

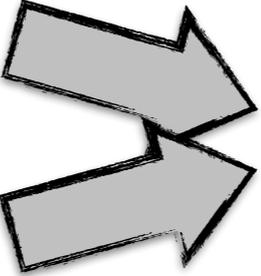
# NPAC course on Astroparticles

## I - ASTRONOMY: the Milky Way

# Electromagnetic waves (photons)

Most of the information we have on celestial objects comes from the study of the electromagnetic radiation (photons) they emit

photons move at the speed of light:  $c = 3 \times 10^{10} \text{ cm/s}^*$

are characterised by a wavelength:  $\lambda$    $\nu \lambda = c$   
a frequency:  $\nu$

and an energy:  $\epsilon = h\nu^{**}$

Planck's constant:  $h = 6.6 \times 10^{-27} \text{ cm}^2 \text{ g s}^{-1}$

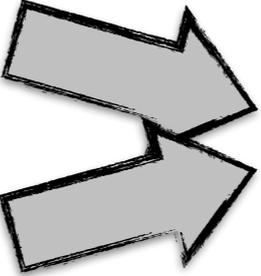
\* astronomers use mostly Gauss units

\*\* very often expressed in eV ( $1 \text{ eV} = 1.6 \times 10^{-12} \text{ erg}$ )

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a body at temperature  $T$  emits a thermal radiation @characteristic energy:  $\epsilon = k T$

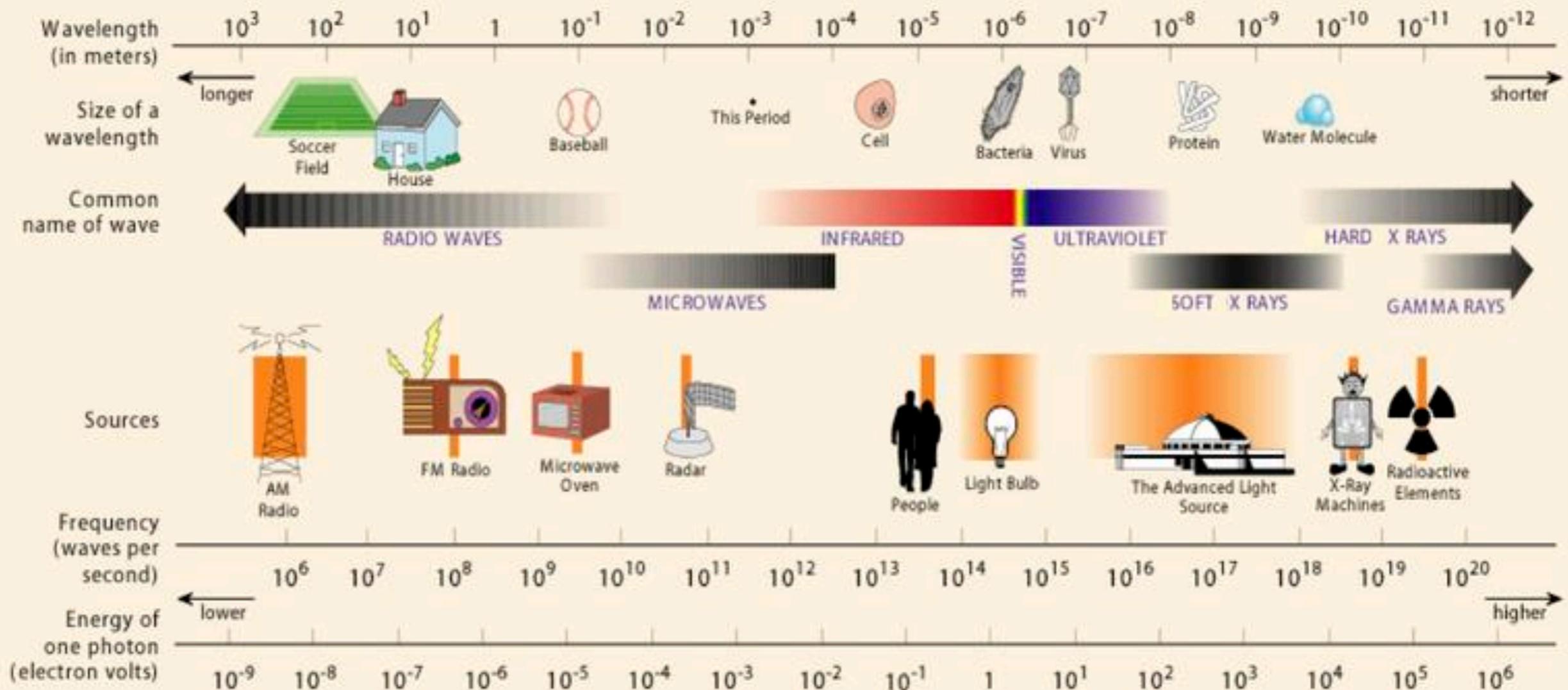
Boltzmann's constant:  $k = 1.4 \times 10^{-16} \text{ erg/K}$

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# The electromagnetic spectrum

## THE ELECTROMAGNETIC SPECTRUM



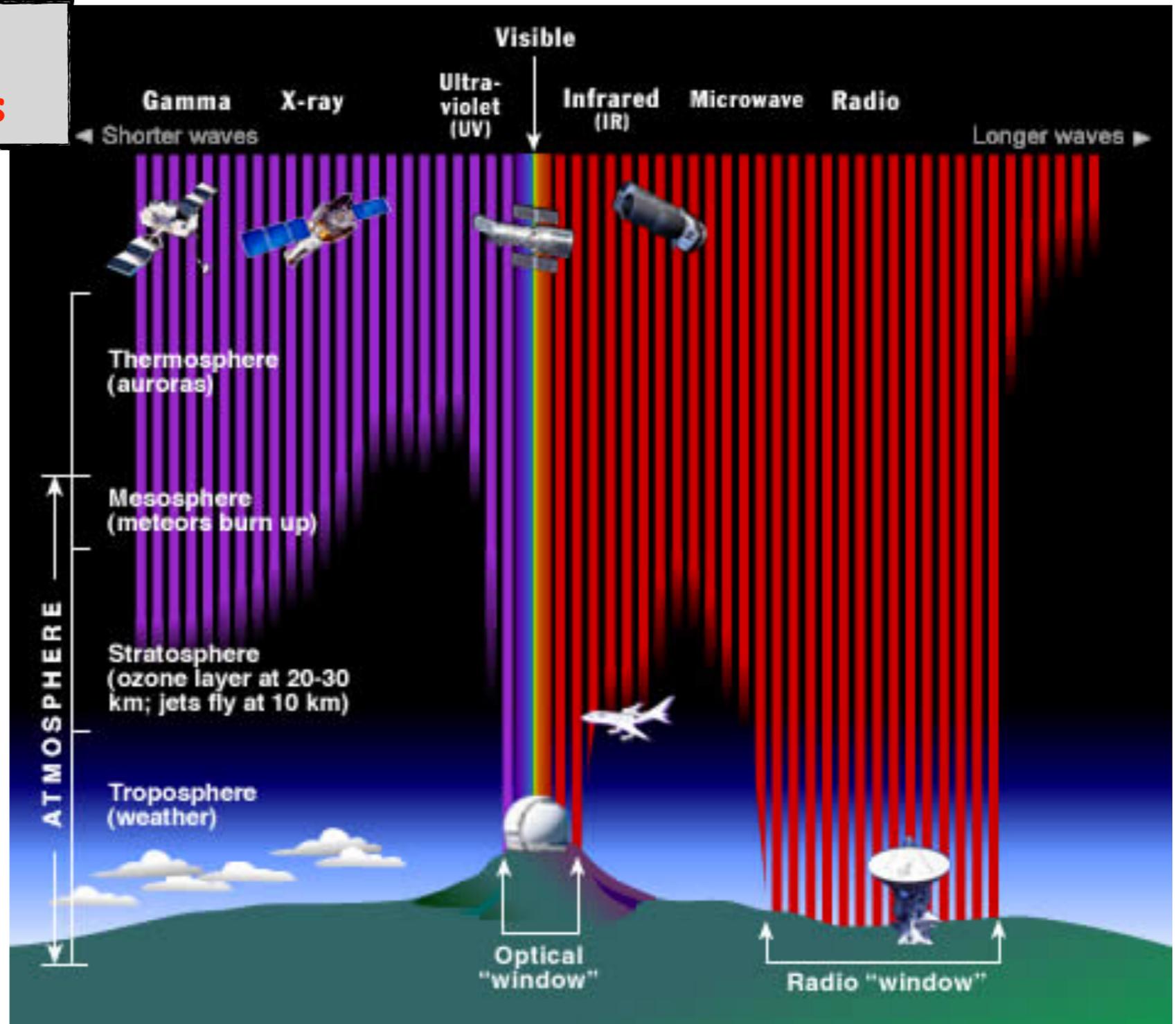
# The electromagnetic spectrum

The Earth's atmosphere is opaque to most frequencies

light (visible by human eye)

3800 – 7600 Å

400 – 790 THz



Optical

Radio

# The Milky Way in the night sky



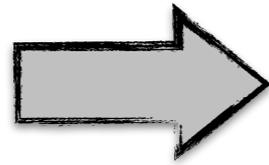
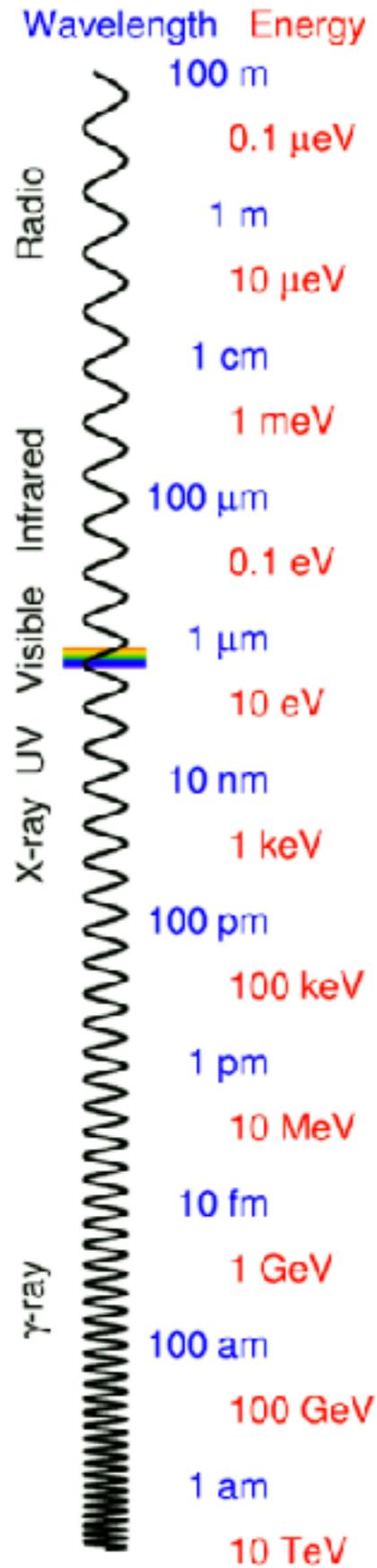
Visible -> stars

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Visible -> stars

# The electromagnetic spectrum

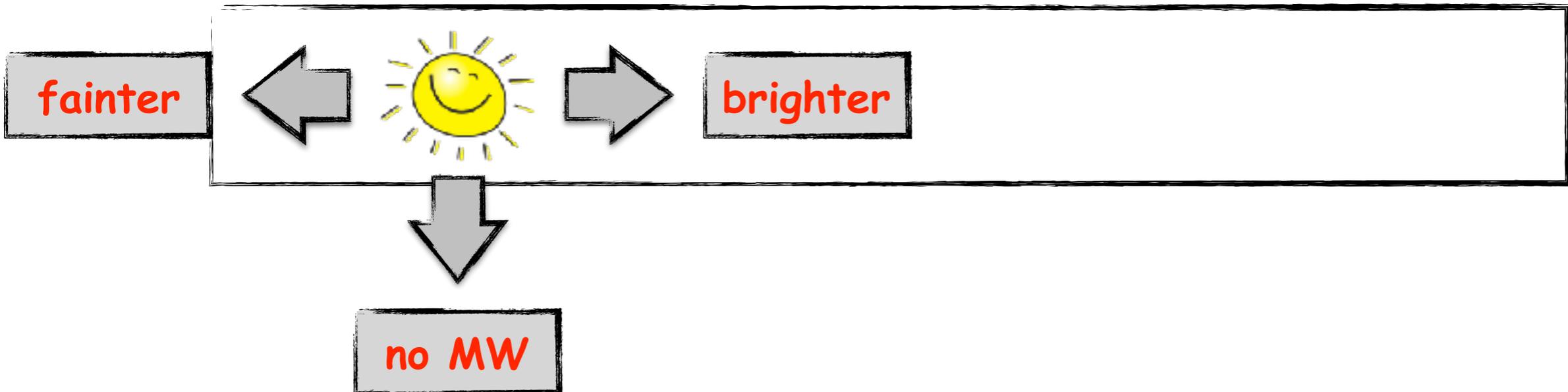


visible light -> stars

# The Milky Way

The Milky Way is brighter when observed from the southern hemisphere

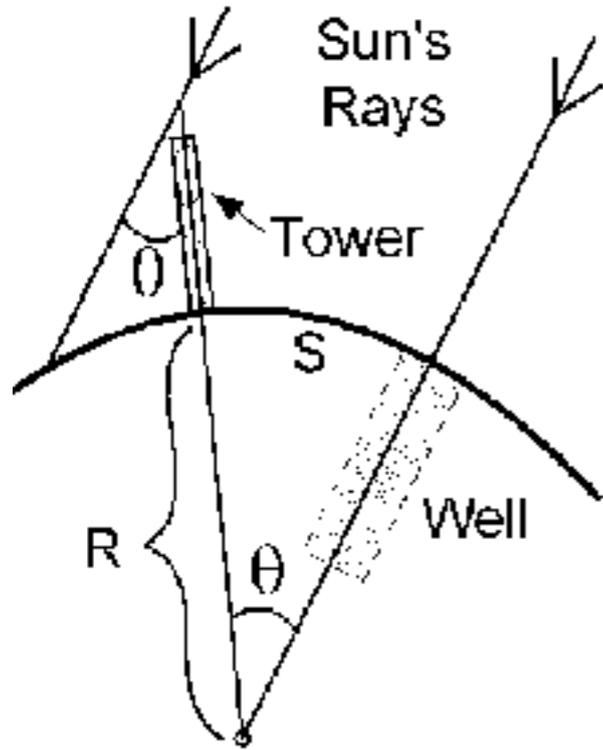
how big (and massive, and bright, ...) is the MW?



Our galaxy is a disk system and we are located away from the centre...

# The Earth radius

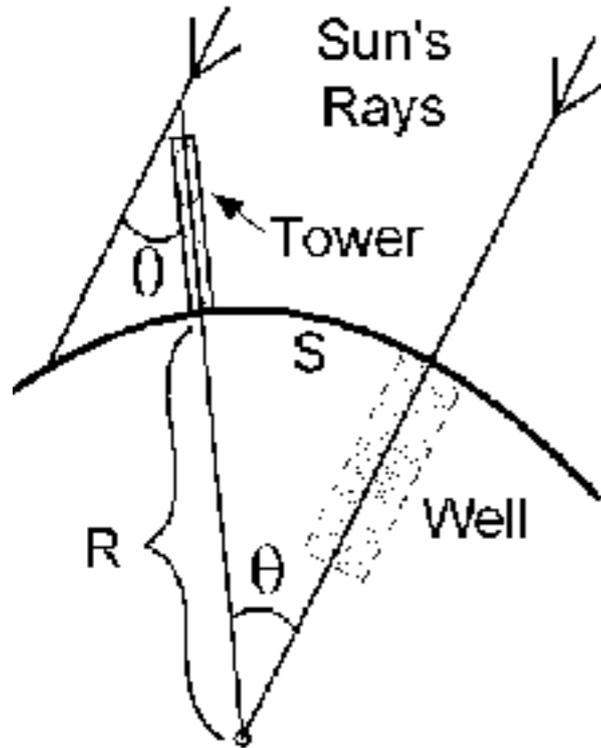
First measured by Eratosthenes in Alexandria in ~200 BC !!!



- Eratosthenes knew that in Syene at noon of the summer solstice the sun was at the zenith
- On the same day at noon the sun was  $\sim 7^\circ$  away from zenith
- He also knew that the distance between Syene and Alexandria is  $\sim 800$  km
- Luckily, Syene and Alexandria are roughly at the same longitude

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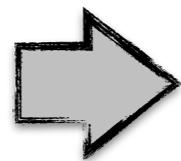


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in radians

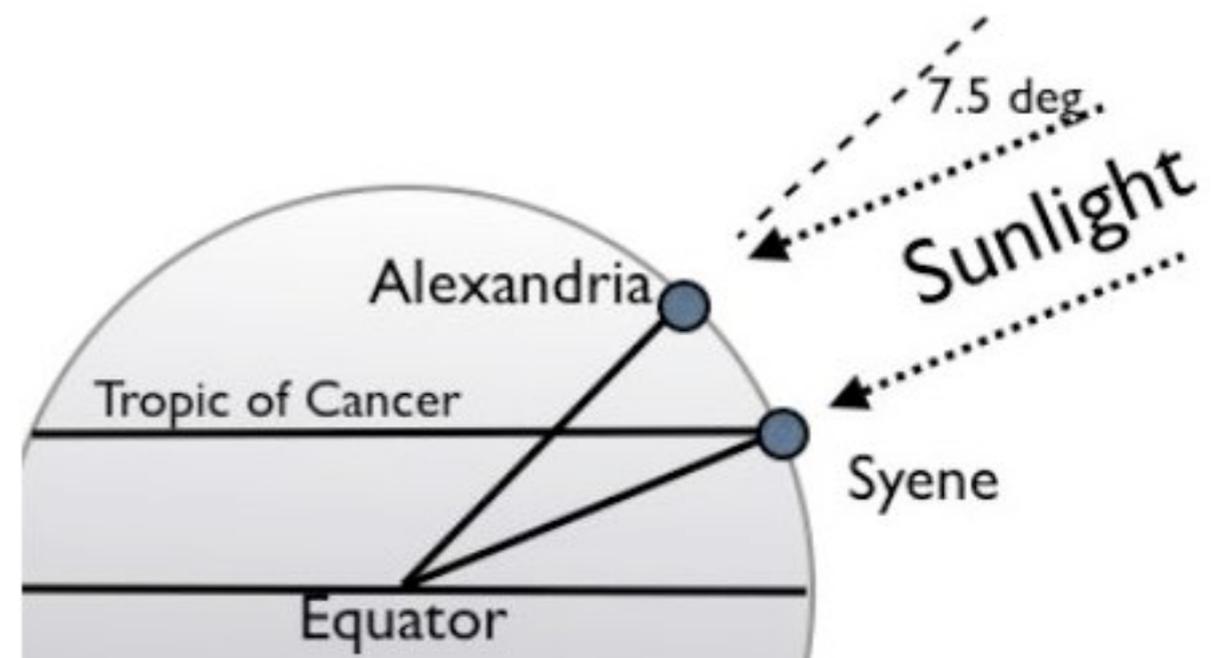
Earth's radius

$$S \approx R \theta$$



$$R \approx 6500 \text{ km}$$

(the real answer is  $\sim 6400$  km !!!)



# The Earth mass and density

Gravitational acceleration on Earth surface  $\rightarrow g \sim 9.8 \text{ m/s}^2$

1687: Newton's law of universal gravity  $\rightarrow F = \frac{GMm}{R^2}$

1798: Henry Cavendish measures Newton's constant  $\rightarrow G = 6.7 \times 10^{-8} \text{ cm}^3/\text{g/s}^2$

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**Earth's mass**

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Earth's mass

$$M = \frac{4\pi}{3} R^3 \rho \longrightarrow \rho \sim 5 \text{ g/cm}^3$$

Earth's density

# Astronomical quantities

Earth

radius ~ 6400 km

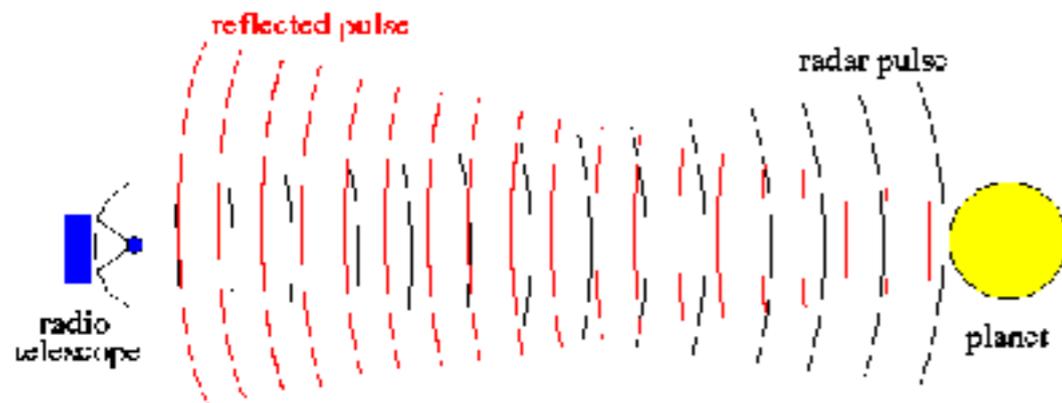
mass ~  $6 \times 10^{27}$  g

density ~  $5 \text{ g/cm}^3$

# How distant is the sun?

A long history of attempts...

in modern times, the distance to planets can be measured with radars

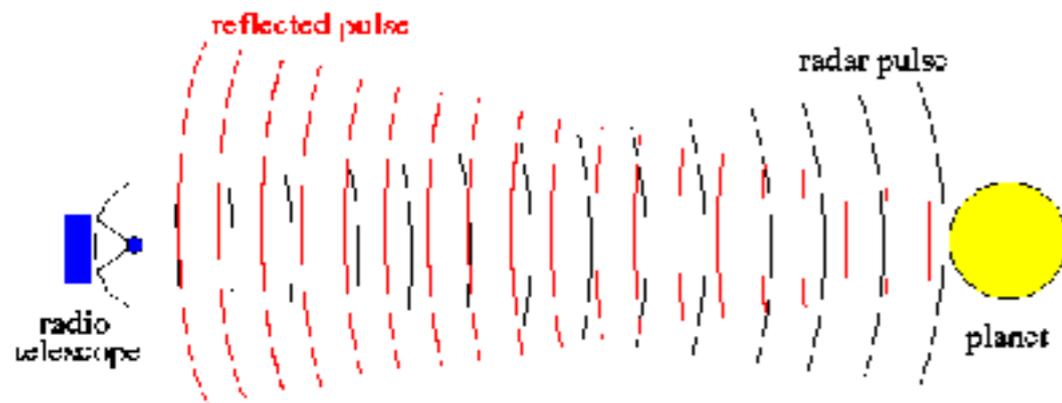


a radio pulse is beamed to the planet in question, and reflected pulse is detected and timed, the time of reflect times the speed of light equals the distance to the planet

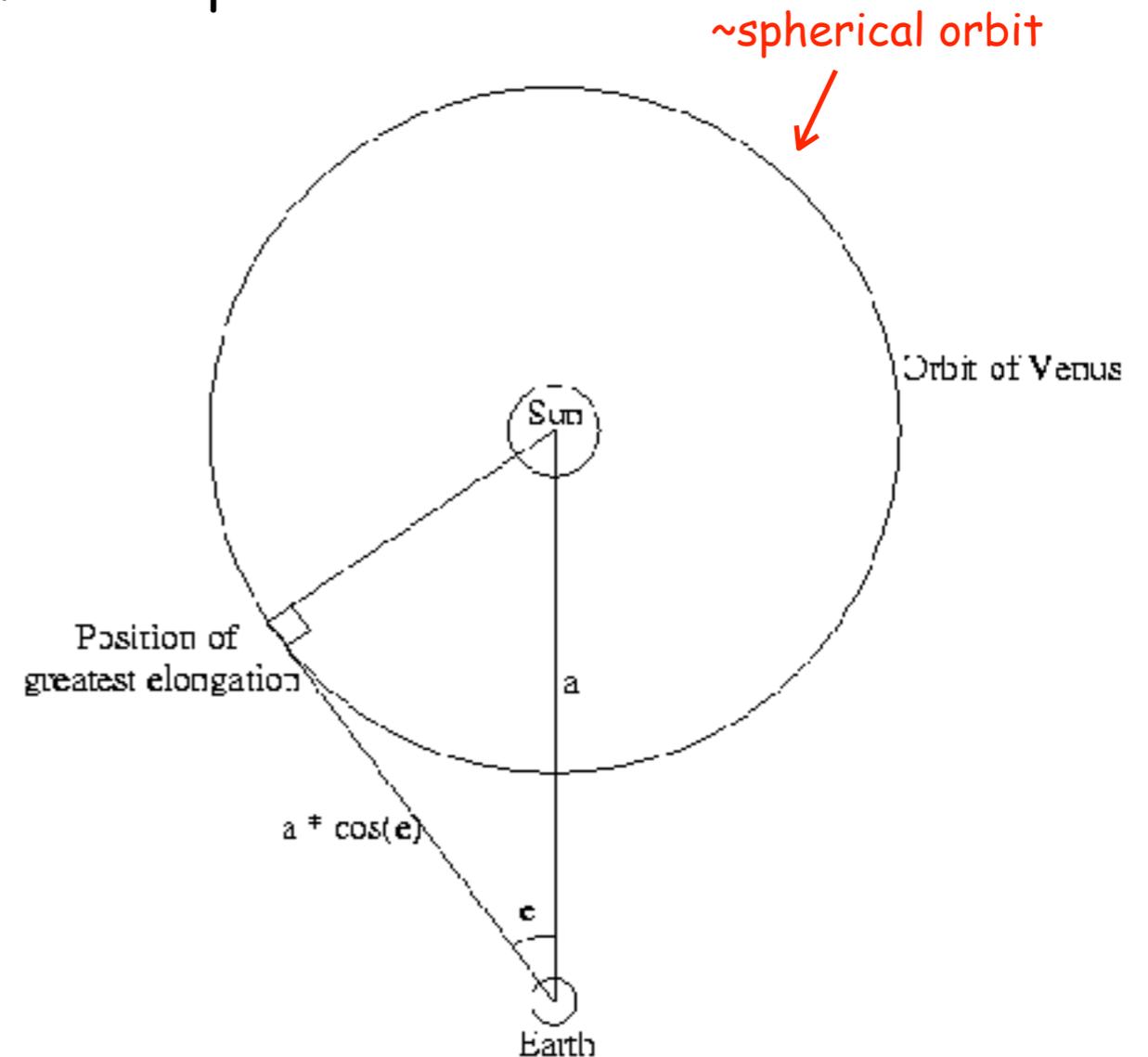
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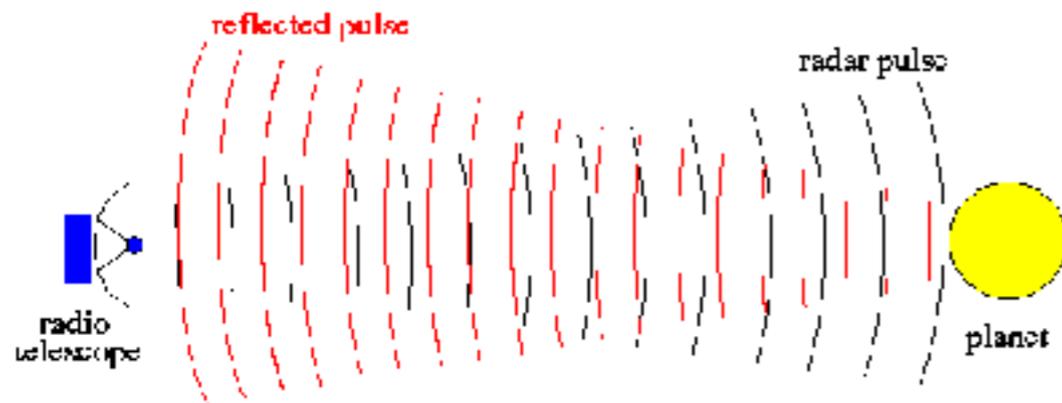


then one can measure the maximum elongation of a planet

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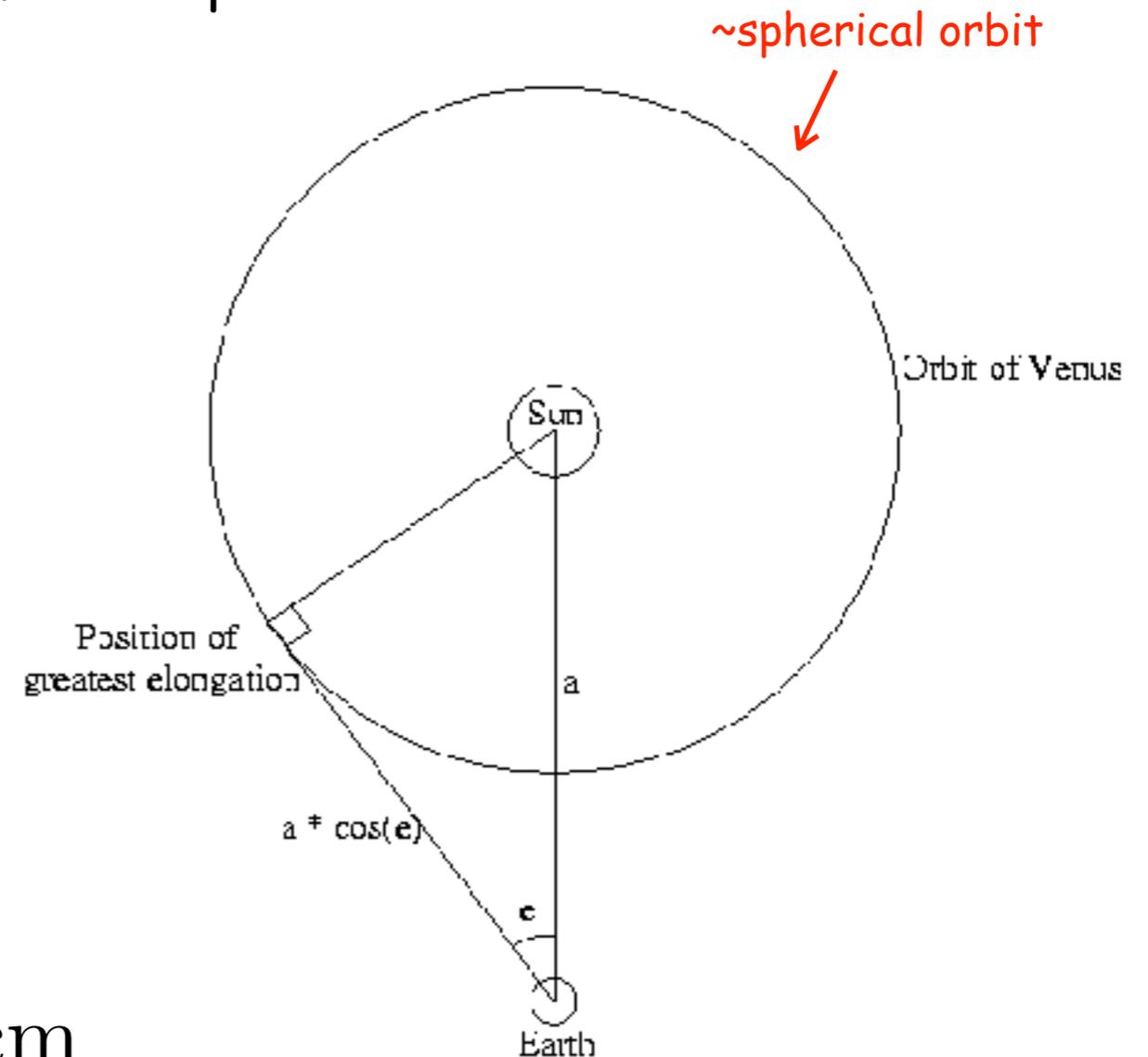
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$$a = 1.5 \times 10^8 \text{ km} = 1.5 \times 10^{13} \text{ cm}$$

**Astronomical Unit**



then one can measure the maximum elongation of a planet

# The sun's radius and mass

The sun's apparent size is  $\theta \sim 0.5^\circ$  in diameter  $\rightarrow R_{\odot} \sim a \frac{\theta}{2} \sim 7 \times 10^{10} \text{cm}$

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Earth's velocity  $\rightarrow v = \frac{2\pi a}{3.15 \times 10^7 \text{ s}} \approx 30 \text{ km/s}$

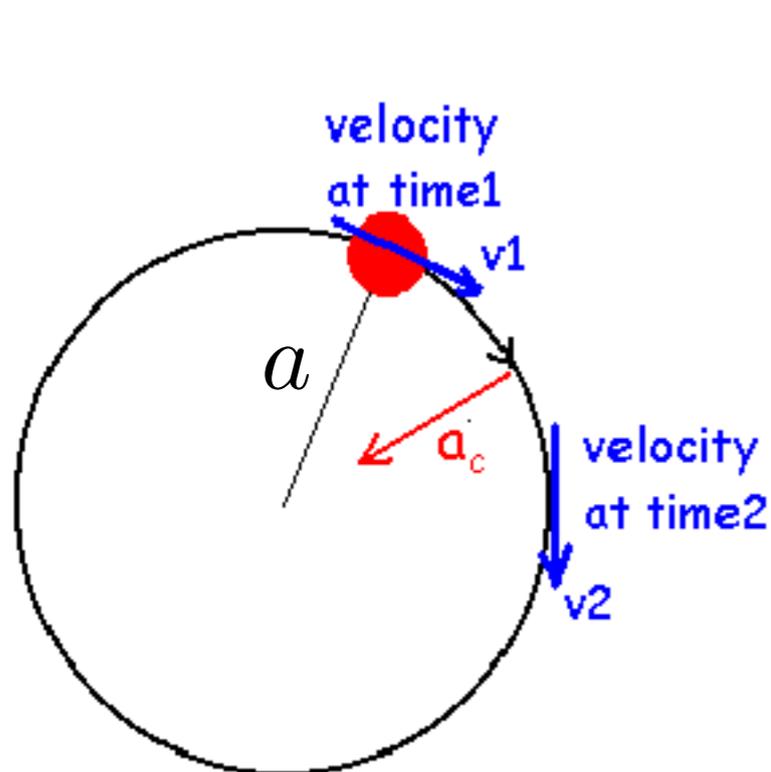


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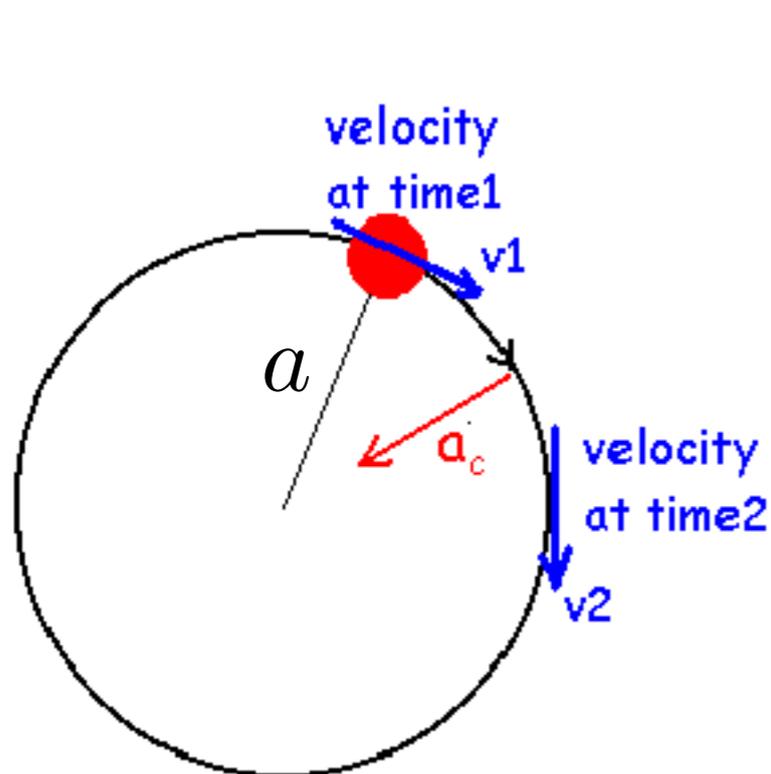
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$$M_{\odot} = 2 \times 10^{33} \text{ g}$$

$$\rho_{\odot} = 1.4 \text{ g/cm}^3$$

# Astronomical quantities

Earth

↑

$1.5 \times 10^{13}$  cm

↓

Sun

radius  $\sim 6400$  km

mass  $\sim 6 \times 10^{27}$  g

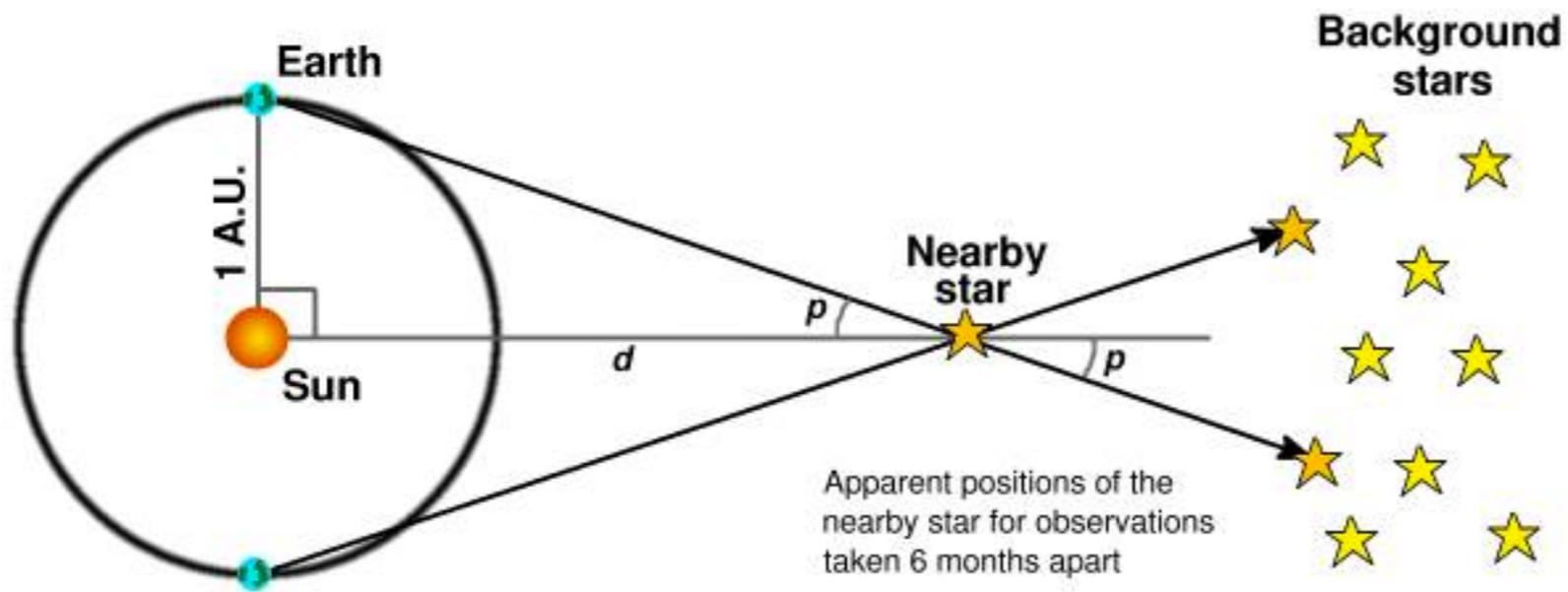
density  $\sim 5$  g/cm<sup>3</sup>

radius  $\sim 7 \times 10^{10}$  cm

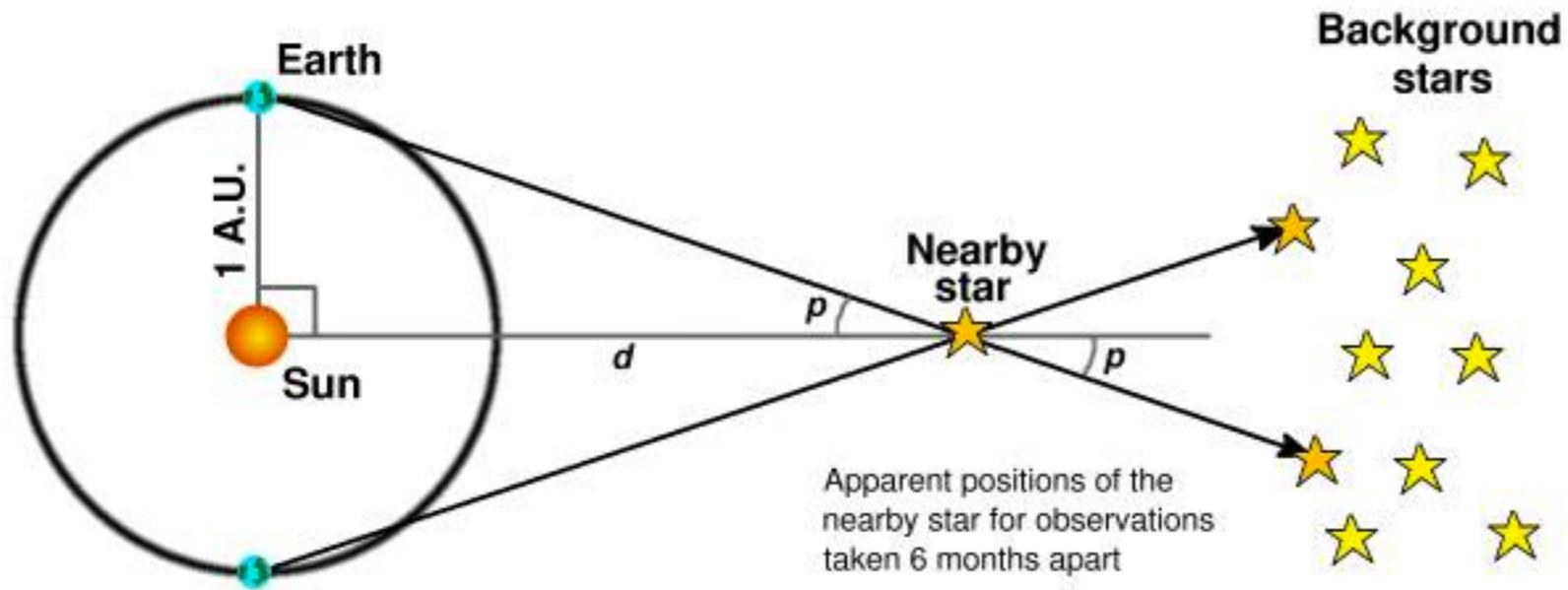
mass  $\sim 2 \times 10^{33}$  g

density  $\sim 1.4$  g/cm<sup>3</sup>

# Parallaxes



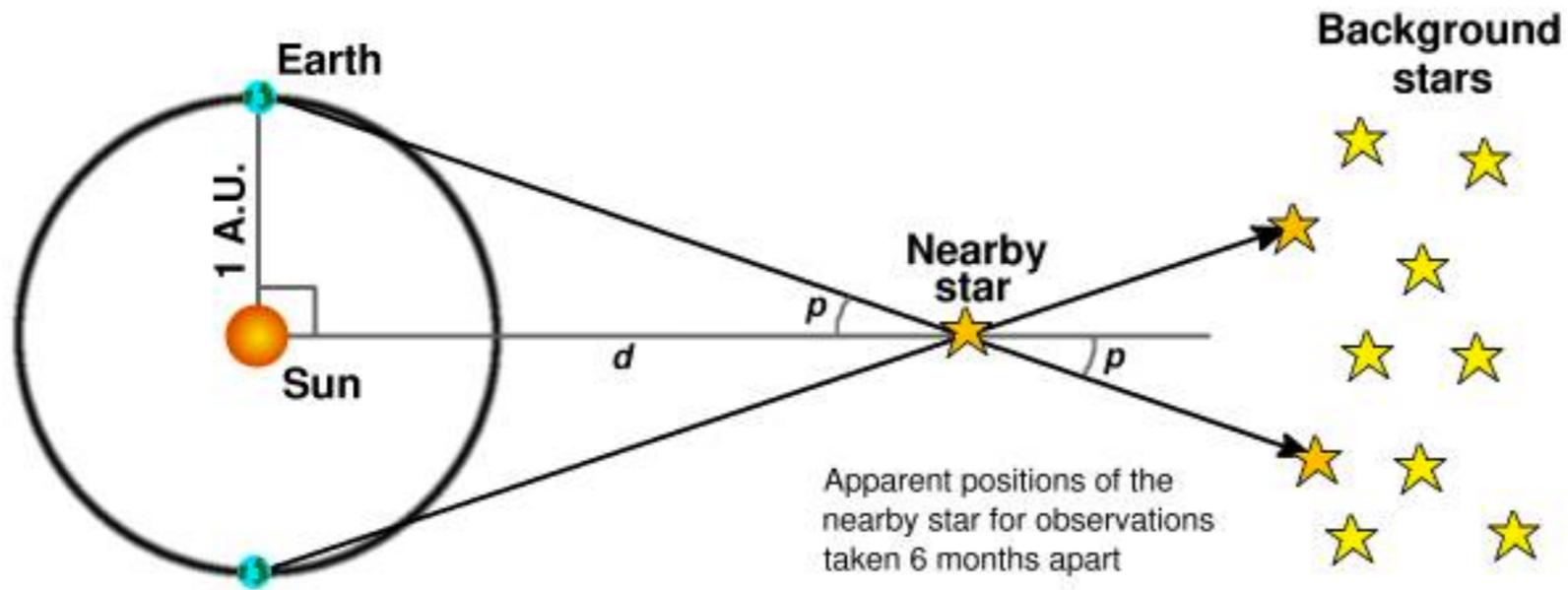
# Parallaxes



$$d \sim \frac{a}{p}$$

$$1 \text{ rad} = 206265 \text{ arcsec}$$

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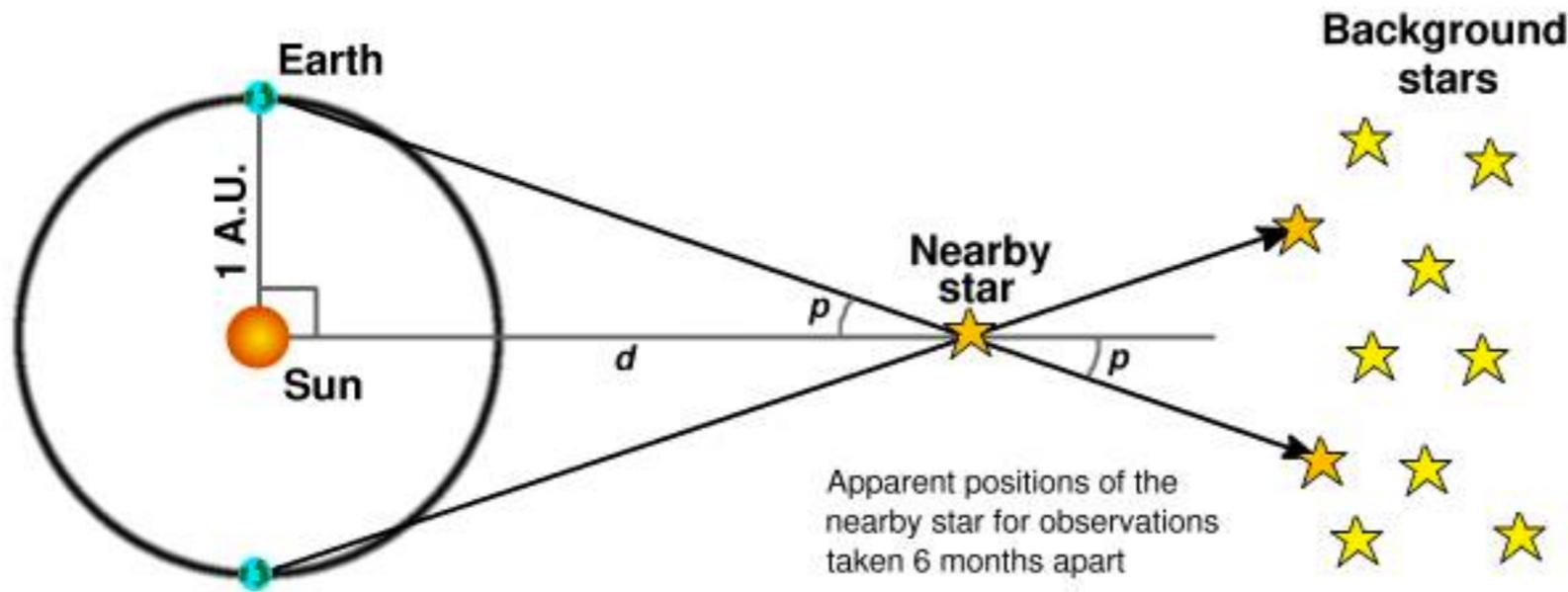


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$$p = 1 \text{ arcsec} \longrightarrow 1 \text{ pc} = 3 \times 10^{18} \text{ cm}$$

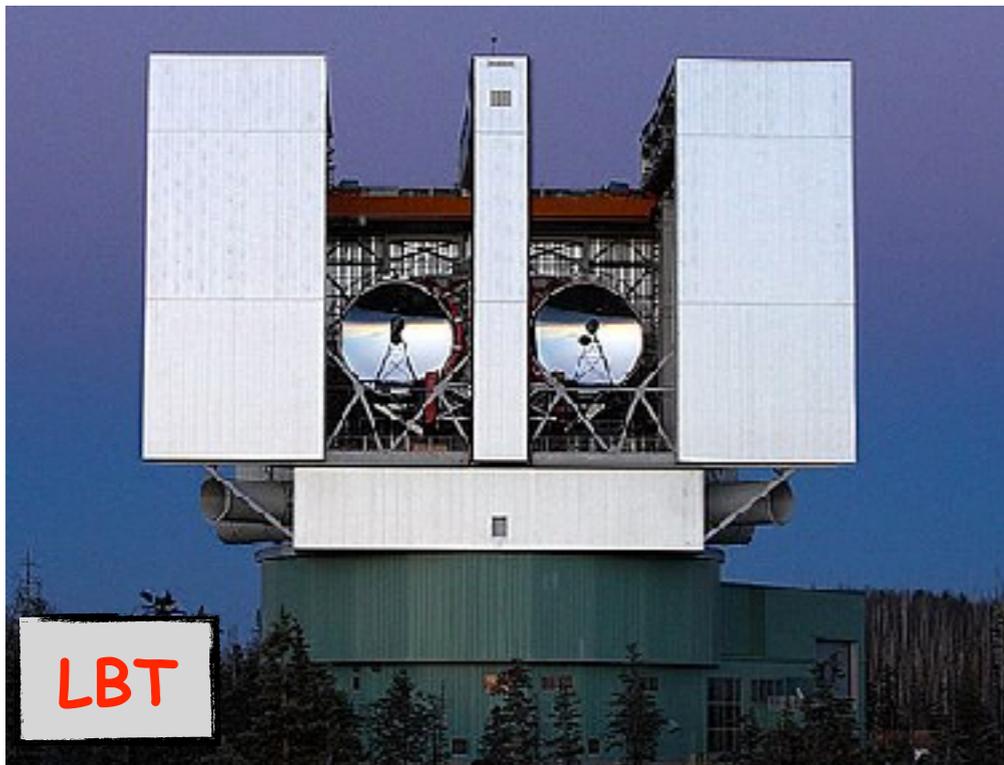
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**angular resolution**

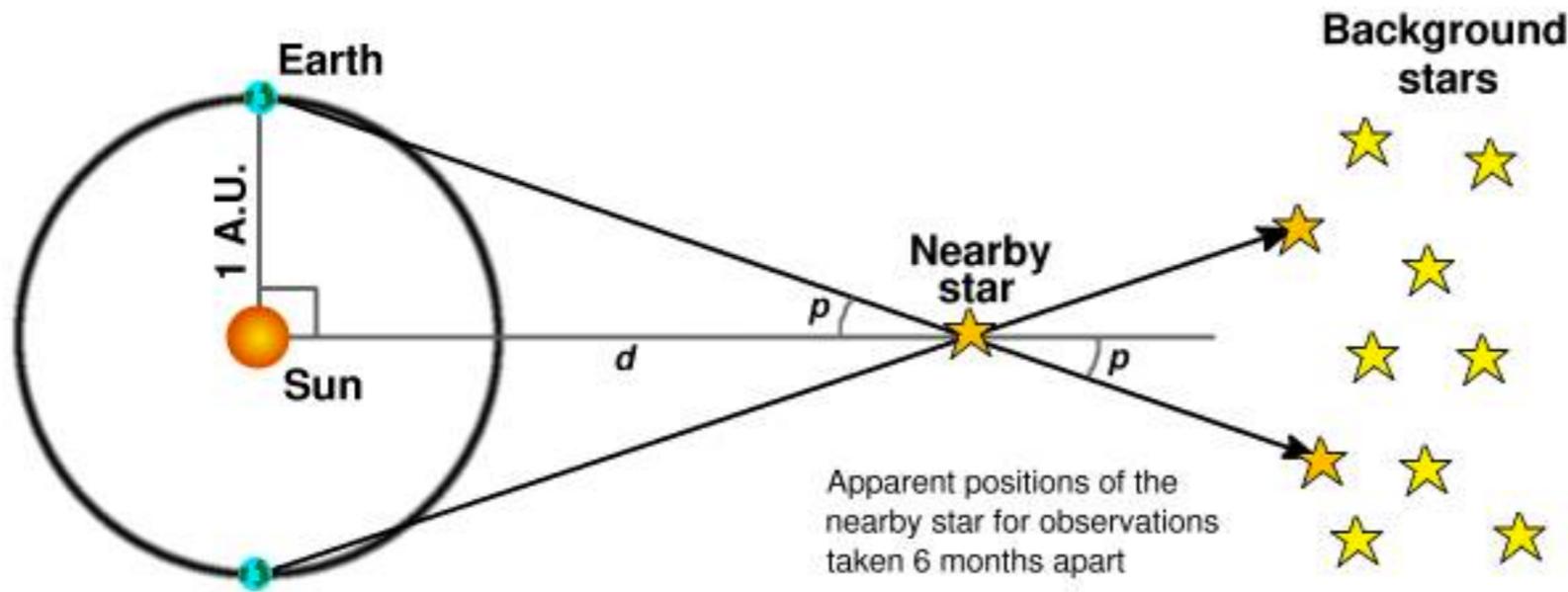
$$\delta\theta \sim \frac{\lambda}{D} \approx 0.02 \text{ arcsec}$$

wavelength

mirror diameter

LBT

# Parallaxes

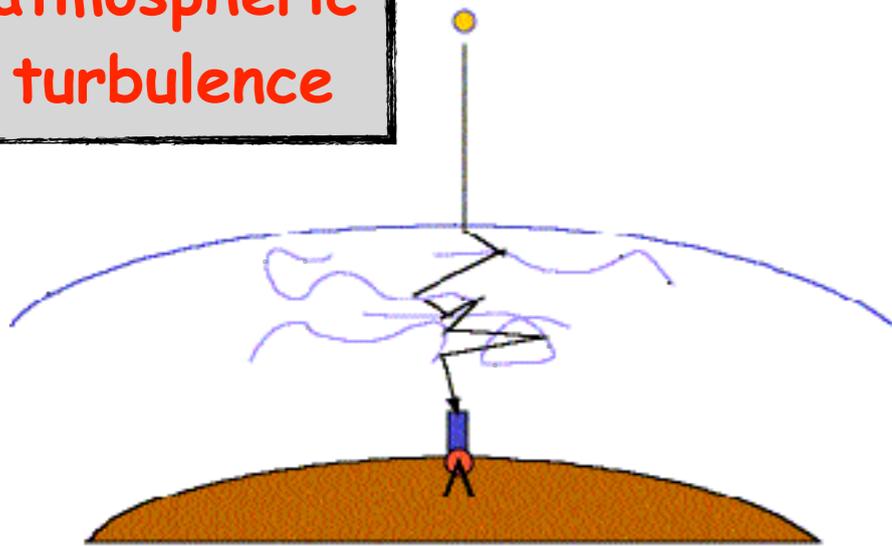


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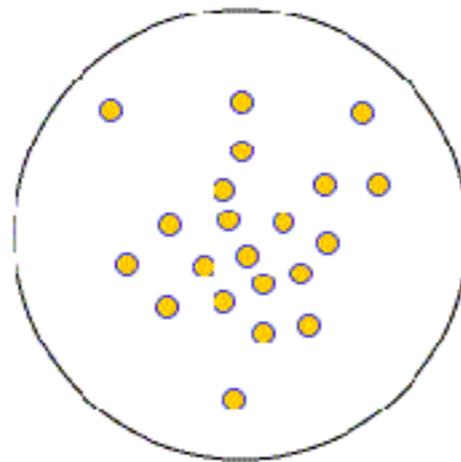
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atmospheric turbulence



atmosphere refracts starlight in random directions very quickly—stars “twinkle”.

telescope view (high magnification)



multiple images created

$$\delta\theta \sim \frac{\lambda}{D} \approx 0.02 \text{ arcsec}$$

wavelength

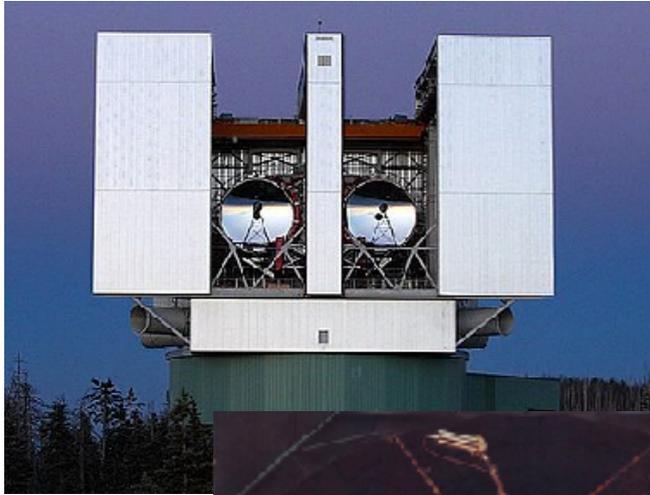
mirror diameter



seeing

# Limitations of parallaxes

The blurring on the image of a star (seeing) limits the accuracy in determining the position of a star

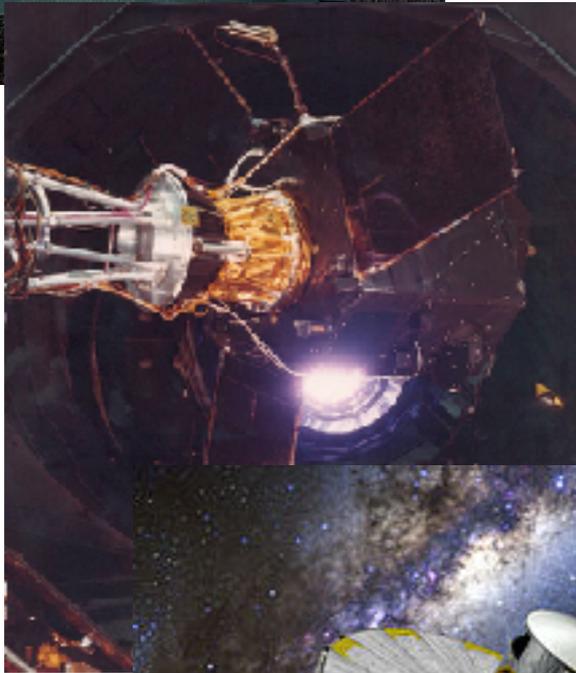


ground based telescopes

$$p_{min} \approx 0.01 \text{ arcsec} \longrightarrow d_{max} \approx 100 \text{ pc}$$

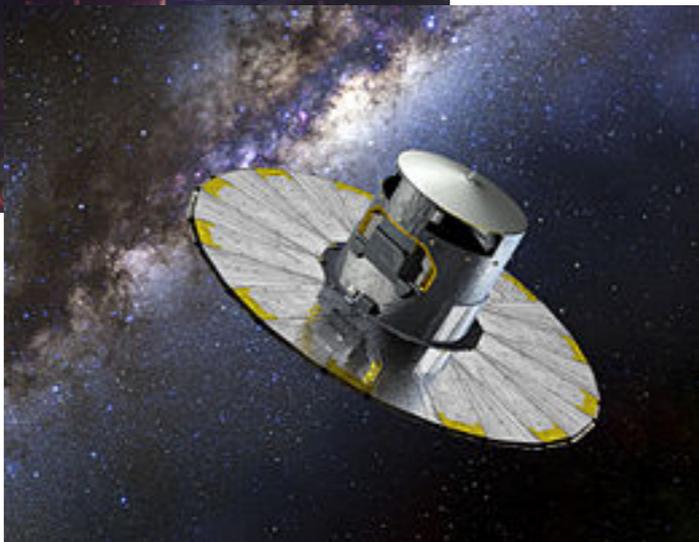
Hypparcos (satellite, 1989-1993)

$$p_{min} \approx 0.001 \text{ arcsec} \longrightarrow d_{max} \approx 1 \text{ kpc}$$



Gaia (satellite, taking data)

$$\longrightarrow d_{max} \approx 10 \text{ kpc}$$



# Astronomical quantities

Earth

radius  $\sim 6400$  km

mass  $\sim 6 \times 10^{27}$  g

density  $\sim 5$  g/cm<sup>3</sup>

↑

$1.5 \times 10^{13}$  cm

↓

Sun

radius  $\sim 7 \times 10^{10}$  cm

mass  $\sim 2 \times 10^{33}$  g

density  $\sim 1.4$  g/cm<sup>3</sup>

1.3 pc

Alpha Centauri

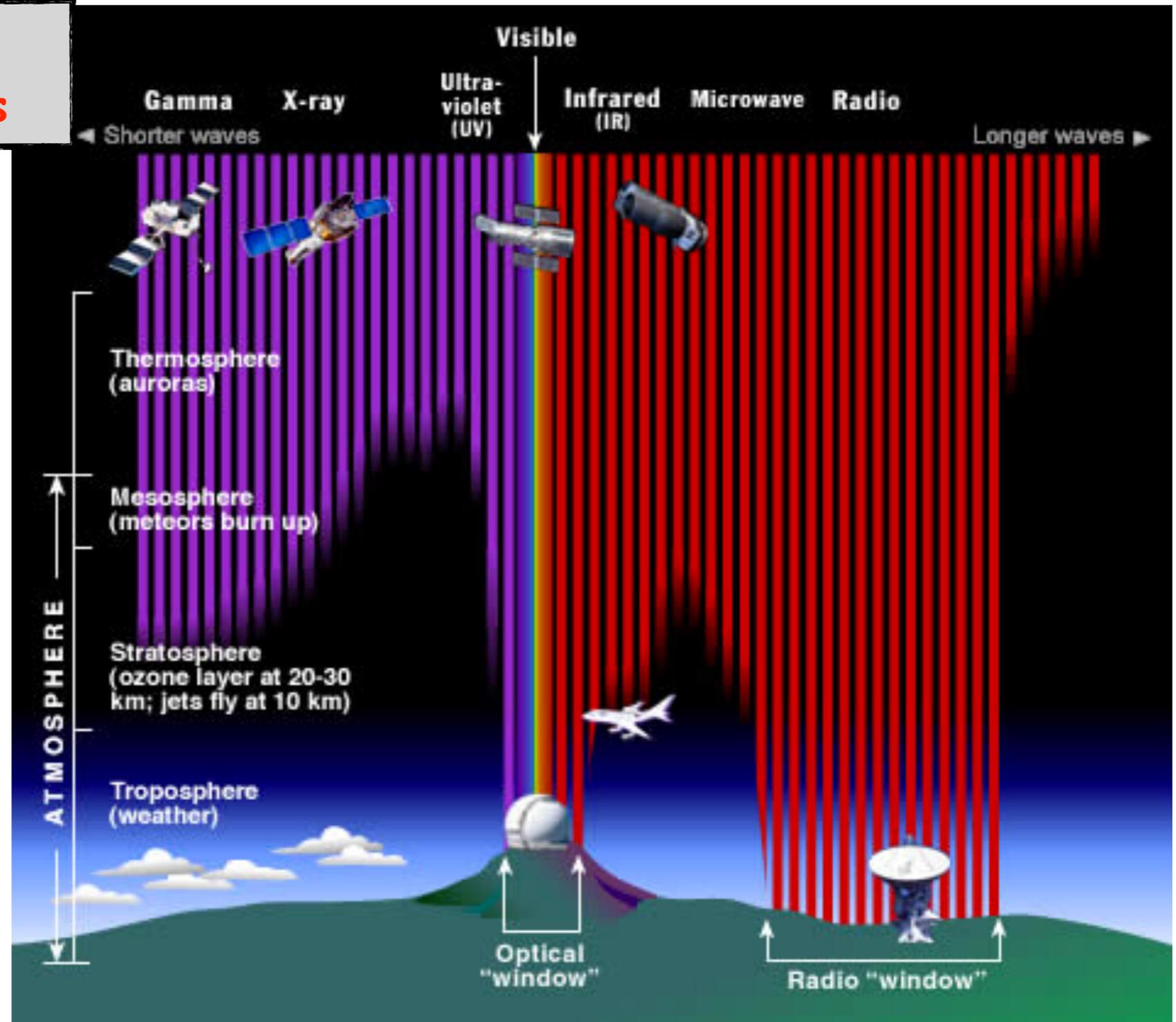
# The radio window

The Earth's atmosphere is opaque to most frequencies

radio window

$\lambda > 1 \text{ cm}$

$\nu < 30 \text{ GHz}$



Optical

Radio

# Radio telescopes and interferometry



Very Long Baseline Interferometry

# Distance to the centre of the Milky Way

At radio frequencies the effect of atmospheric seeing is negligible

$$\delta\theta \sim \frac{\lambda}{D} \longrightarrow \delta\theta \approx \text{milliarcsec}$$

*>1 cm* (pointing to  $\lambda$ )  
*size of the Earth!!!* (pointing to  $D$ )

positions of radio sources can be determined with great accuracy

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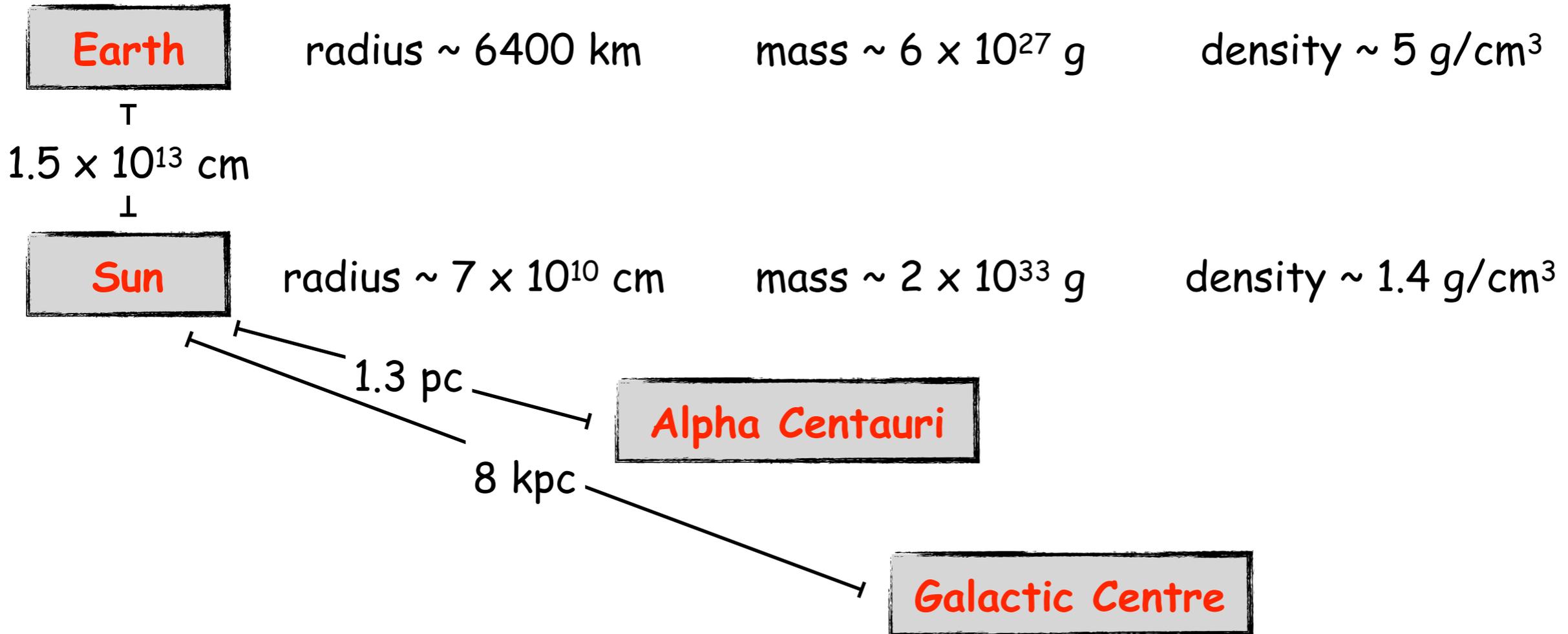
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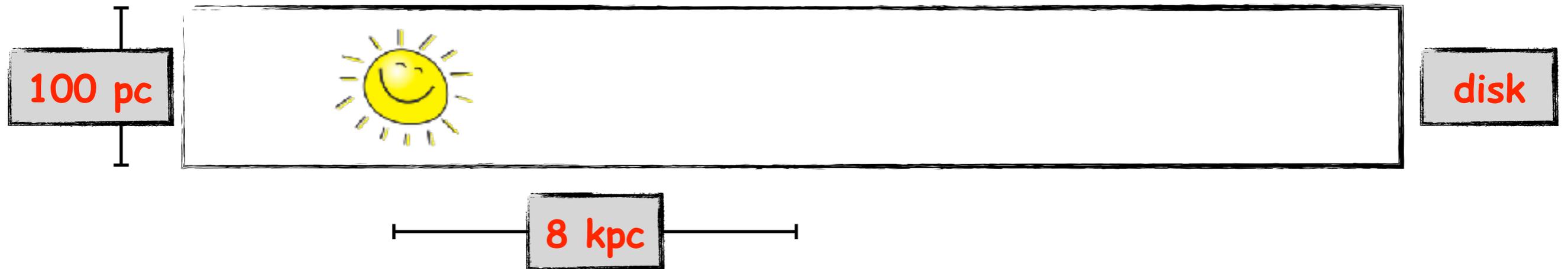
parallaxes from radio observations allowed to measure the distance of the supermassive black hole located at the galactic centre -> ~8 kpc

other (more indirect) methods gave consistent results

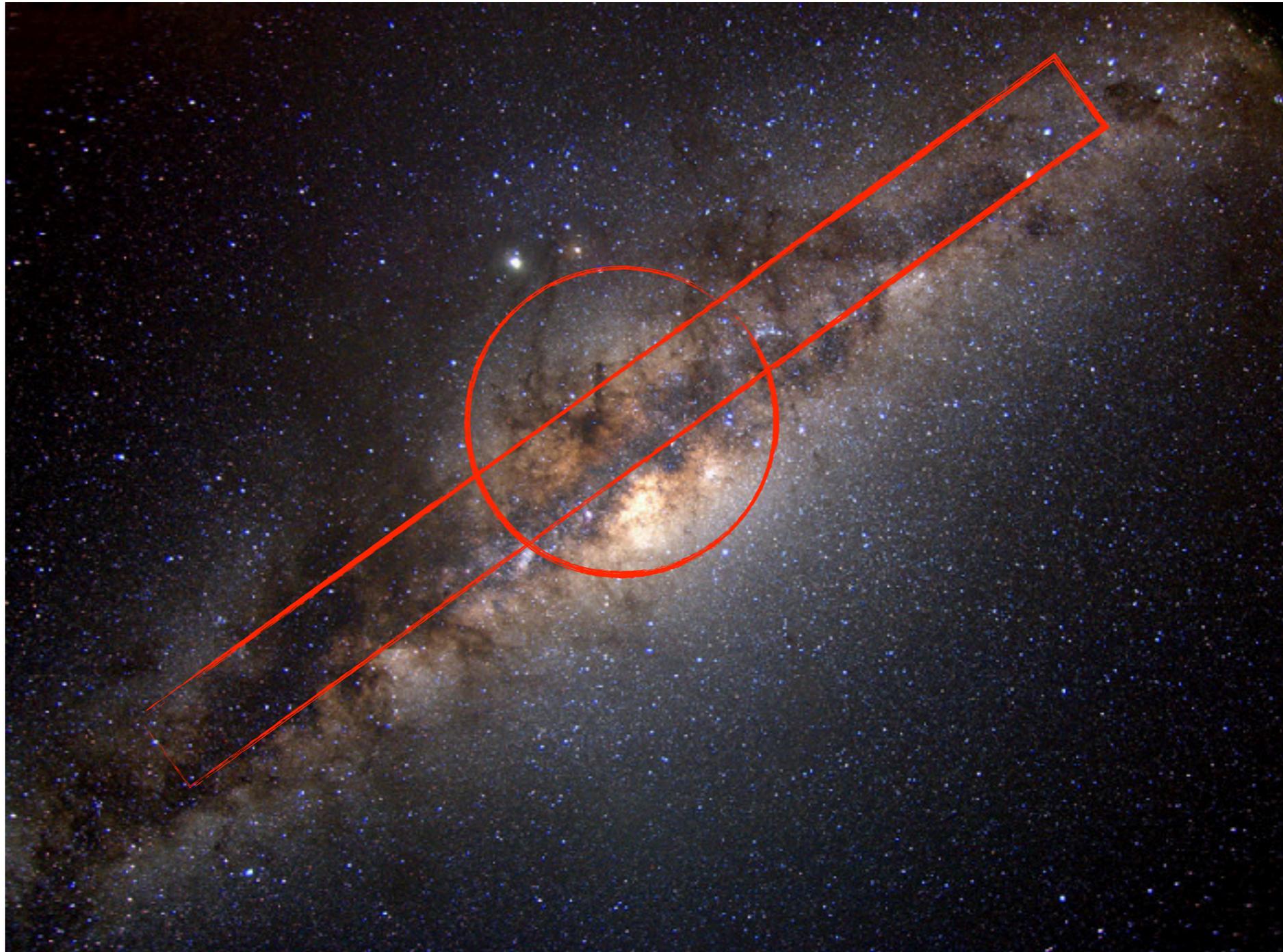
# Astronomical quantities



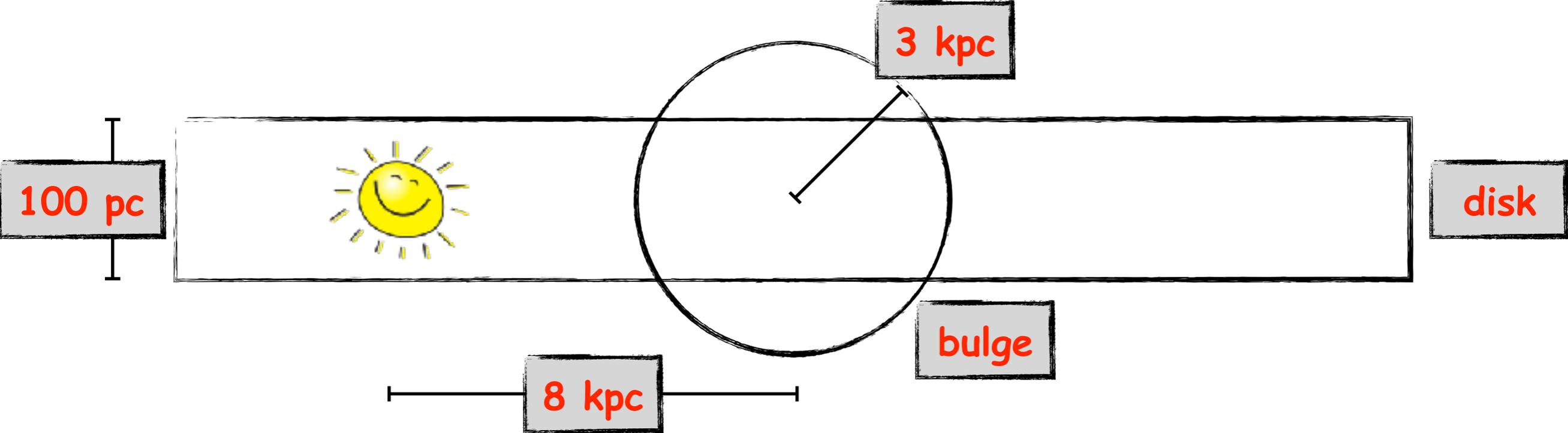
# The Milky Way



# The galactic bulge

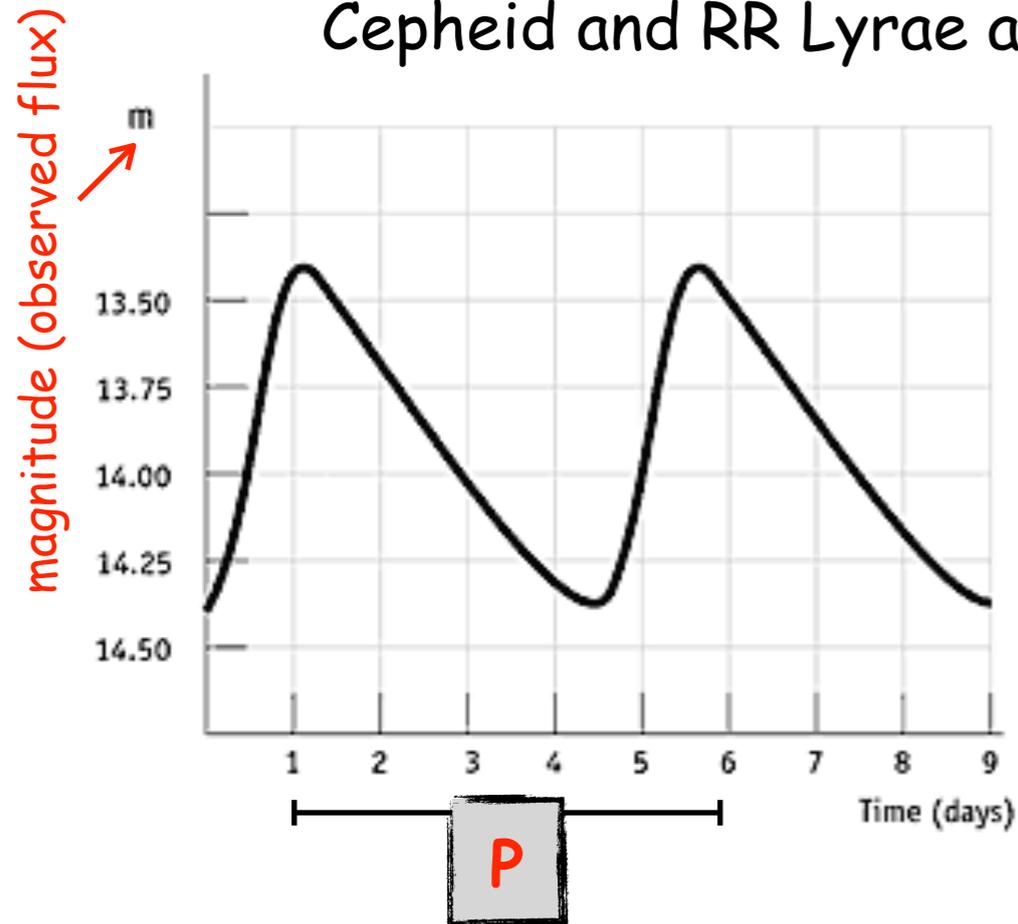


# The Milky Way



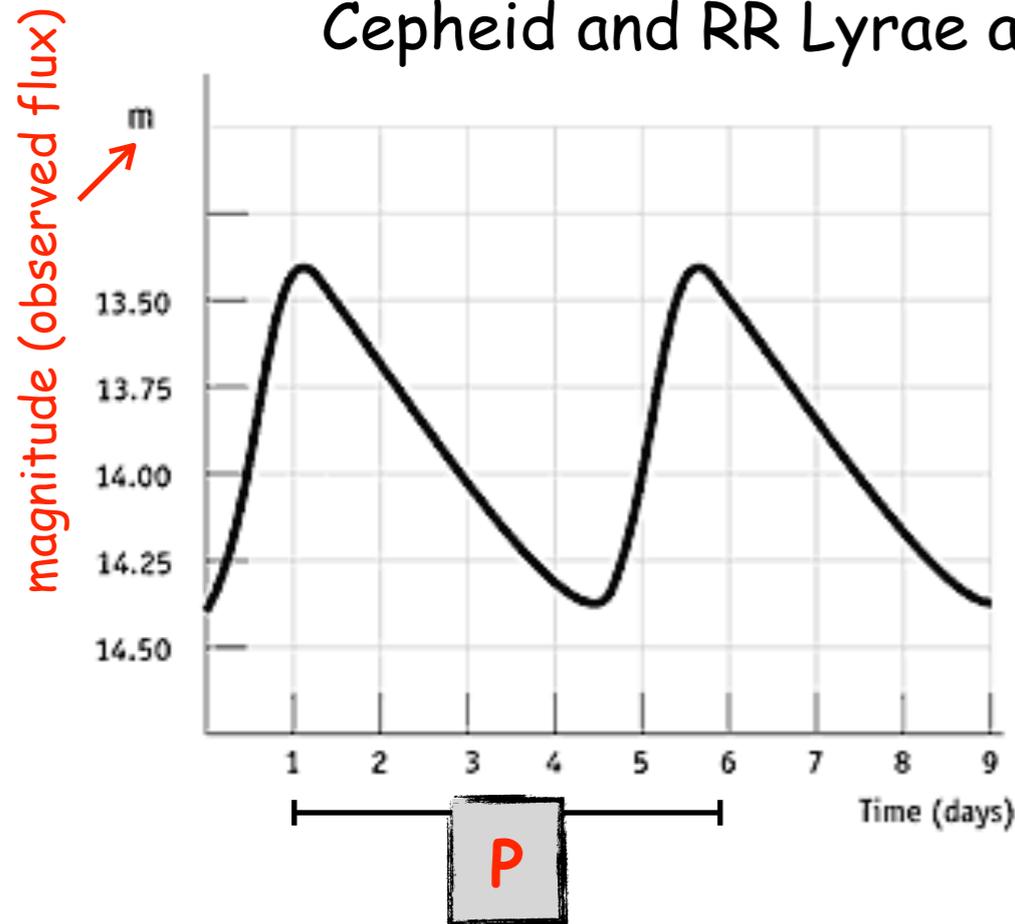
# Larger distances: Cepheids and RR Lyrae

Cepheid and RR Lyrae are variable stars characterised by a period  $P$

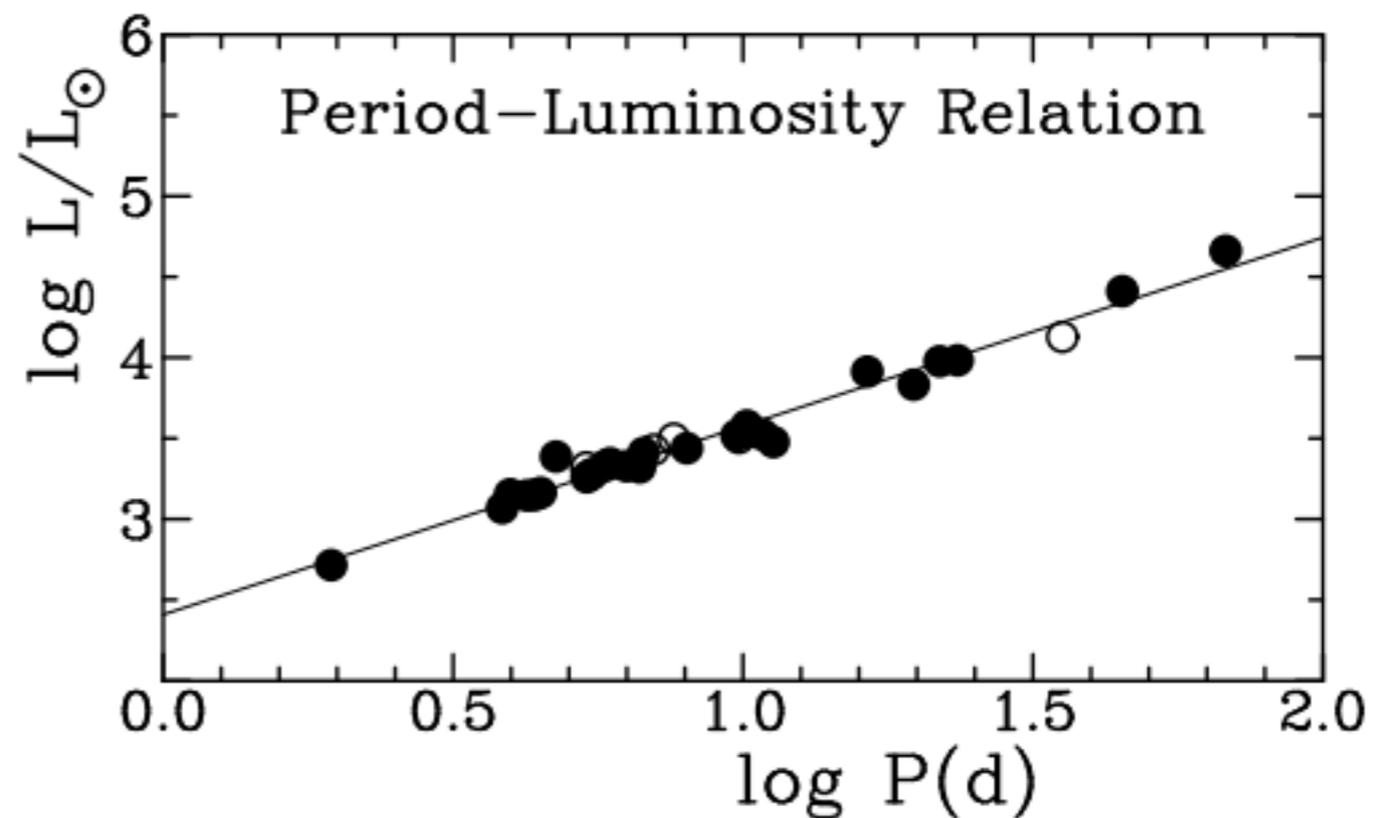


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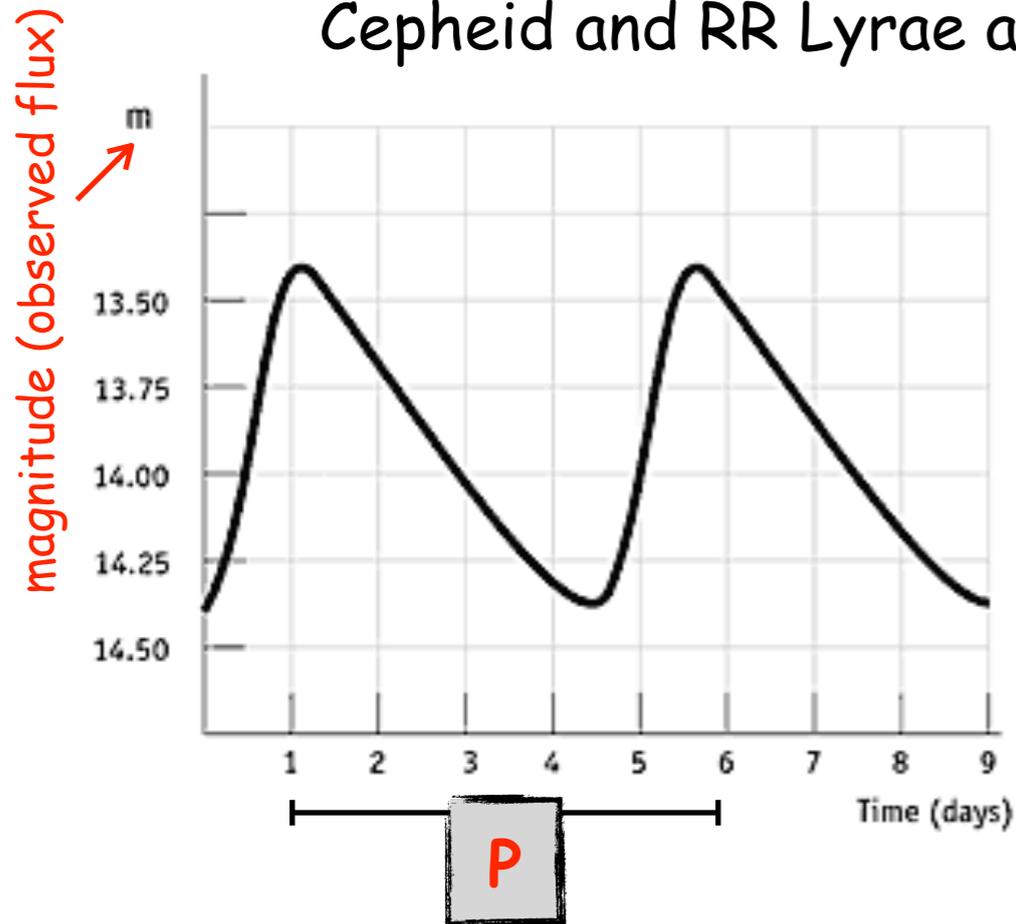


- Take some Cepheids of known distance  $D$  (for example, measured from parallaxes)
- from their observed flux, measure the luminosity  $L = F \times (4\pi D^2)$
- build a period-luminosity diagram



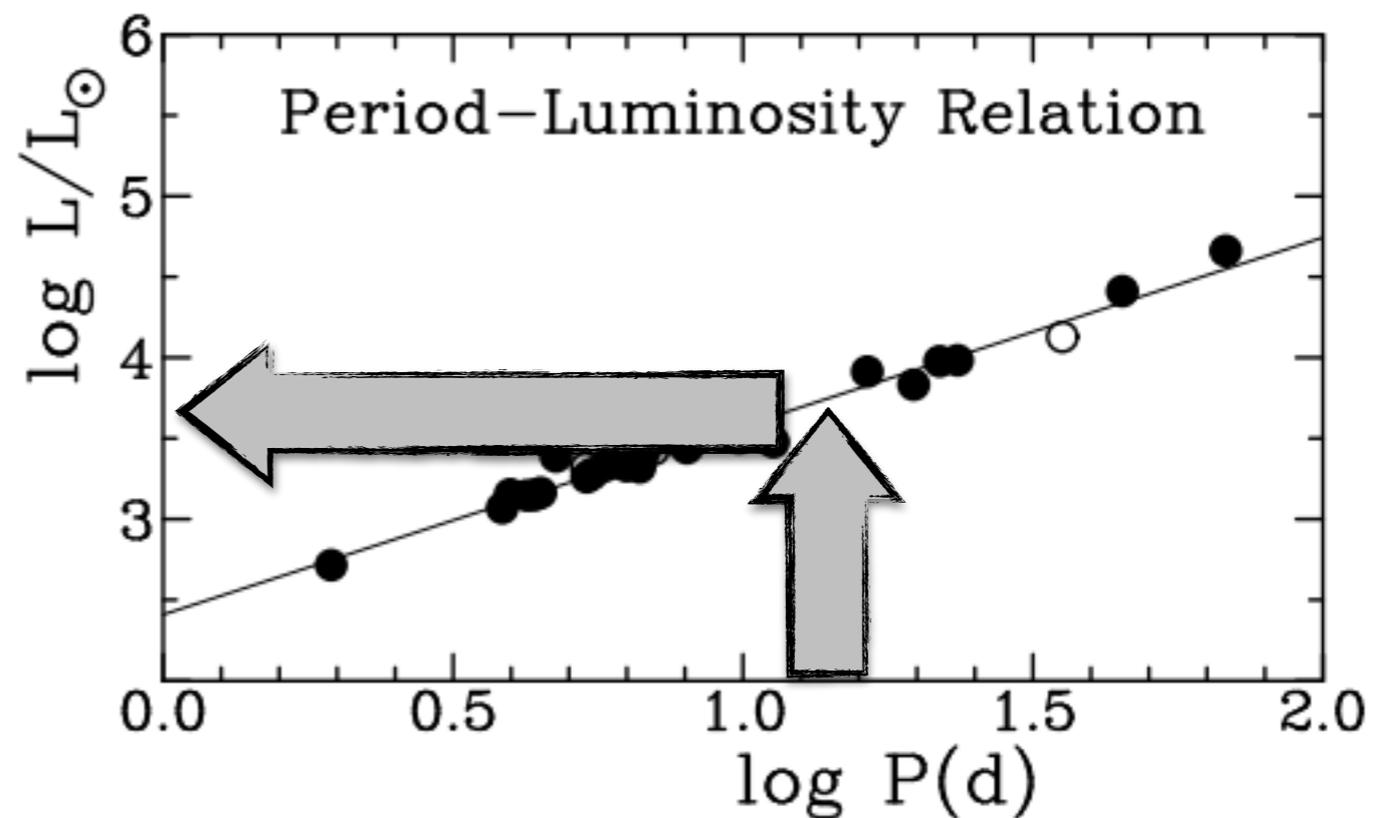
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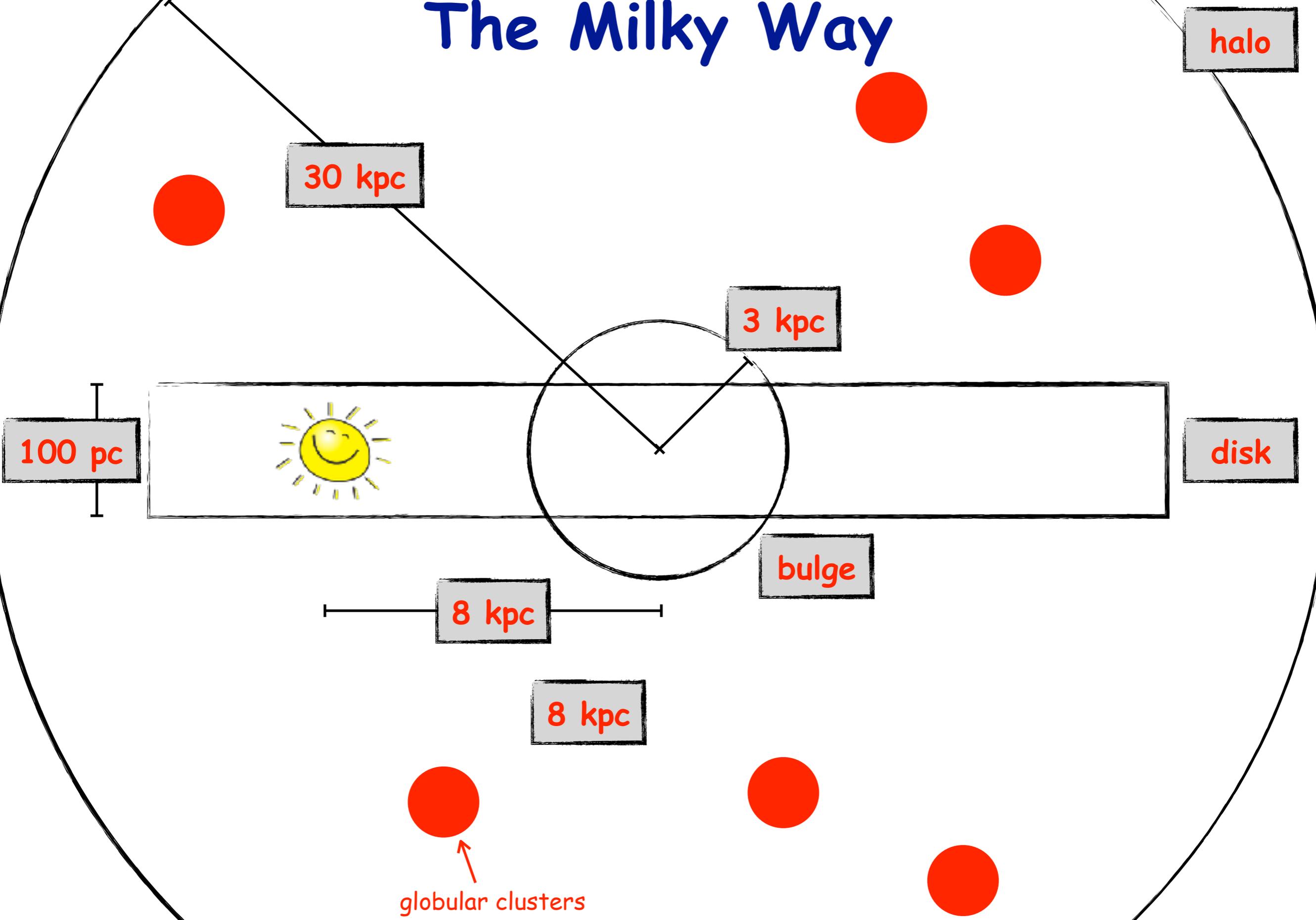


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- now take a Cepheid of unknown distance, and from its period derive the luminosity  $L$
- the distance can be derived as  $D = (F/4\pi L)^{1/2}$
- Cepheids are **STANDARD CANDLES**



# The Milky Way



halo

30 kpc

3 kpc

100 pc

disk

bulge

8 kpc

8 kpc

globular clusters

# The mass of the Milky Way: stars

Solar luminosity

total photon energy output

$$L_{\odot} \sim 4 \times 10^{33} \text{ erg/s}$$

Solar mass

$$M_{\odot} \sim 2 \times 10^{33} \text{ g}$$

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MW luminosity

$L_{MW} \rightarrow$  can be measured

MW stellar mass

$M_{MW} \rightarrow ?$

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Assumption

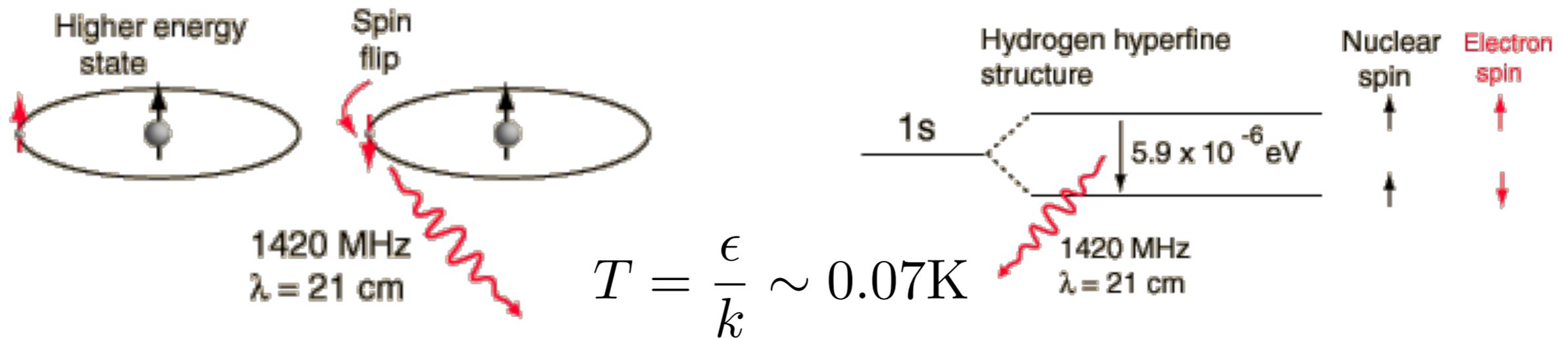
$$\frac{M_{\odot}}{L_{\odot}} \approx \frac{M_{MW}}{L_{MW}} \longrightarrow M_{MW} \approx 10^{11} M_{\odot}$$

very very rough



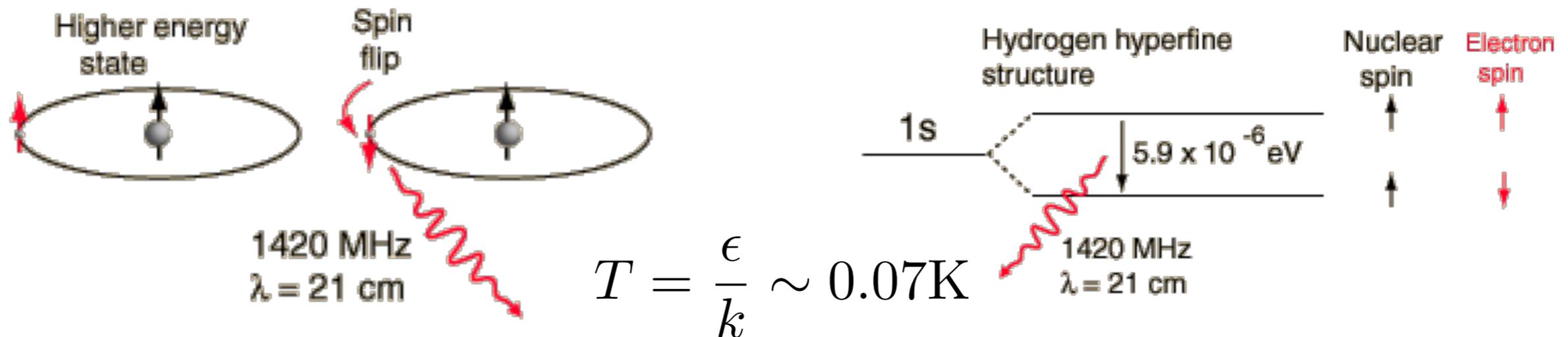
# Atomic H in the Milky Way

## Hydrogen hyperfine structure

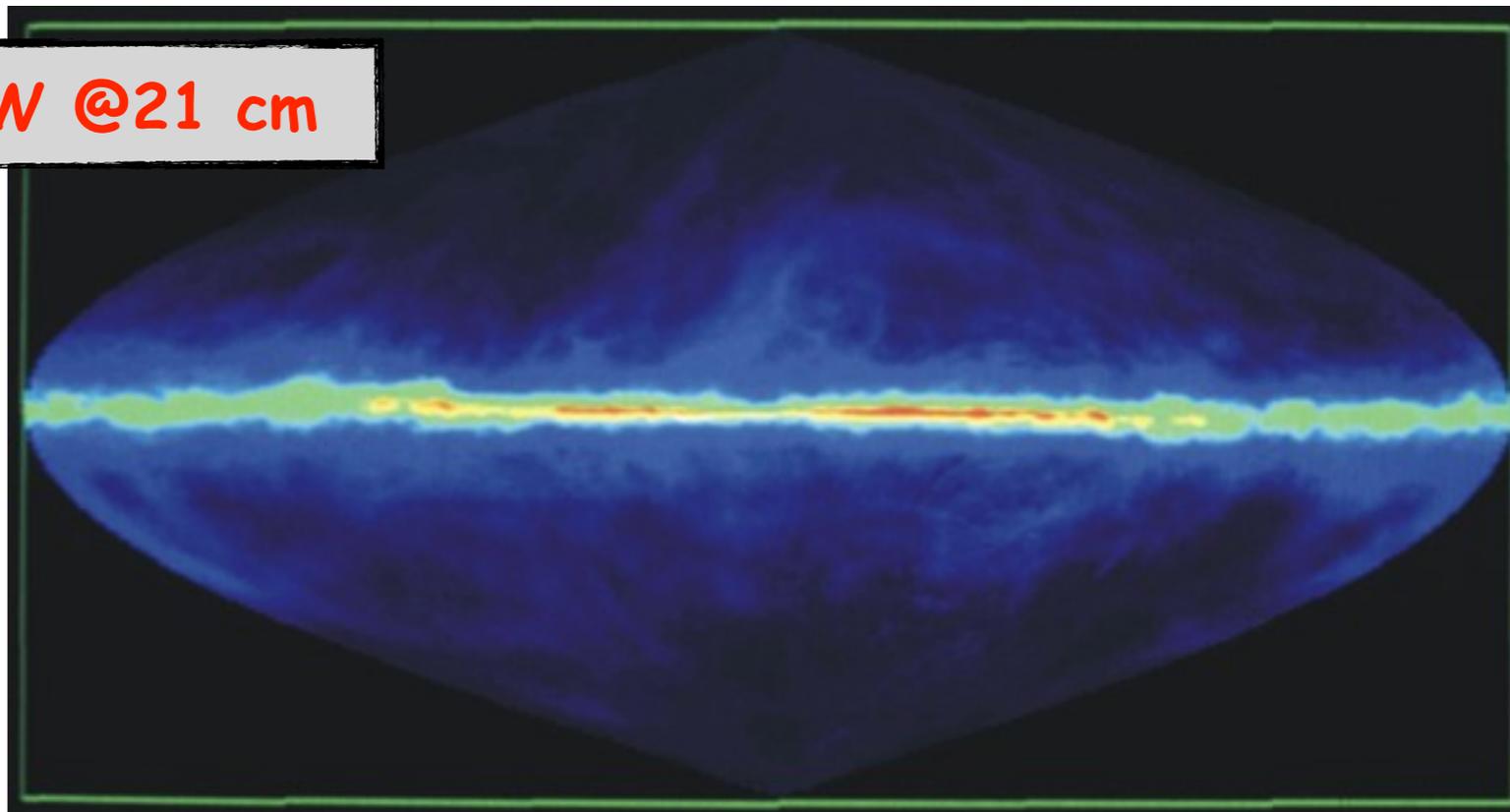


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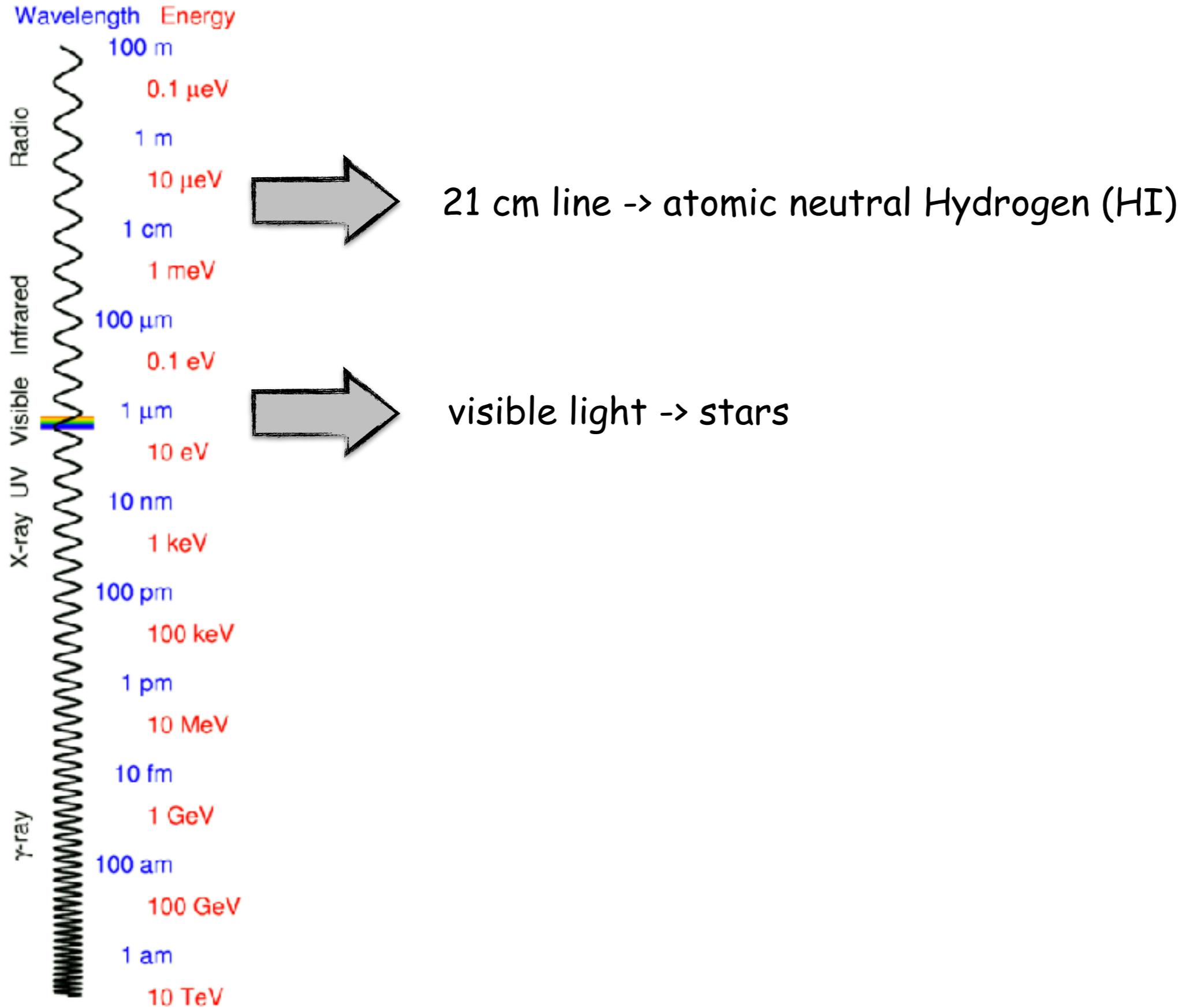


MW @21 cm



The disk of the Milky Way is filled with a diffuse gas of neutral H

# The electromagnetic spectrum



# The interstellar medium (ISM)

Is the diffuse matter that exists in the space between stars

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Observations of the MW in the 21 cm and in other lines (especially CO) revealed that:

- more than 90% (in number) of the ISM particles are Hydrogen
- 80% of which are atomic Hydrogen, either neutral (HI) or ionised (HII)
- the remaining 20% is molecular Hydrogen (H<sub>2</sub>)

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$$M_{ISM} \approx 10^{10} M_{\odot} \quad \sim 10\% \text{ of the stellar mass}$$

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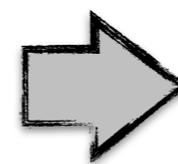
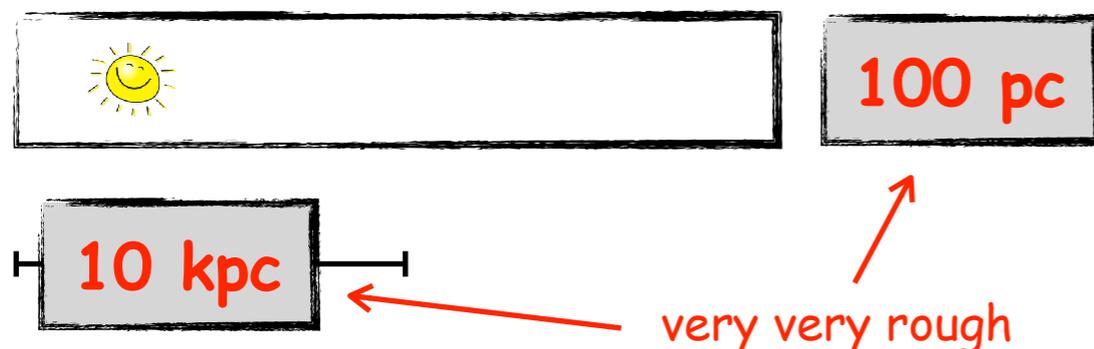
Is the diffuse matter that exists in the space between stars

Observations of the MW in the 21 cm and in other lines (especially CO) revealed that:

- more than 90% (in number) of the ISM particles are Hydrogen
- 80% of which are atomic Hydrogen, either neutral (HI) or ionised (HII)
- the remaining 20% is molecular Hydrogen (H<sub>2</sub>)

$$M_{ISM} \approx 10^{10} M_{\odot}$$

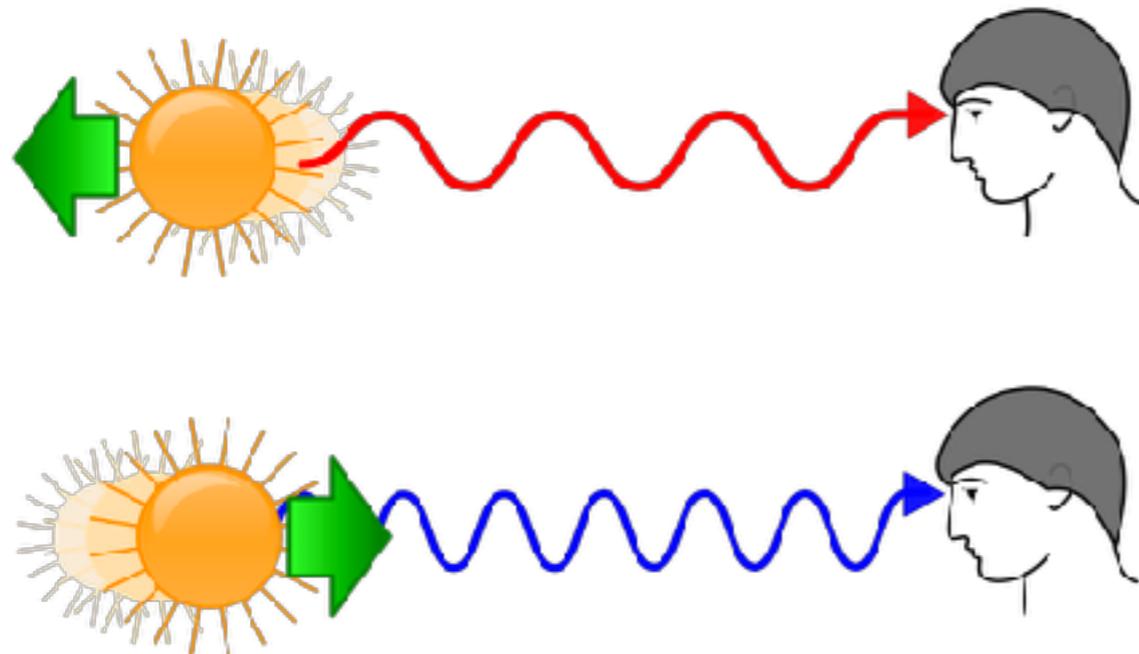
~10% of the stellar mass



$$n_{ISM} \approx 1 \text{ cm}^{-3}$$

# The MW rotation curve

Doppler effect



wavelength shift

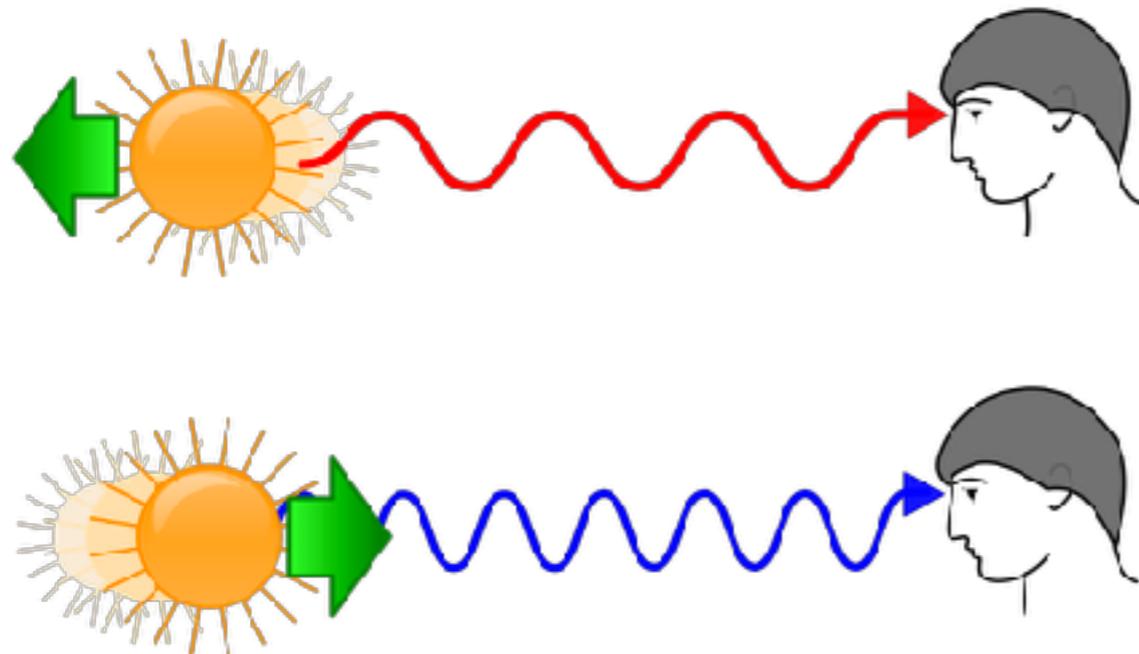
velocity along the  
line of sight

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

wavelength in the rest  
frame of the source

# The MW rotation curve

## Doppler effect

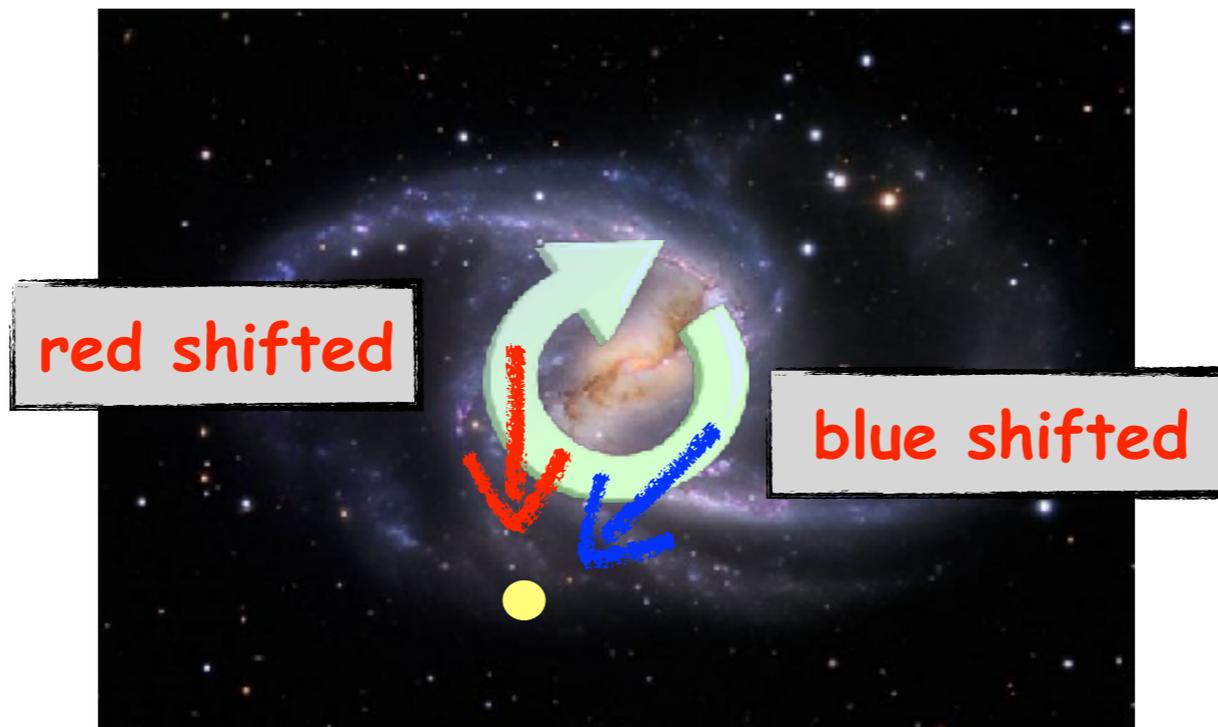


wavelength shift

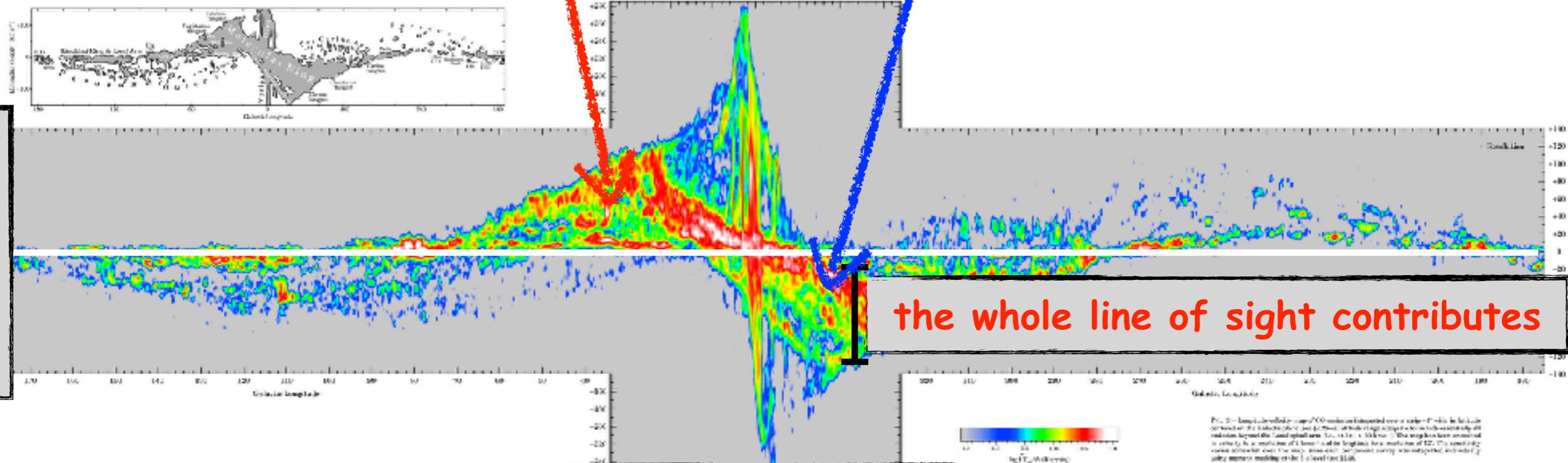
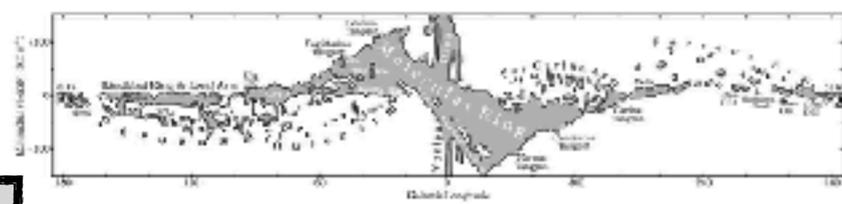
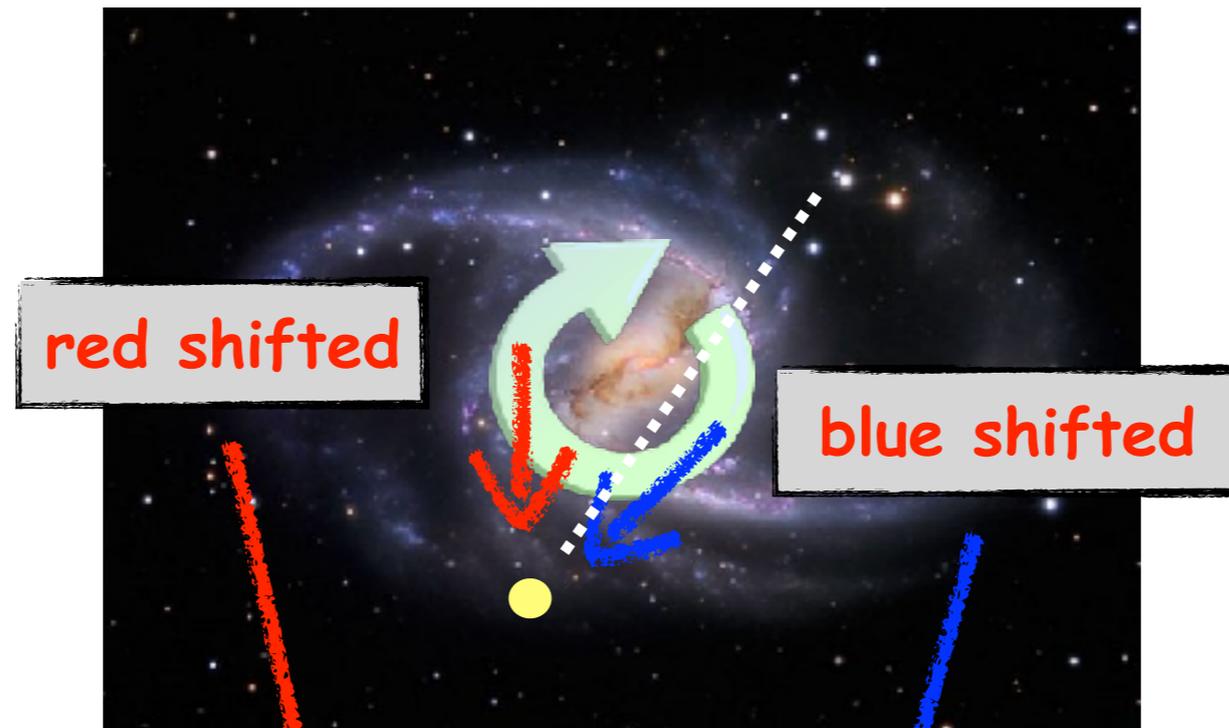
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# The MW rotation curve

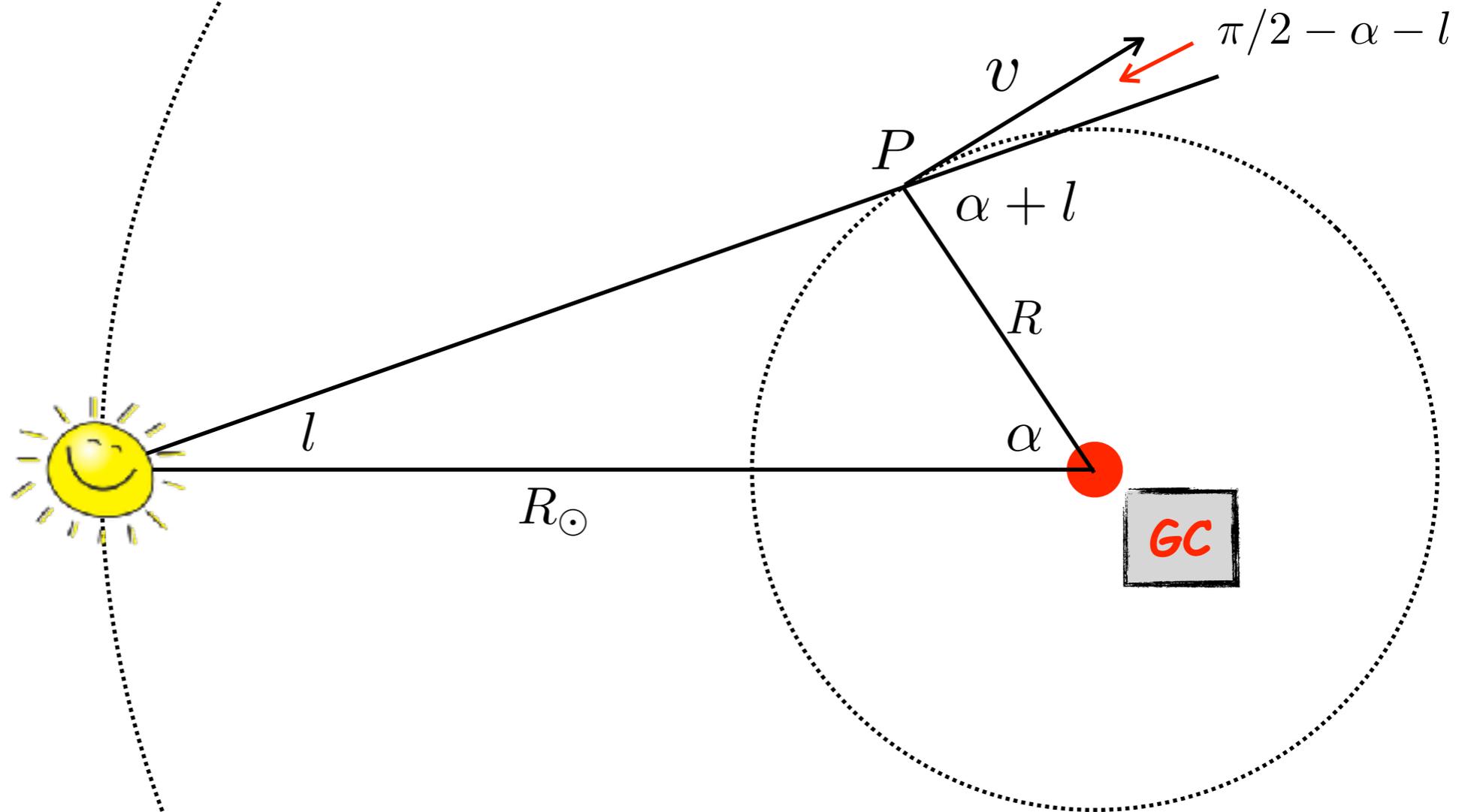


galactic longitude

FIG. 2.— Longitude-resolved mean CO emission line centered over a value of 0 with its 1- $\sigma$  level (100 level) at 100 km/s (0.001) and 2- $\sigma$  level (0.0001) at right. The color scale is logarithmic and all emission beyond the 100 km/s level is at 100 km/s. The map has been smoothed to a velocity resolution of 1 km/s and the longitude resolution is 12'. The resulting mean velocity over the whole line-of-sight is shown in gray. The color bar at the bottom right shows the intensity scale in units of  $\log(I_{\nu}/Jv \text{ km/s})$ .

# The MW rotation curve

Simplifying hypothesis: circular orbits



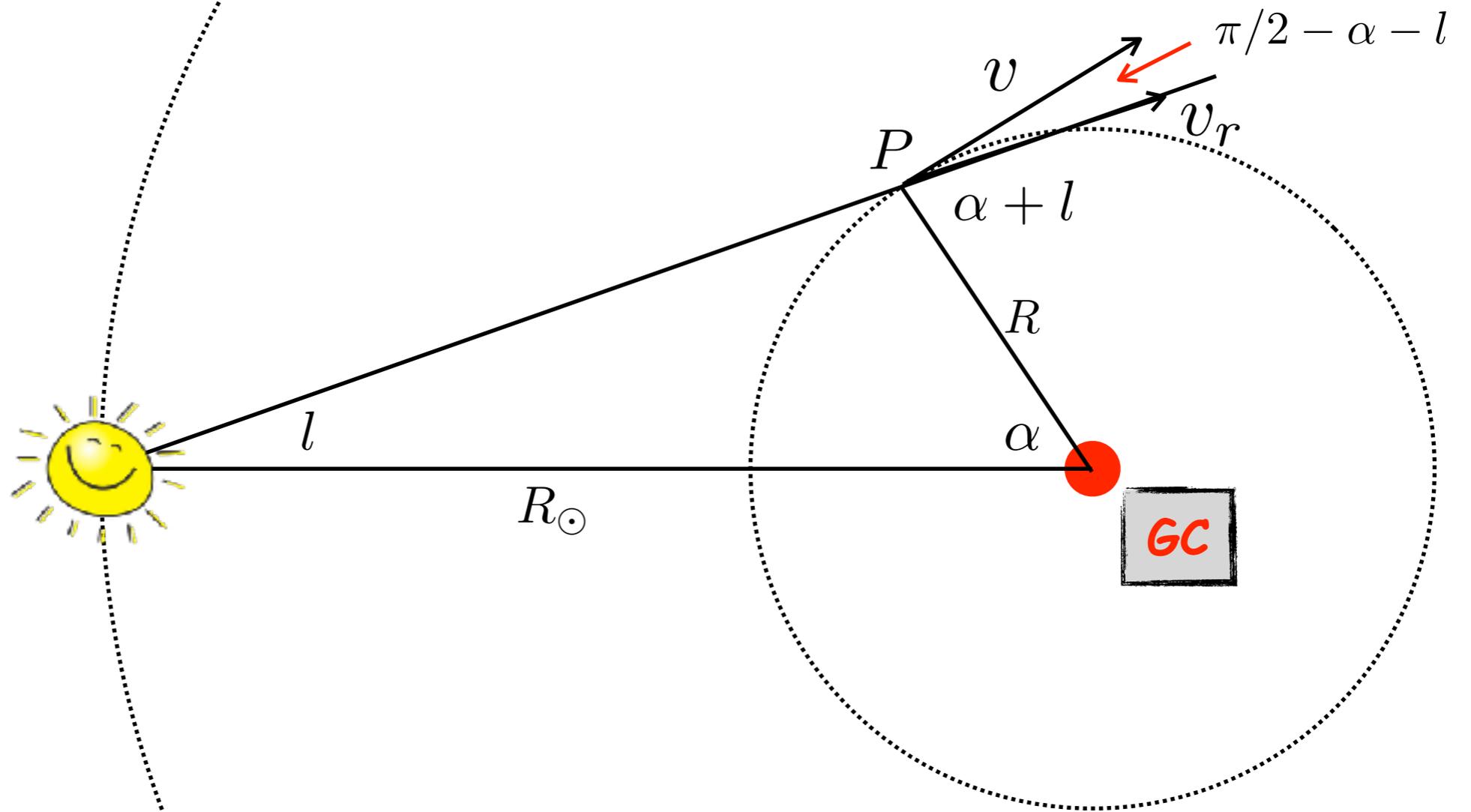
$$\omega_{\odot} = \omega(R_{\odot})$$

$$v = (\omega - \omega_{\odot})R$$

this depends on R

# The MW rotation curve

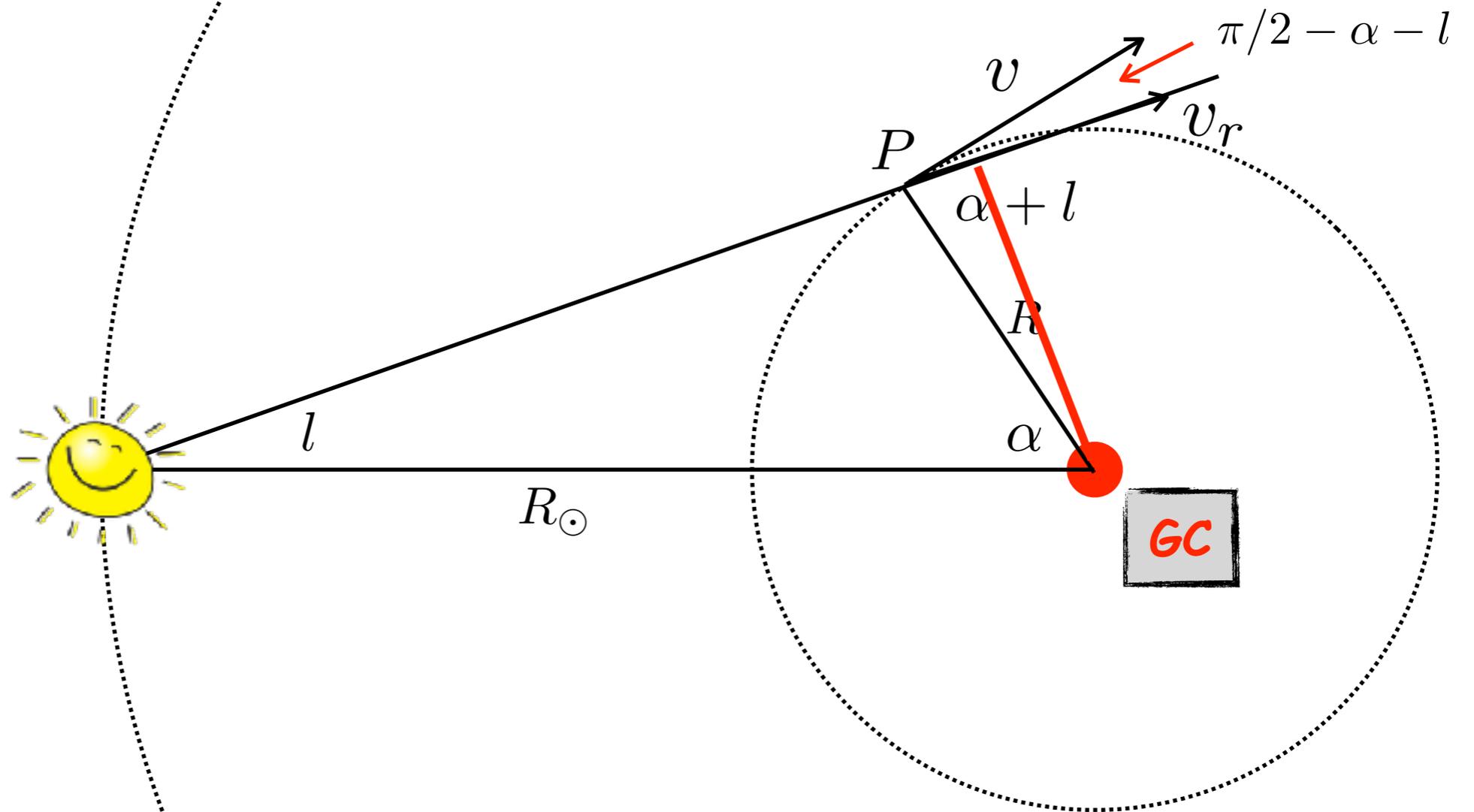
What we measure are RADIAL velocities



$$v_r = (\omega - \omega_{\odot})R \cos(90 - l - \alpha) = (\omega - \omega_{\odot})R \sin(l + \alpha)$$

# The MW rotation curve

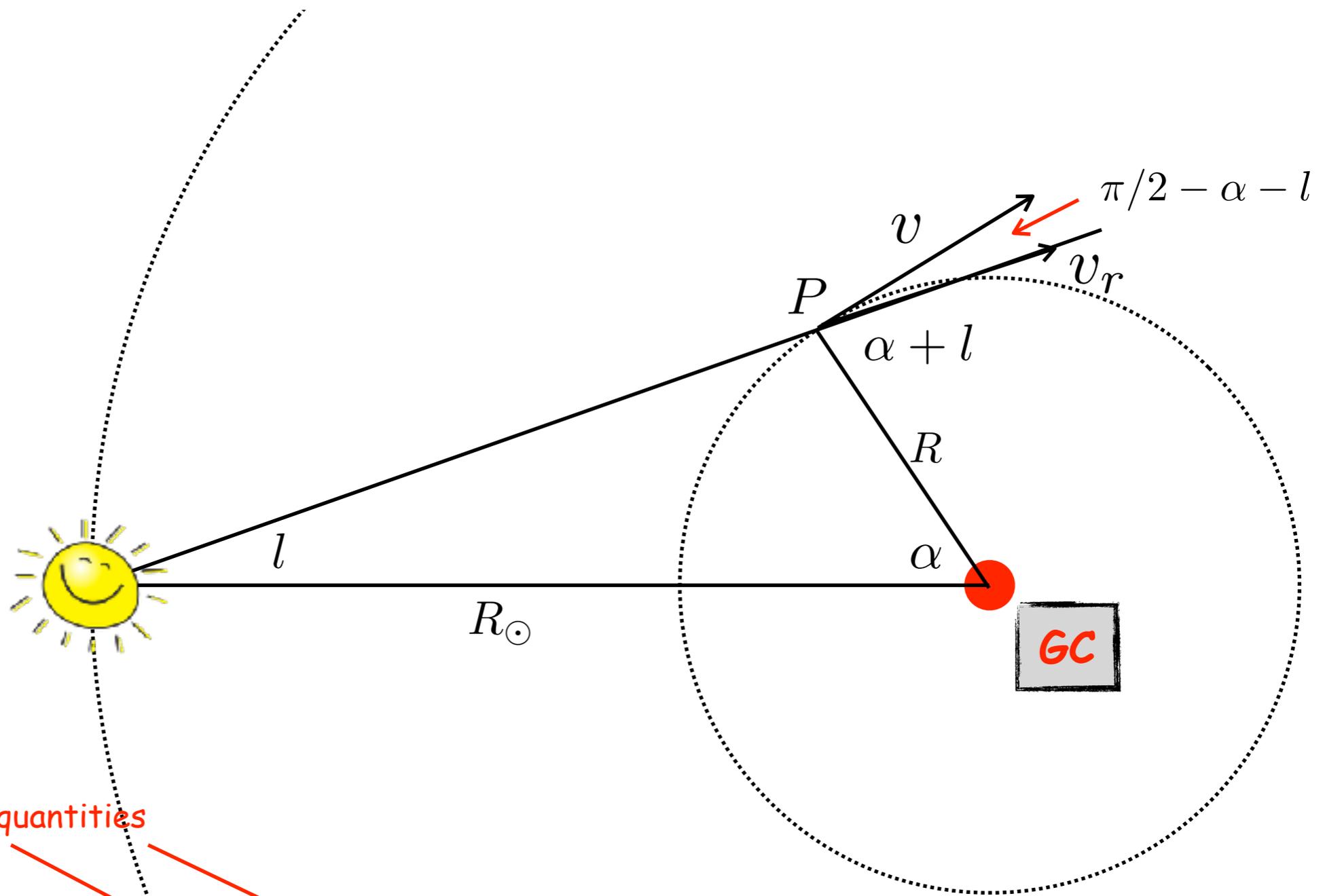
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$$= R_{\odot} \sin(l)$$

# The MW rotation curve



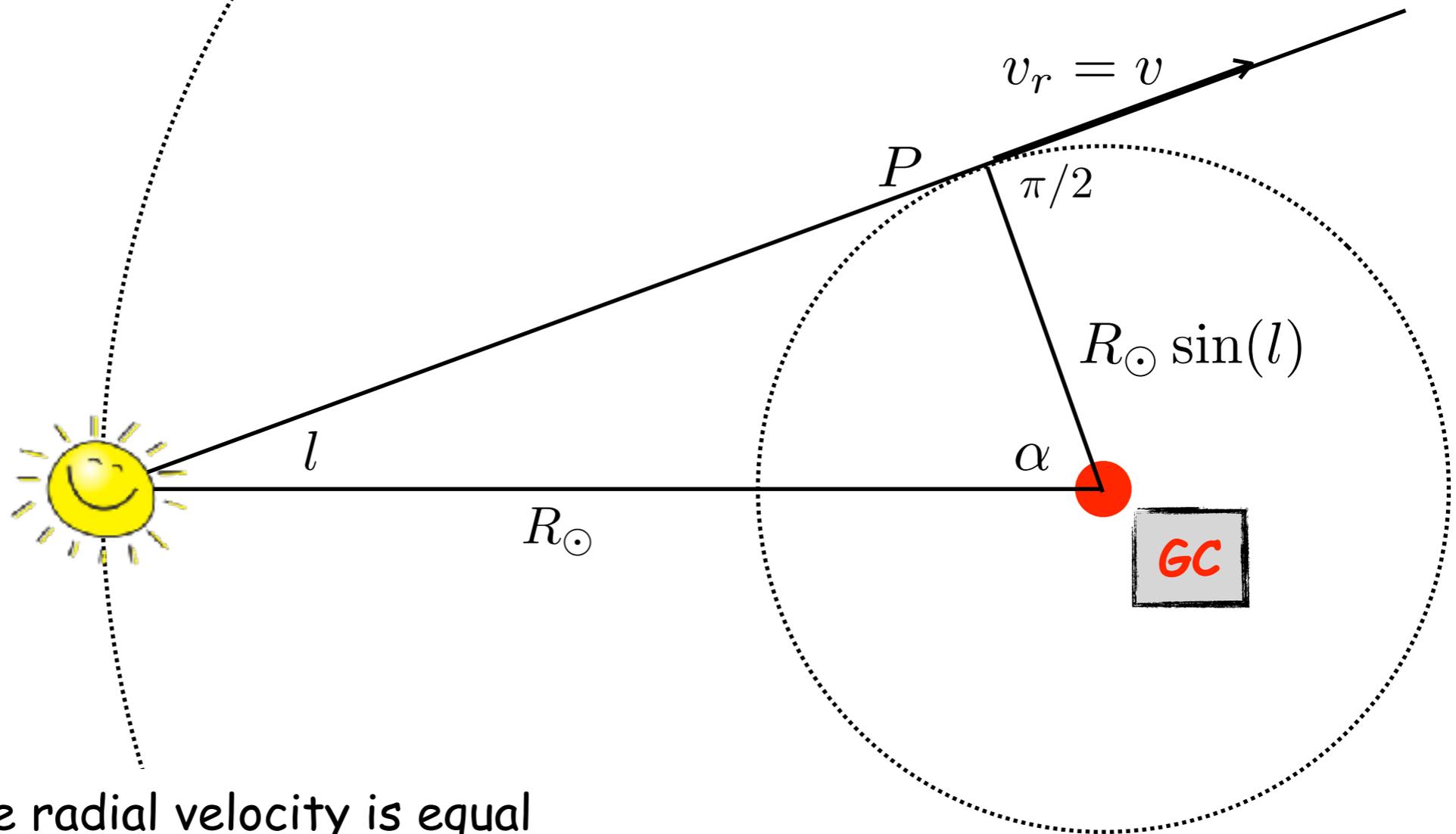
measured quantities

$$v_r = (\omega - \omega_{\odot}) R_{\odot} \sin(l)$$

this depends on  $R$

# The MW rotation curve

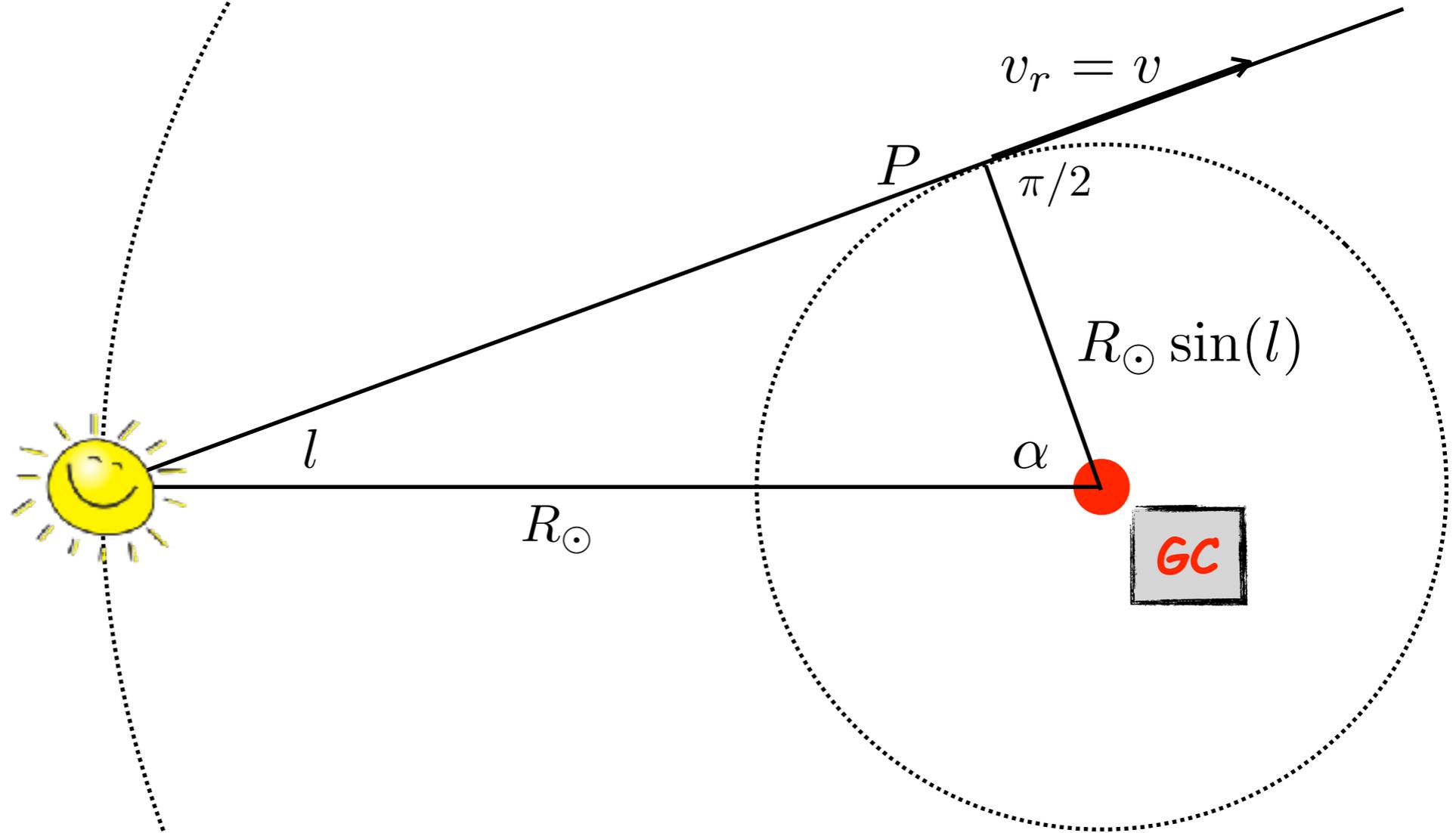
Consider now the particular case  $\alpha + l = \pi/2$



in this case the radial velocity is equal to the orbital velocity around the galactic centre

# The MW rotation curve

WARNING! We measure a range of radial velocities all along a given line of sight

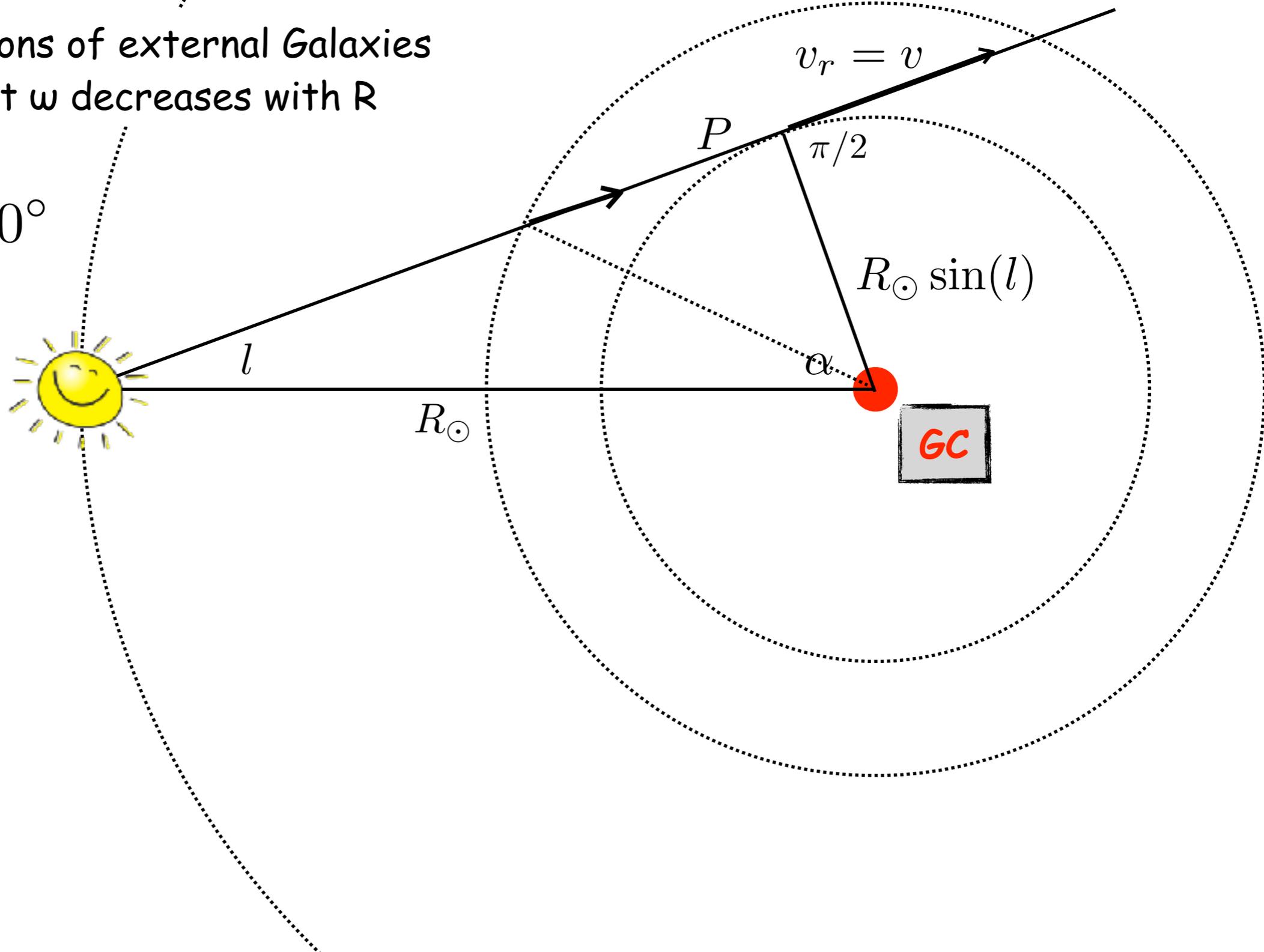


# The MW rotation curve

**WARNING!** We measure a range of radial velocities all along a given line of sight

from observations of external Galaxies  
we know that  $w$  decreases with  $R$

$$0^\circ < l < 90^\circ$$

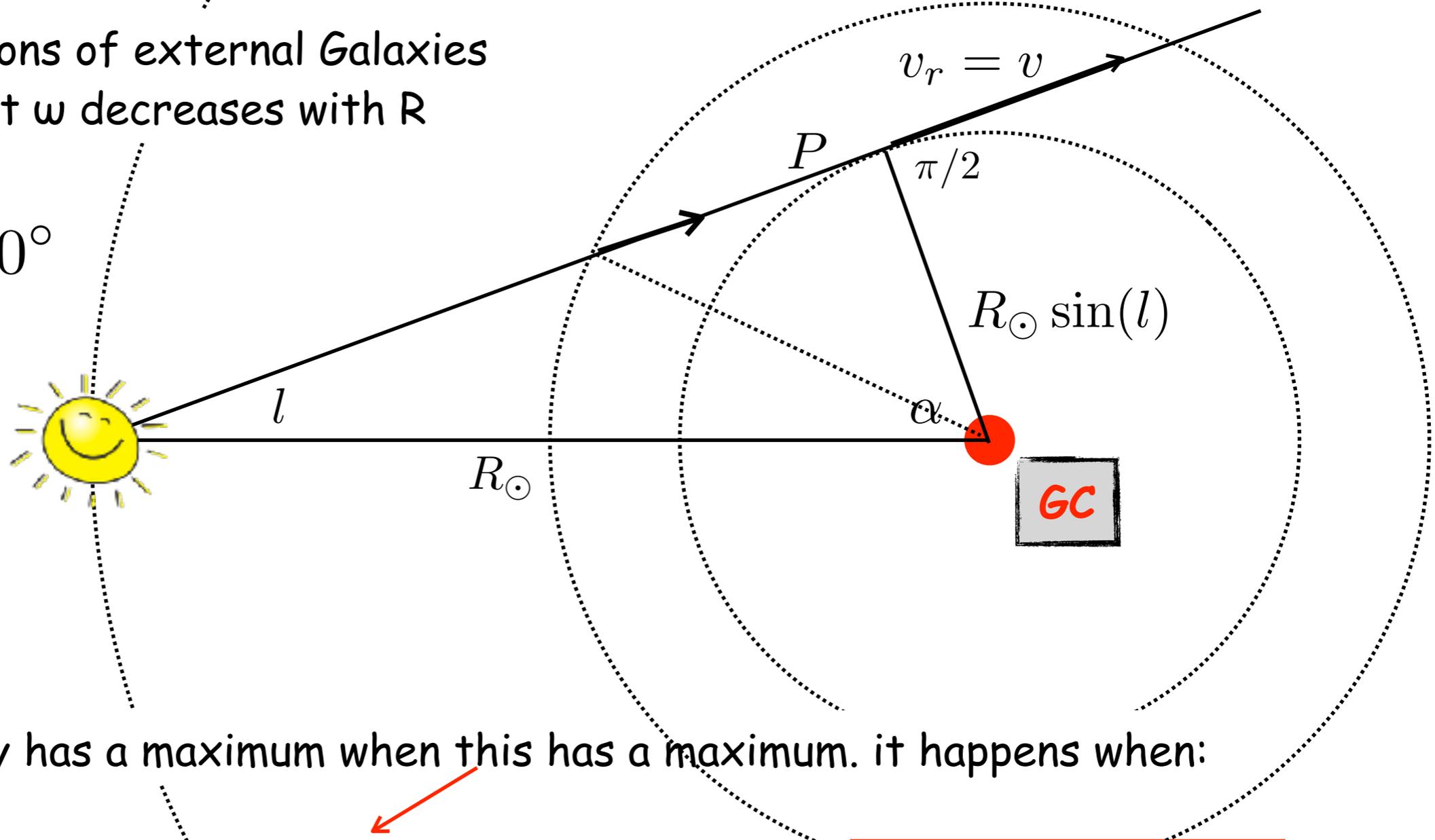


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the radial velocity has a maximum when this has a maximum. it happens when:

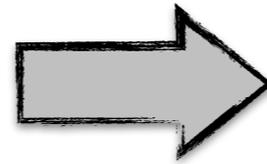
$$v_r = (\omega - \omega_\odot) R_\odot \sin(l)$$

$$R = R_\odot \sin(l)$$

in this way we can measure the rotation curve for  $0 < R < R_\odot$

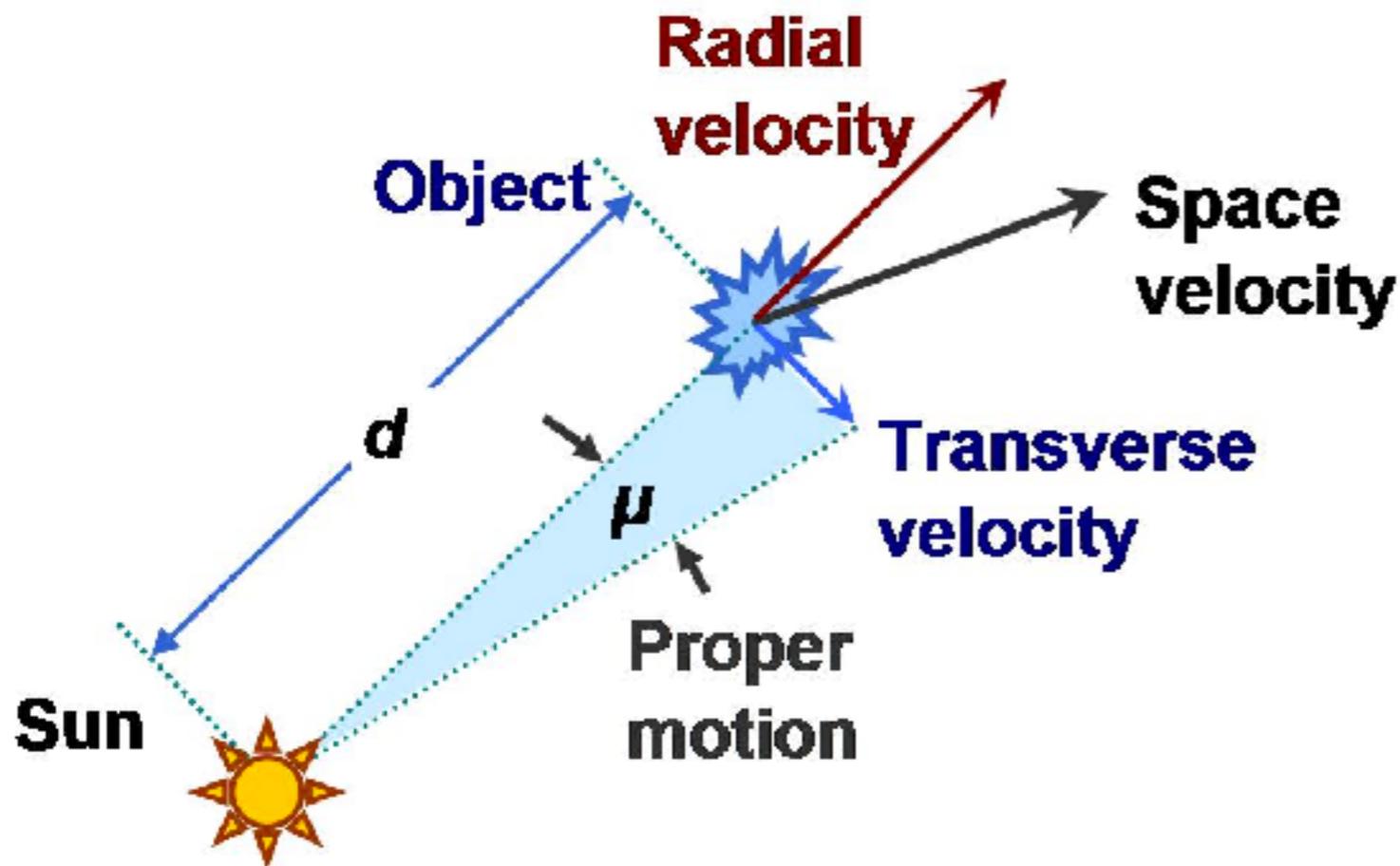
# Proper motions of stars

apparent motion of the star  $\rightarrow \mu$   
doppler effect  $\rightarrow$  radial velocity  
parallax  $\rightarrow$  distance



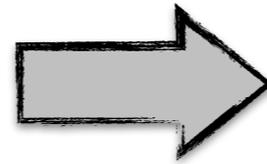
transverse velocity  $v_t = \mu d$

total velocity  $v^2 = v_t^2 + v_r^2$



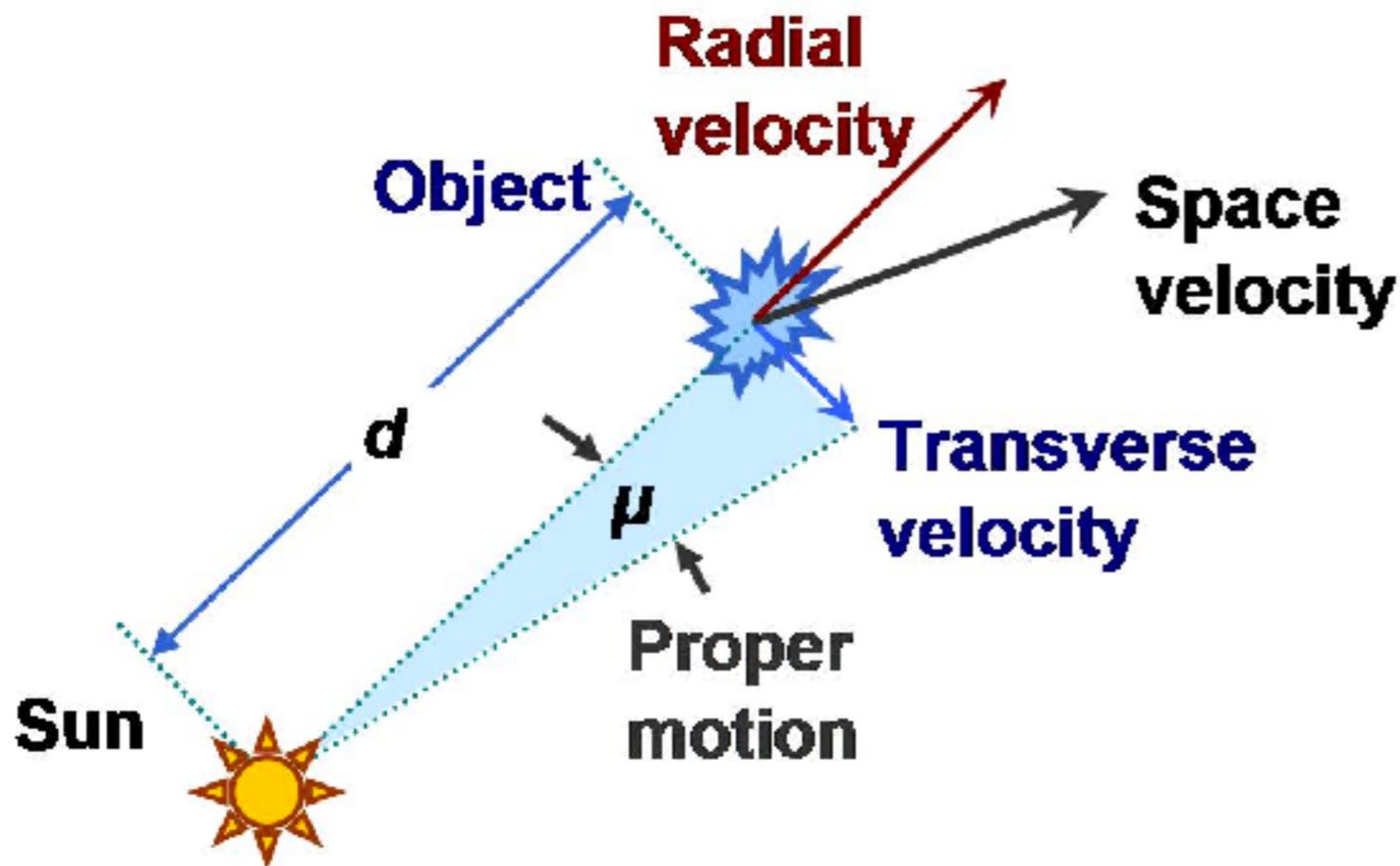
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- Stars in the vicinity of the sun have typical random velocities  $\sim 10$  km/s
- the average of these velocities is different from zero
- this is due to the fact that the sun has a proper motion with respect to the population of neighbouring stars ( $\sim 10$ - $15$  km/s)
- Local Standard of Rest  $\rightarrow$  the rest frame where the average of star's random velocities is zero

# The velocity of the sun

very few stars in the vicinity of the sun are characterised by very large proper motions

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## interpretation

- cold stellar component -> disk
- hot stellar component -> halo
- the galactic disk is supported by rotation
- the halo is supported by velocity dispersion (no rotation)
- the LSR (i.e. all the stars close to the sun) rotates around the GC at a speed of  $\sim 220$  km/s

$$v_{\odot} \sim 220 \text{ km/s}$$

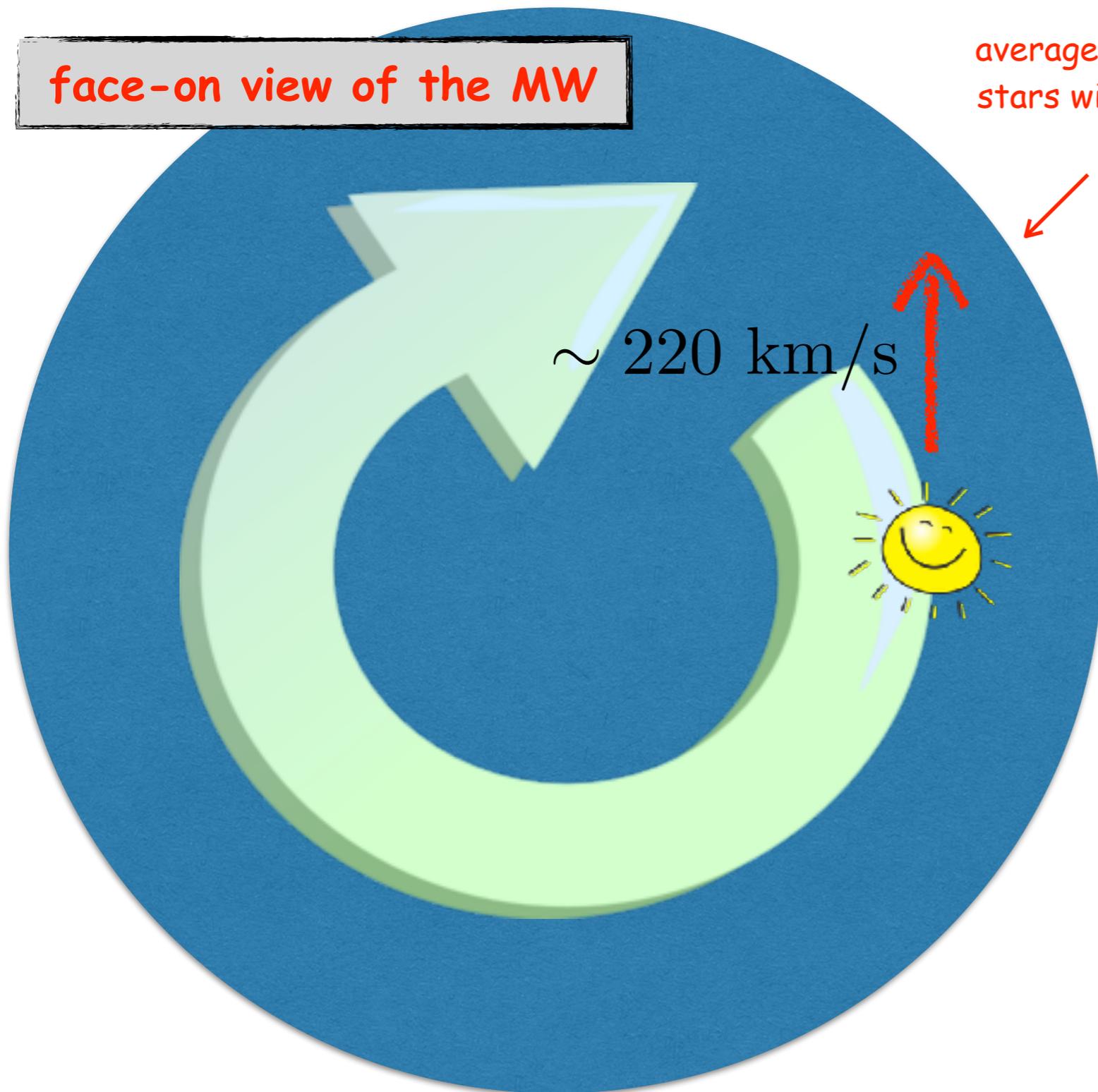
# The velocity of the sun

very few stars in the vicinity of the sun are characterised by very large proper motions

face-on view of the MW

average speed of high velocity stars with respect to the LSR

interpretation



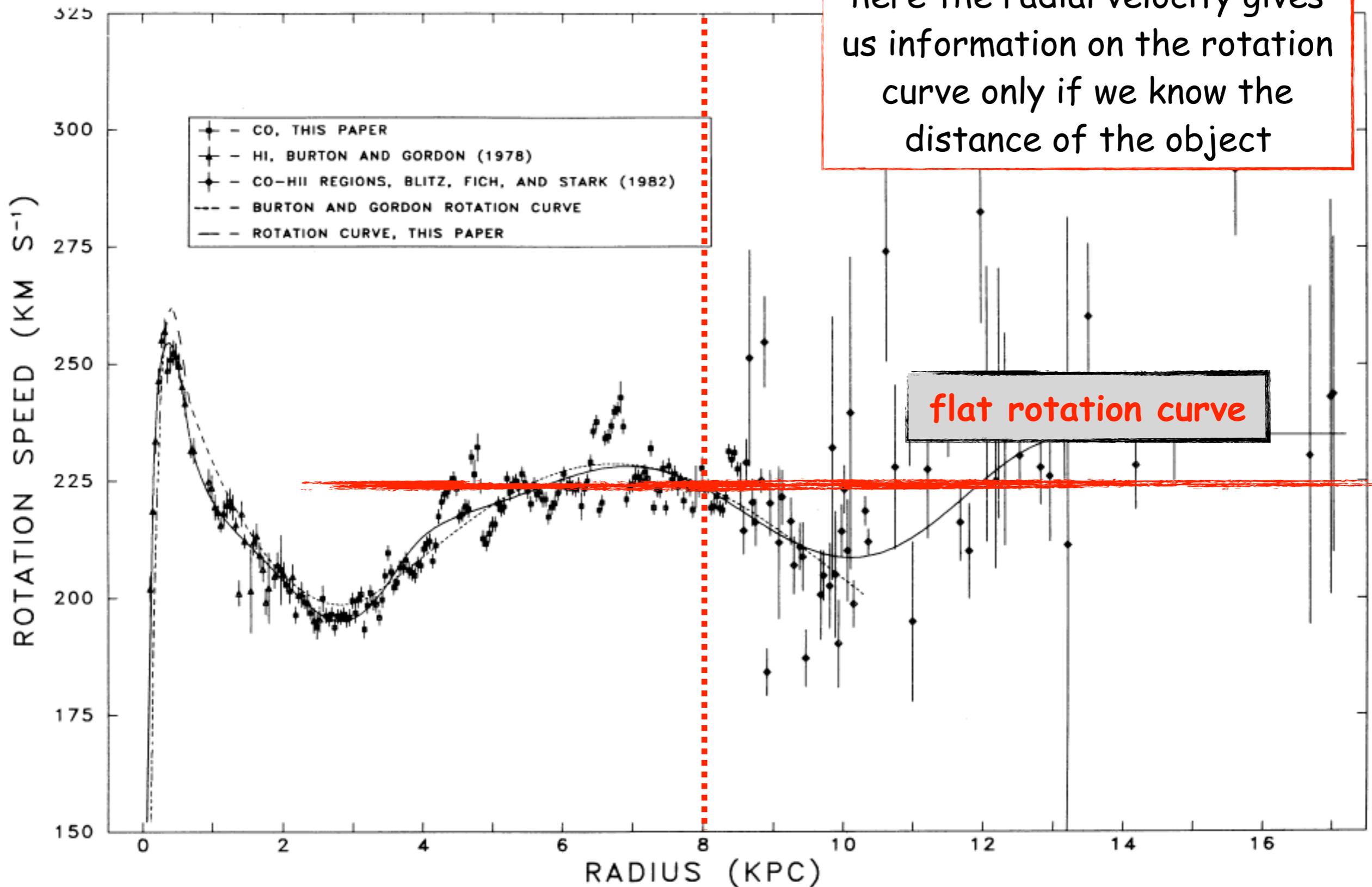
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$$v_{\odot} \sim 220 \text{ km/s}$$

$$t_{\odot} = \frac{2\pi R_{\odot}}{v_{\odot}} \sim 220 \text{ Myr}$$

# The rotation curve of the MW



# Galactic dynamics

assumption: the surface density of the disk follows the distribution of light

$$\Sigma = \Sigma_0 e^{-R/R_d} \quad \text{the mass is concentrated towards the centre}$$

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far away from the centre...

$$\frac{v^2}{R} = \frac{GM}{R^2} \longrightarrow v = \left( \frac{GM}{R} \right)^{1/2}$$

**this is not flat! -> evidence for the existence of matter which is not traced by light (dark matter)**

# Dark matter

simplest assumption: spherical distribution of dark matter  $M(R)$

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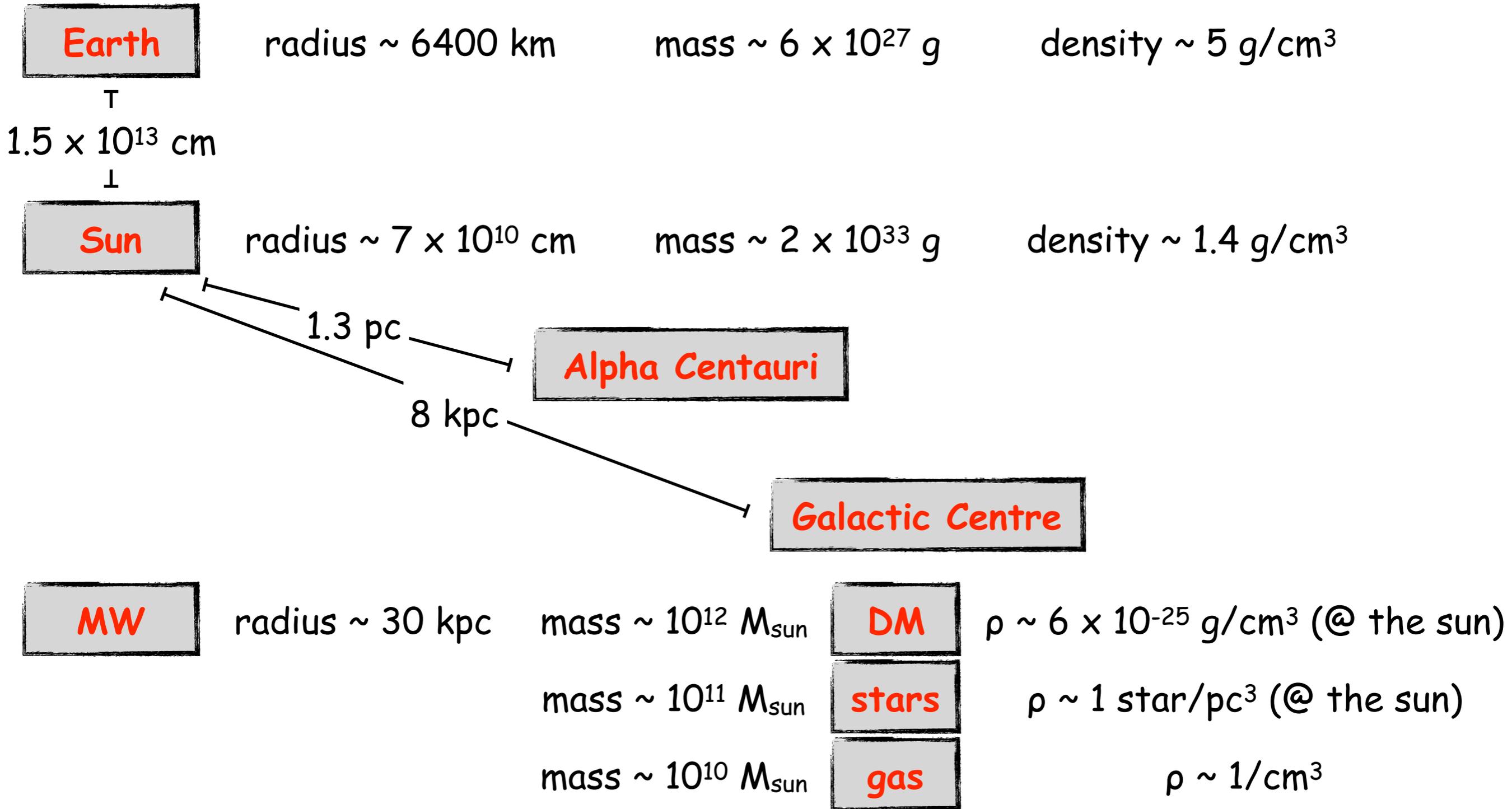
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$$M(R) = 10^{12} \left( \frac{R}{100 \text{ kpc}} \right) M_{\odot}$$

mass of the MW

# Astronomical quantities



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