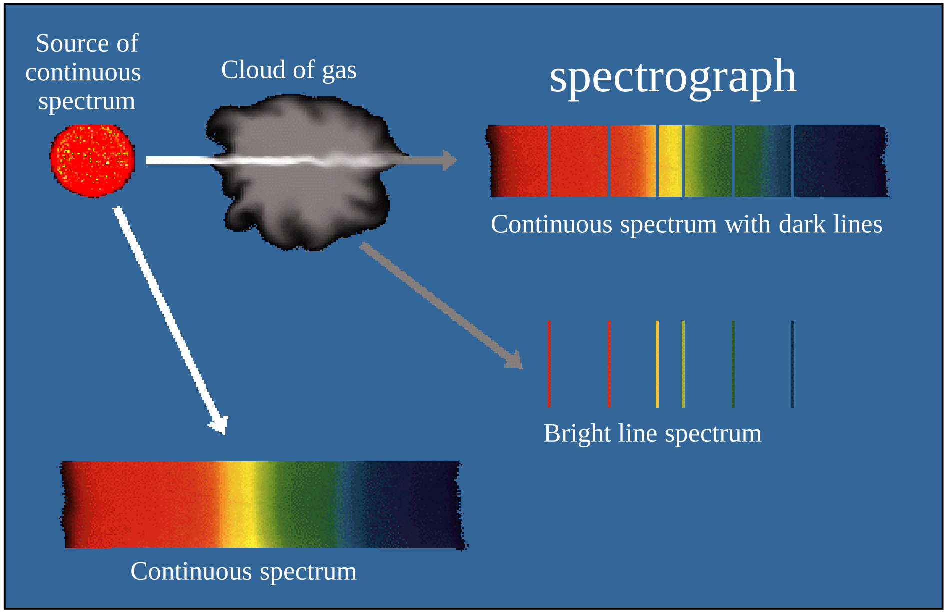
The observed properties of stars: The electromagnetic spectrum

➢ Stellar spectra possess much more information than simple color… the presence /strengths of the absorption lines characterize the nature of the outer layers of the star & their temperature

- In 1901, Annie Cannon showed that stars can be classified into seven groups (spectral-type) according to strengths of absorption lines: O B A F G K M
- In 1921, Cecilia Payne showed that all stars are composed mostly of H and He; spectral differences reflect differences of temperature, not only composition



The observed properties of stars: The electromagnetic spectrum



- ➢ Each element absorbs light of a particular frequency
  → a particular color

Stellar spectra like barcodes

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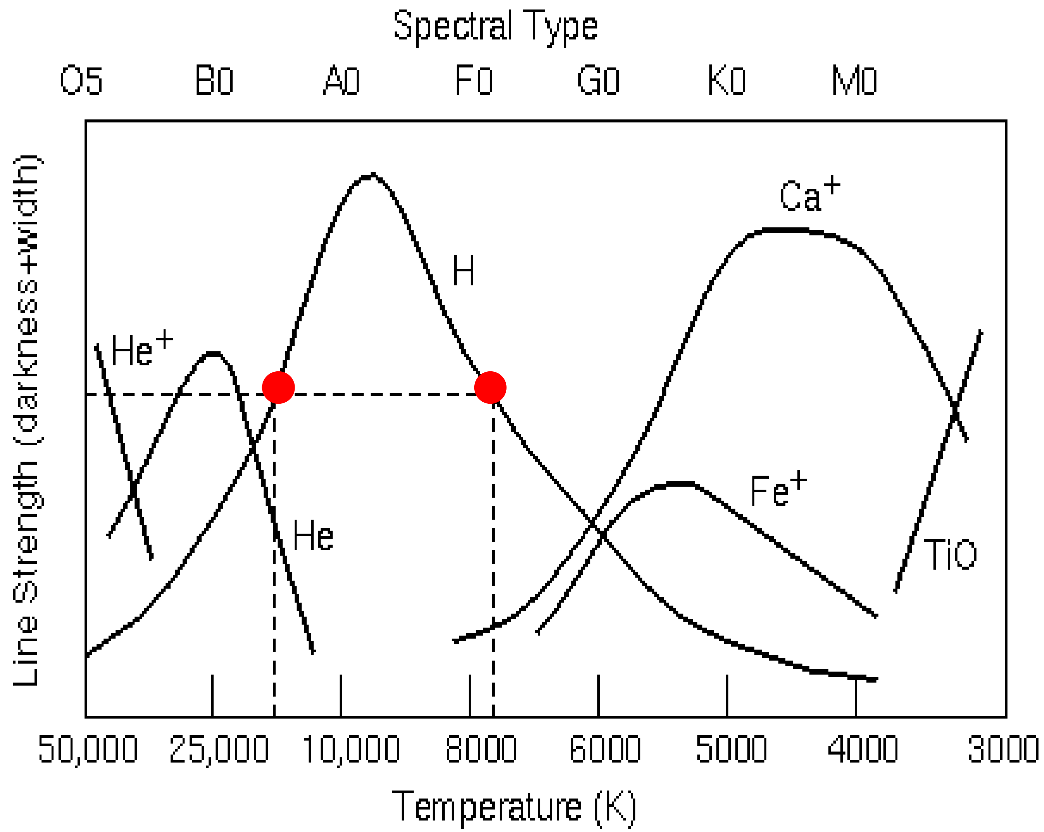
Informations on:

surface temperature
- chemical composition
- excitation/ionization degree
- gas pressure and density
- relative velocity of source
- rotation or expansion
- …

- The strength and pattern of the absorption lines does vary among the stars. Some stars have strong (dark) hydrogen lines, other stars have no hydrogen lines but strong calcium and sodium lines.
- The temperature of the star's photosphere determines what pattern of lines you will see $\Rightarrow$ you can determine the temperature of a star from the pattern of absorption lines you see and their strength.

- Cross-referencing each elements' line strengths narrows the possible temperature range. It gives an accurate temperature with an uncertainty of only 20 to 50 K.



The observed properties of stars: Chemical composition & Spectral class

➢ From absorption and emission lines the chemical composition in the photosphere of the star can be determined. We define:

X = the relative mass fraction of hydrogen
Y = the relative mass fraction of helium
Z = the relative mass fraction of all other elements.

For the Sun

X~0.75, Y~0.23 & Z~0.02

Class	Teff	Colour	Absorption lines
O	>25000 K	blue	Nitrogen, carbon, ionized helium & oxygen
B	10000 – 25000 K	Blue-white	Neutral helium, hydrogen
A	7500-10000 K	white	hydrogen
F	6000-7500 K	Yellow-white	Metals: Fe, Ti, Ca, Mg
G	5000 – 6000 K	Yellow (Sun)	Ca, hydrogen, metals
K	3500 – 5000 K	Yellow-orange	Metals & titanium oxide
M	< 3500 K	Red	Metals & titanium oxide

- Metal (astronomy) : every chemical element heavier than helium

- Metallicity of a star: the mass fraction Z of elements heavier than He

- Metallicity :
$$[Fe/H] = \log_{10}\left(\frac{n_{Fe}}{n_H}\right)_{star} - \log_{10}\left(\frac{n_{Fe}}{n_H}\right)_{Sun}$$

n_H and n_{Fe} : numbers of H and Fe per unit volume (density)

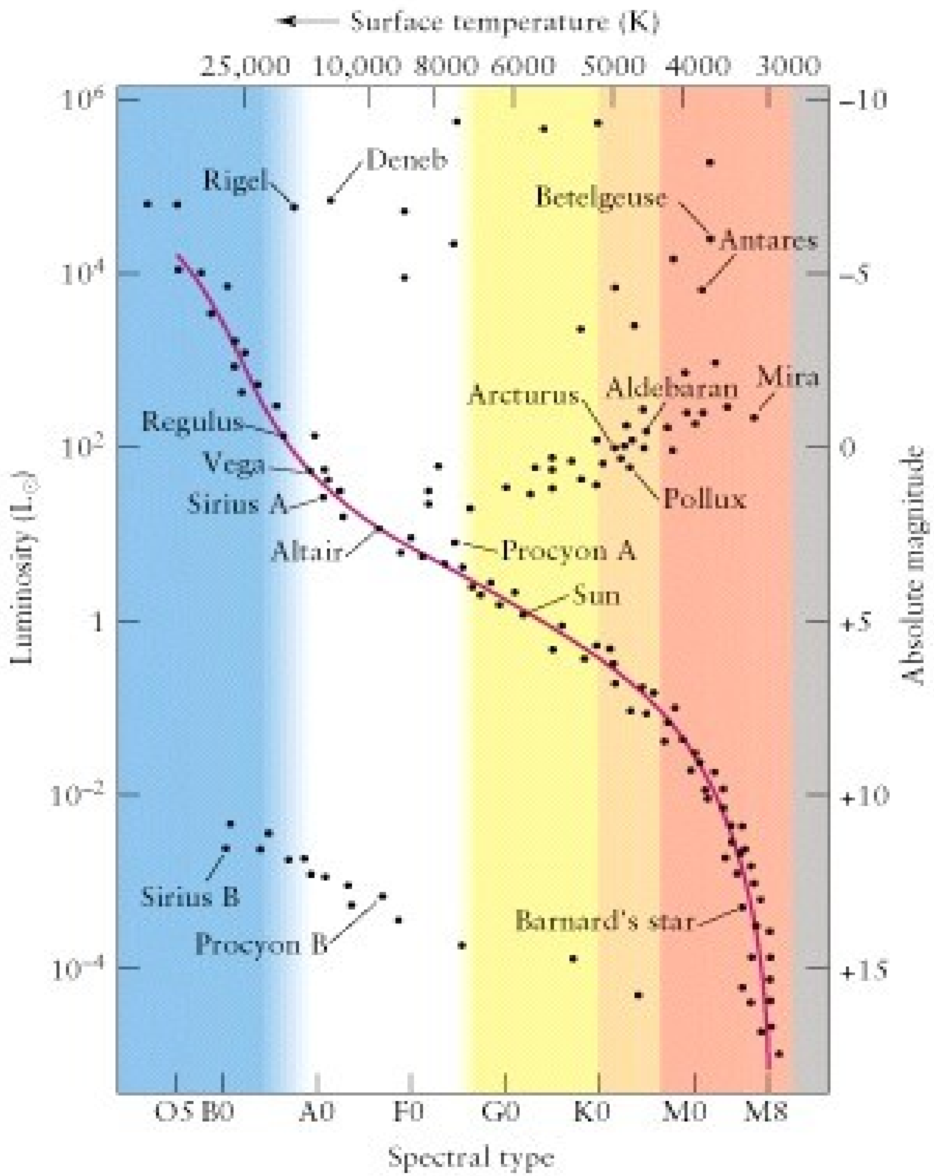
- The Fe abundance (n_{Fe}/n_H) is one of the most simple to measure in stellar optical spectra

- $[Fe/H] = 0$ (metallicity of the proto-solar cloud 4.6×10^9 yrs ago)

- Population I (Pop I) : "metal–rich" stars. Relatively "young" stars (our sun)

- Population II (Pop II) : "metal–poor" stars. Older than Pop I.

The observed properties of stars: The Hertzprung Russel (H-R) diagram



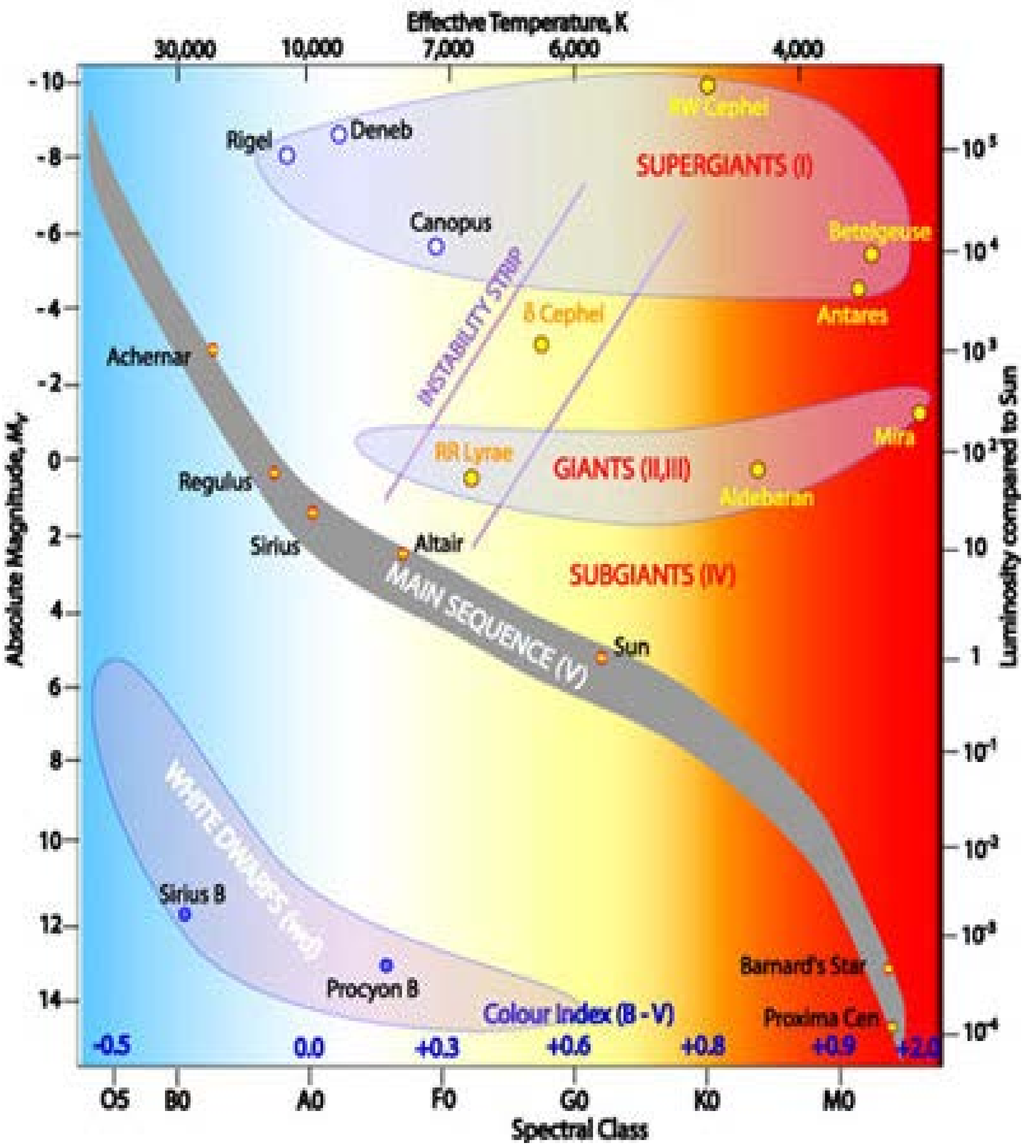
In 1905, Hertzsprung & Russell, noticed that the luminosity of stars decreased from spectral type O to M → Plot absolute magnitude or luminosity for a star versus its spectral type or effective temperature to look for families of stellar type

→ **The H-R diagram**

- Hot stars are plotted on the left, and cool stars on the right
- Bright stars at the top, faint stars at bottom
- Our sun seats near the middle
- Luminosity is displayed as fractions of a star's absolute luminosity compared to the Sun.

The observed properties of stars: The Hertzprung Russel (H-R) diagram

→ "Rosetta stone" of stellar astronomy



➢About 90% of the stars are located on a diagonal band, which goes from cool/faint to hot/bright.
→The main sequence (MS).

➢ The Sun is a **G2** main sequence star.

Luminosity classes :

I → Supergiants

II & III → Giants

IV → Subgiants

V → Main sequence

VI → White dwarfs

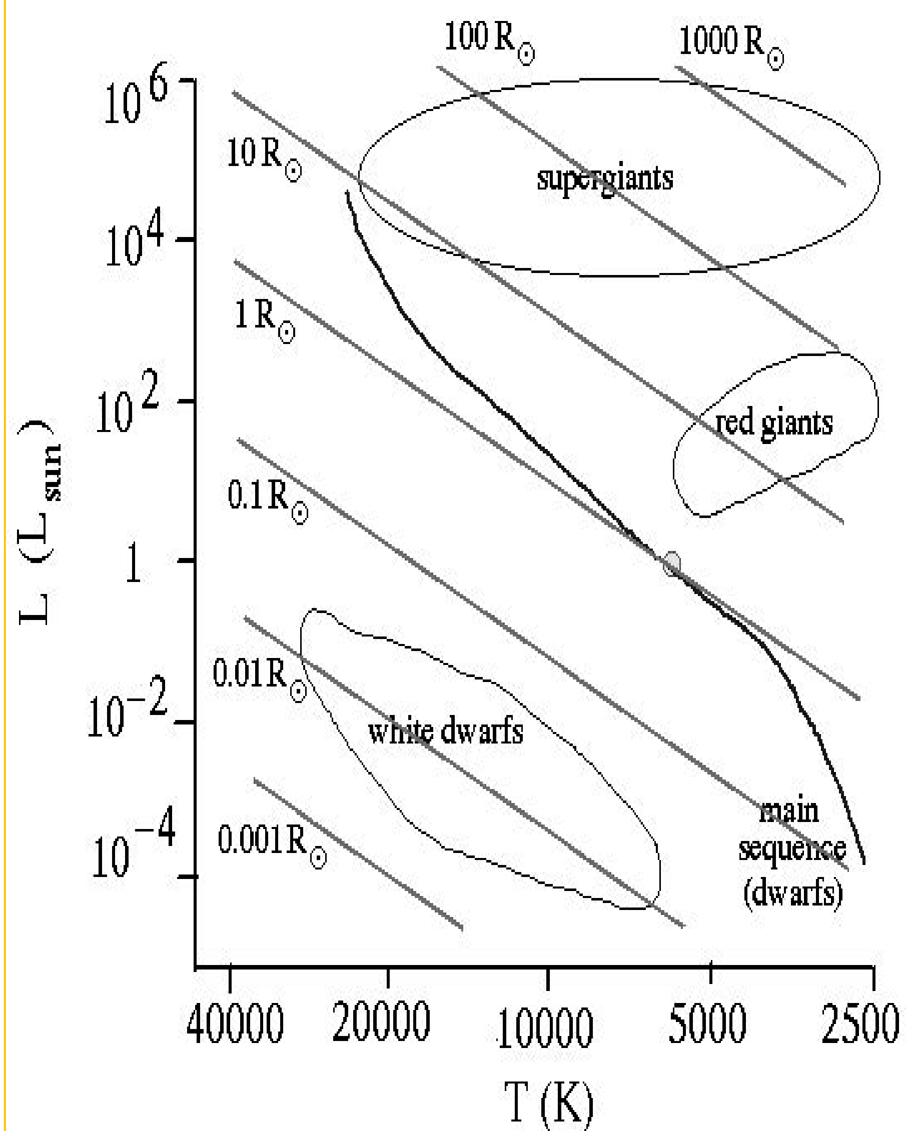
The observed properties of stars: HR-diagram & the size of stars

- $L = 4 \pi R^2 \sigma T^4$ $\Rightarrow$ hot things emit more light. But a star's brightness also depends on its size: the larger the area, the more cm^2 are emitting and the more light you get.

- Some stars are very cool, but also very bright. Since cool objects don't emit much light, these stars must be huge → red giants

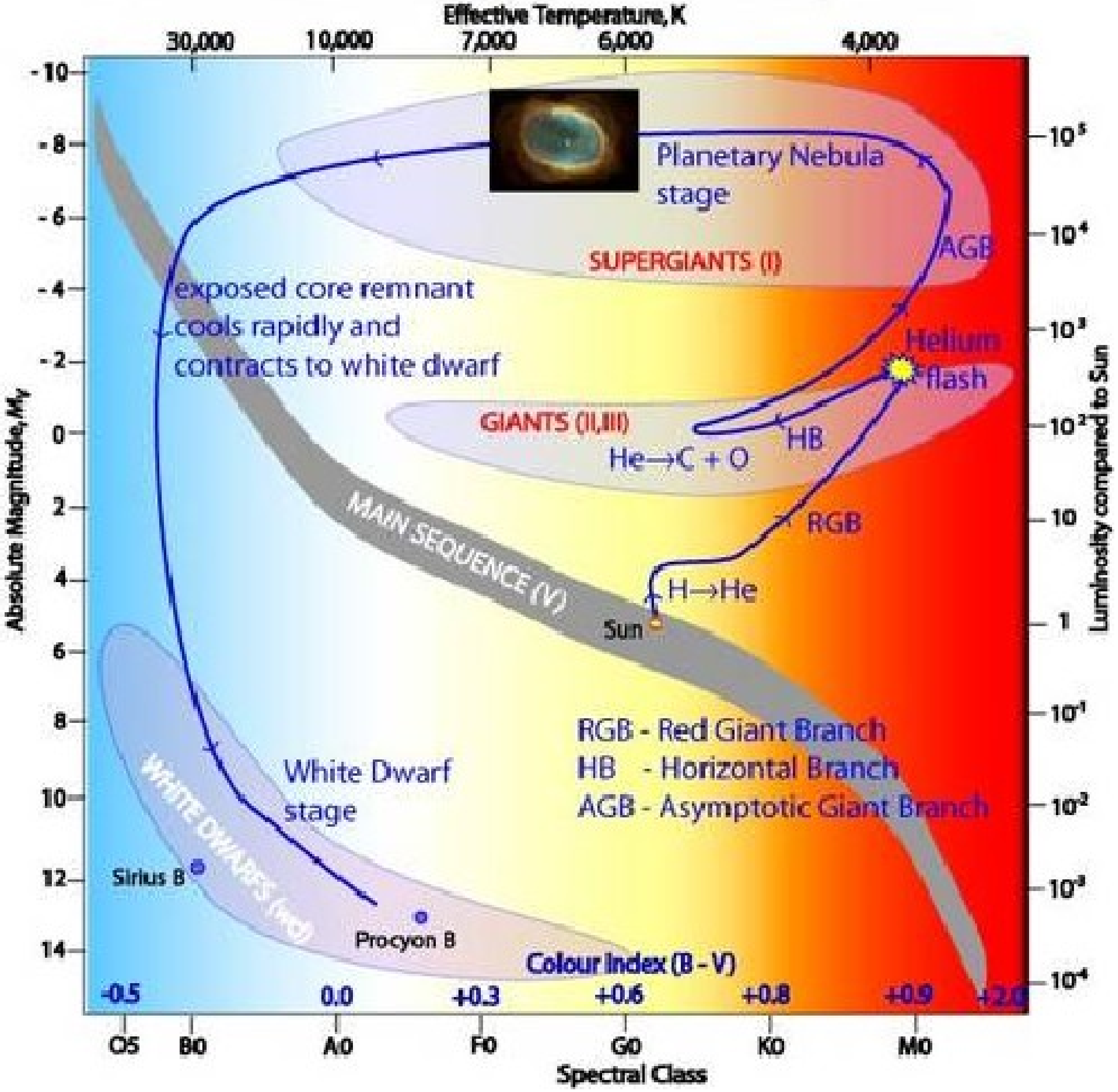
- Some stars are faint, but very hot. These must therefore be very small →white dwarf stars.

- The R^2 term is a straight line on an HR diagram → star's size easy to read one L and T (or color) are known



The observed properties of stars: The H-R diagram & stellar evolution

→ Key tool in tracing the evolution of stars (we will come back to HR-Diagram in lecture II)



Chemical Abundances

Chemical abundances: How solar abundances are determined?

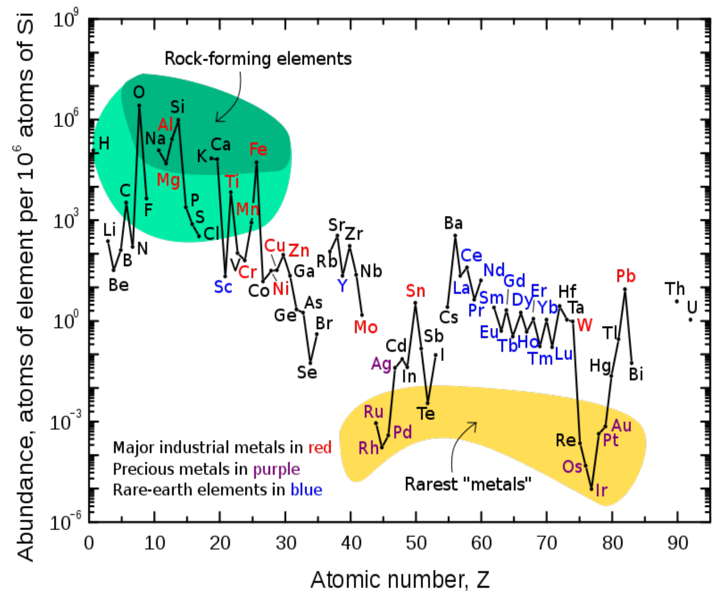
1. Earth material (crusts,…)

Problem: chemical fractionation modified the local composition strongly compared to pre-solar nebula and overall solar system.

for example: Quartz is 1/3 Si and 2/3 Oxygen and not much else. This is not the composition of the solar system.

But: Isotopic compositions mostly unaffected (as chemistry is determined by number of electrons (protons), not the number of neutrons).

→ **main source for isotopic composition of elements**



2. Solar spectra

Sun formed directly from presolar nebula - (largely) unmodified outer layers create **spectral features**

3. Unfractionated meteorites

Certain classes of meteorites formed from material that never experienced high pressure or temperatures and therefore was never fractionated.
These meteorites directly sample the pre-solar nebula

Chemical abundances: from stellar spectra

corona
up to 2 MK

hot thin gas
→ **emission lines**

---

chromosphere
~ 10,000 km
up to 25,000 K

hot thin gas
→ **emission lines**

---

photosphere
~ 500 km
~ 6000 K

photons escape freely

still dense enough for photons to excite atoms when frequency matches
→ **absorption lines**

---

convective zone

radiation transport (short photon mean free path)

continuous spectrum

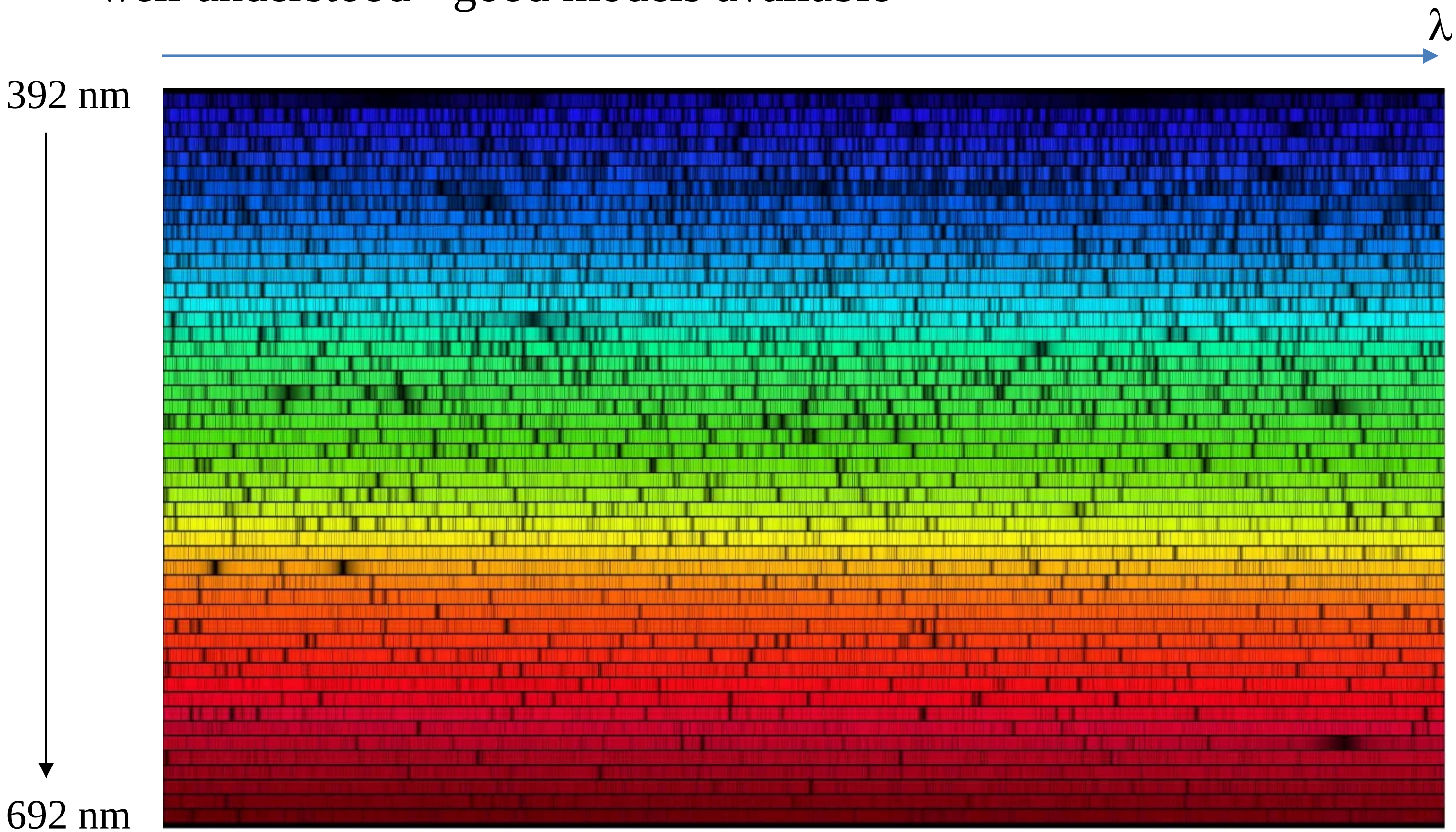
Emission lines from atomic deexcitations
Absorption lines from atomic excitations

Wavelength → Atomic Species
Intensity → Abundance

Absorption Spectra:

provide majority of data because:

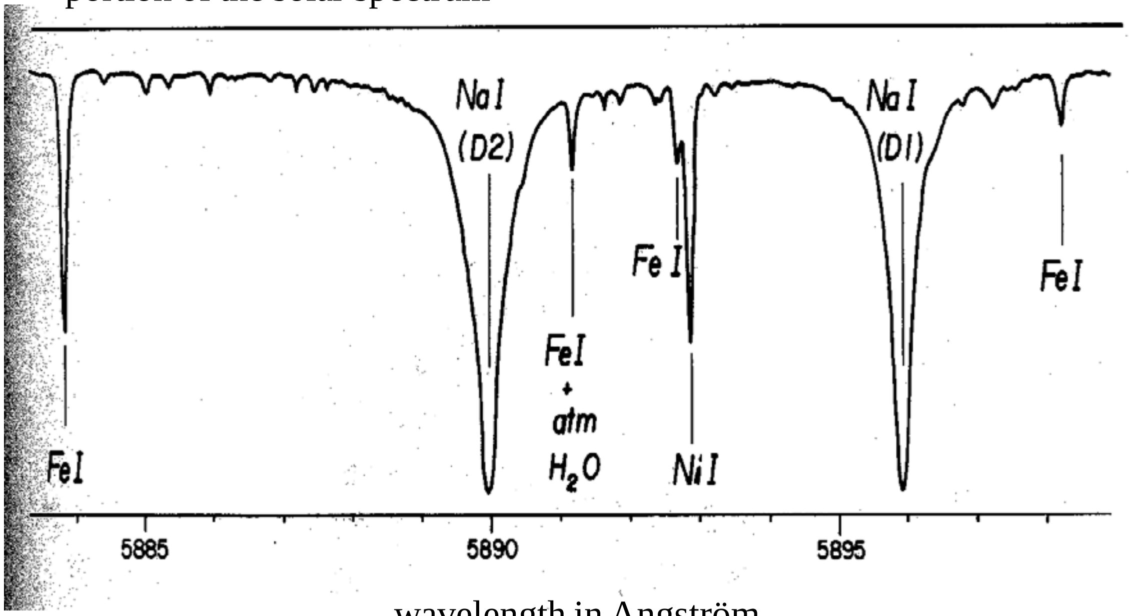
- by far the largest number of elements can be observed
- least fractionation as right at end of convection zone - still well mixed
- well understood - good models available



solar spectrum (R. Kurucz, KittPeak National Observatory)

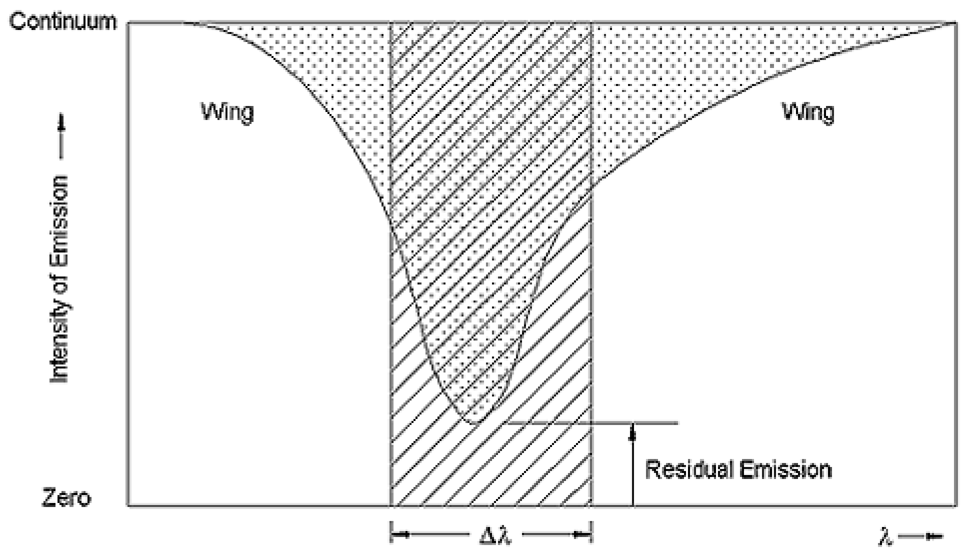
Each line originates from absorption from a specific atomic transition in a specific atom/ion:

portion of the solar spectrum



wavelength in Angström

Fe I: neutral ion
FeII: singly ionized iron ion
…



effective line width ~ total absorbed intensity

Simple model consideration for absorption in a slab of thickness Δx:

$$I = I_0 \ e^{-\sigma n \Delta x}$$

I, I_0 = observed and initial intensity
σ = absorption cross section
n = number density of absorbing atom

So if one knows σ one can determine n and get the abundances.

There are 2 complications:

Complication (1) Determine σ

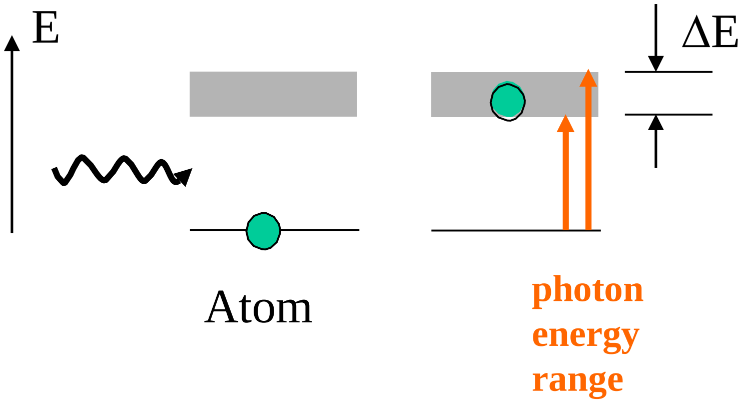
The cross section is a measure of how likely a photon gets absorbed when an atom is bombarded with a flux of photons (more on cross section later …)

It depends on:

•**Oscillator strength** a quantum mechanical property of the atomic transition. It expresses the probability of absorption or emission of e.m radiation in transitions between energy levels of an atom → Needs to be measured in the laboratory - not done with sufficient accuracy for a number of elements.

• **Line width**

the wider the line in wavelength, the more likely a photon is absorbed (as in a classical oscillator).



excited state has an energy width ΔE. This leads to a range of photon energies that can be absorbed and to a line width

Heisenbergs uncertainty principle relates that to the **lifetime** τ of the excited state

$$\Delta E \cdot \tau = \hbar$$

→ need lifetime of final state

The lifetime of an atomic level in the stellar environment depends on:

•**The natural lifetime** (natural width)

lifetime that level would have if the atom is left undisturbed

•**Frequency of Interactions of atom with other atoms or electrons**

Collisions with other atoms or electrons lead to deexcitation, and therefore to a shortening of the lifetime and a broadening of the line

Varying electric fields from neighboring ions vary level energies through Stark Effect

$\rightarrow$ depends on **pressure**

$\rightarrow$ need local **gravity**, or **mass/radius** of star

•**Doppler broadening** through variations in atom velocity

- thermal motion $\longrightarrow$ depends on **temperature**
- micro turbulence

Need detailed and accurate model of stellar atmosphere !

Complication (2)

Atomic transitions depend on the state of ionization !

The number density n determined through absorption lines is therefore the number density of ions in the ionization state that corresponds to the respective transition.

to determine the total abundance of an atomic species one needs the fraction of atoms in the specific state of ionization.

Notation: I = neutral atom, II = one electron removed, III=two electrons removed …..

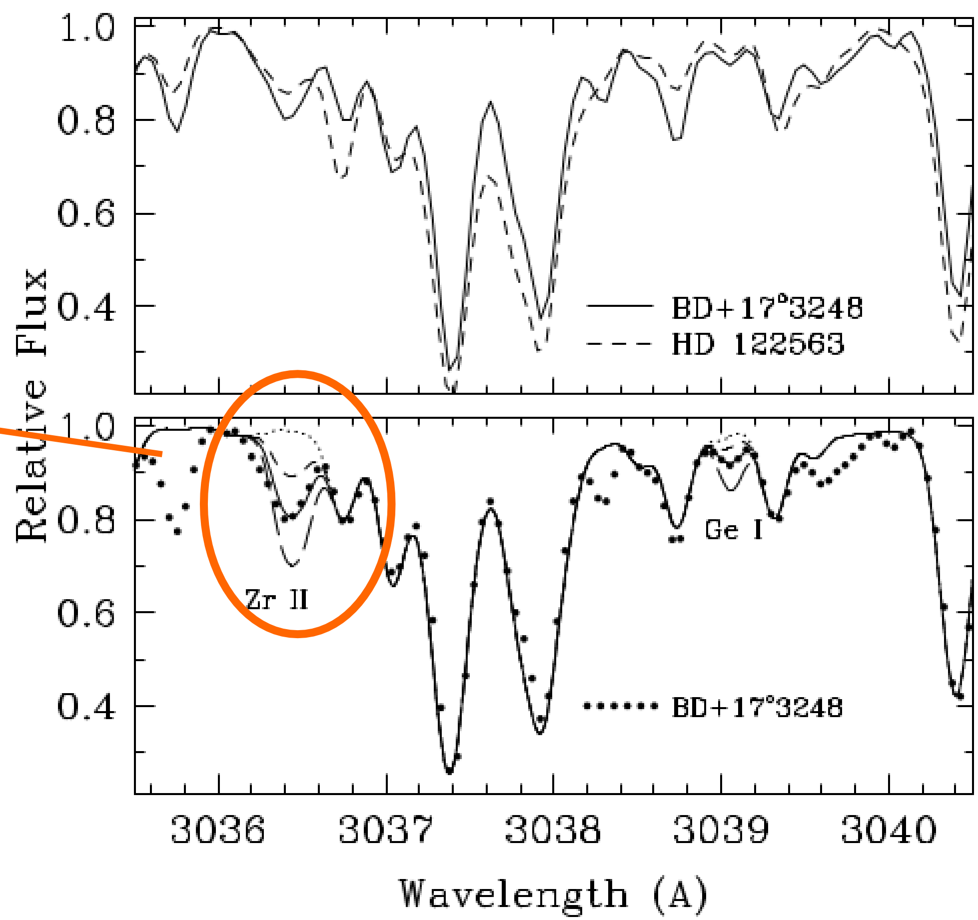
Example: a CaII line originates from singly ionized Calcium

Chemical abundances: from stellar spectra

Practically, one sets up a stellar atmosphere model, based on star type, effective temperature etc. Then the parameters (including all abundances) of the model are fitted to best reproduce all spectral features, incl. all absorption lines (can be 100's or more) .

Example: Spectrum of a BD +17° 3248 halo star obtained with Hubble space telescope

varied ZrII abundance



(Cowan et al. ApJ 572 (2002) 861)

Emission Spectra:

Disadvantages:
- **less understood, more complicated solar regions (Chromosphere & Corona)** (it is still not clear how exactly these layers are heated)
- **some fractionation/migration effects** for example : species with low first ionization potential are enhanced in respect to photosphere possibly because of fractionation between ions and neutral atoms

Therefore abundances less accurate

But there are elements that cannot be observed in the photosphere (for example helium is only seen in emission lines)

this is how Helium was discovered by Sir Joseph Lockyer of England in 20 October 1868.

Solar Chromosphere red from Hα emission lines

<u>Complication (3)</u>

All solar spectroscopic methods determine the **PRESENT DAY** composition on the surface of the sun

The solar abundances are defined as the composition of the presolar nebula

Diffusion effects modify the surface composition !!!
(can be accounted for by solar models that calculate the evolution from the initial bulk composition of the sun to the present day surface composition)

Meteorites can provide accurate information on elemental abundances in the presolar nebula. More precise than solar spectra if data are available …

Bus some gases escape and cannot be determined this way (for example hydrogen, or noble gases

Not all meteorites are suitable - most of them are fractionated and do not provide representative solar abundance information.

One needs primitive meteorites that underwent little modification after forming.

Classification of meteorites:

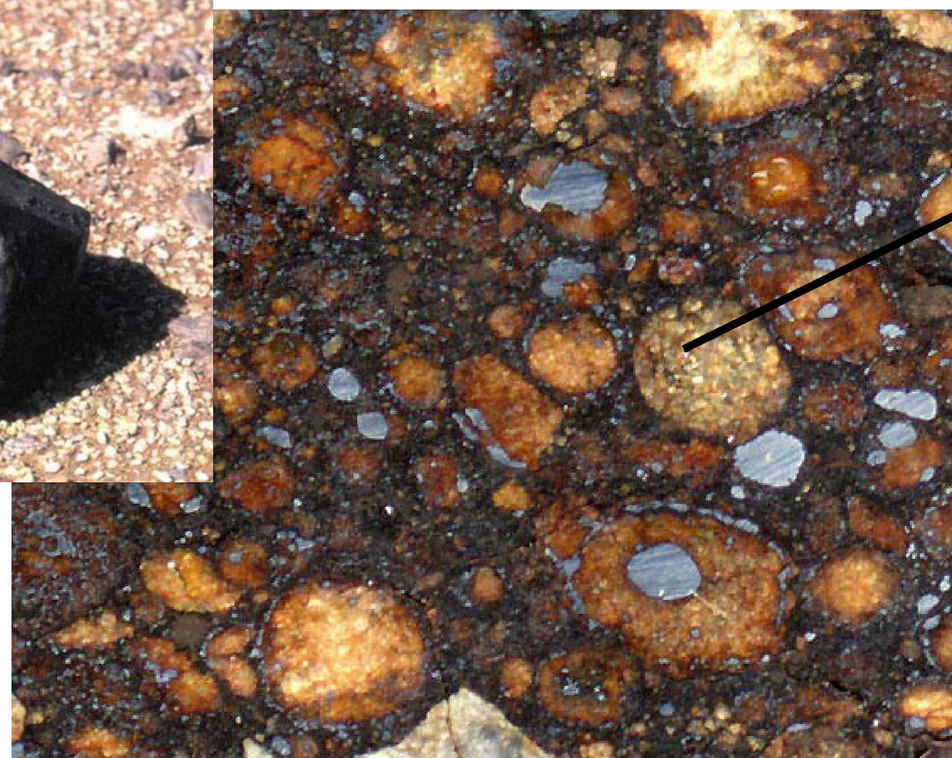
Group	*Subgroup*	*Frequency*
Stones	Chondrites	86%
	Achondrites	7%
Stony Irons		1.5%
Irons		5.5%

Chemical abundances: Meteorites

<u>carbonaceous chondrites</u> <u>(~6% of falls)</u>

Chondrites: Have Chondrules - small ~1mm size spherical inclusions in matrix believed to have formed very early in the presolar nebula accreted together and remained largely unchanged since then.

Carbonaceous Chondrites have lots of organic compounds that indicate very little heating (some were never heated above 50 degrees)



Chondrule

How find them ?

How can we find carbonaceous chondrites?

Explorer par catégories | carbonaceous chondrite | Toutes les catégories | Rechercher

Rare NWA 2086 Stony Meteorite (Carbonaceous Chondrite CV3) .371g

11,27 EUR
0 enchère
+21,15 EUR de frais de livraison

Il reste 17 h (Vendredi, 5:05)
Provenance : États-Unis
Formalités douanières et suivi international fournis

2 Rare NWA 1934 Stony Meteorites (Carbonaceous Chondrite CV3) - 1.47g & 1.2g

20,81 EUR
0 enchère
+17,81 EUR de frais de livraison

Il reste 17 h (Vendredi, 5:11)
Provenance : États-Unis
Formalités douanières et suivi international fournis

Tagish Lake meteorite (Canada) - Unique C2 carbonaceous chondrite

17,35 EUR
1 enchère
+8,63 EUR de frais de livraison

Il reste 1 j 5 h (Vendredi, 17:09)

64.2g end cut NWA 5546 CV3 CARBONACEOUS CHONDRITE METEORITE, CRUSTED

260,20 EUR
ou Offre directe
+12,14 EUR de frais de livraison

Provenance : États-Unis

In the desert

more on meteorites http://www.meteorite.fr

Chemical abundances: Meteorites

Not all carbonaceous chondrites are equal (see http://www.daviddarling.info/encyclopedia/C/carbchon.html)

There are CI, CM, CV, CO, CK, CR, CH, CB, and other chondrites

CI Chondrites (~3% of all carbonaceous chondrites)

- They have the same chemical composition than the sun's photosphere, except H and He
- Are considered to be the least altered meteorites available
- Some chemical alterations but assumed to occur in closed system so no change of overall composition
- Named after Ivuna Meteorite (Dec 16, 1938 in Ivuna, Tanzania, 705g)

- **Only 5 known meteorites contain CIs chondrites– only 4 suitably large (Alais, Ivuna, Orgueil, Revelstoke, Tonk)**
- See Lodders et al. Astrophysical Journal. 591 (2003) 1220 for a recent analysis